

Using Cauchy Distribution To Estimate Survival Function

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Abstract: This paper intends to estimate the unlabeled two parameters for Cauchy distribution model depend on employing the maximum likelihood estimator method to obtain the derivation of the point estimators for all unlabeled parameters depending on iterative techniques, as Newton – Raphson method, then to derive “Lindley approximation estimator method and then to derive Ordinary least squares estimator method. Applying all these methods to estimate related probability functions; death density function, cumulative distribution function, survival function and hazard function (rate function)”.

“When examining the numerical results for probability survival function by employing mean squares error measure and mean absolute percentage measure, this may lead to work on the best method in modeling a set of real data”.

Introduction:

The first publication of Cauchy distribution was in 1824 by the French mathematician Poisson; then it was only associated with the name Augustin Cauchy during an academic debate in 1853. Physicists call it the Lorentz distribution due to Hendrick Lorents.

Cauchy distribution is a continuous probability distribution of curves that has heavier tails than the normal distributions; and one of the most popular practical applications for this distribution is modeling the ratio of two normal random variables. (1,2)

“Introduce a tree-structured Bayesian network suitable for modelling directional data with bivariate wrapped Cauchy distributions. We describe the structure learning algorithm used to learn the Bayesian network”. (2)

Because Cauchy distribution does not have limited moments greater than or equal to one and has no moment generating function, it is a stable distribution with a probability density function that can be explained analytically and therefore, Cauchy distribution might be considered as an example of a well-accepted results and concept in statistics. (1)

This paper is divided as follows:- The objective of this paper, theoretical section, practical section, results and conclusions.

The objective of this paper:

This paper aims to examine sample type Breast cancer by using maximum likelihood estimator method (MLEM), Lindley approximation estimator method (LAEM) and Ordinary least squares estimator method; and by comparing the three methods.

Definition and properties:

“The p.d.f for Cauchy distribution. Is”:

$$f_p(t; \gamma, \delta) = \frac{\gamma}{\pi[\gamma^2 + (t - \delta)^2]} \quad ; \quad t \geq 0 \dots(1)$$

$$\Omega = \left\{ (t; \gamma, \delta) ; \gamma > 0, \delta > 0 \right\}$$

Where γ : “is scale parameter”

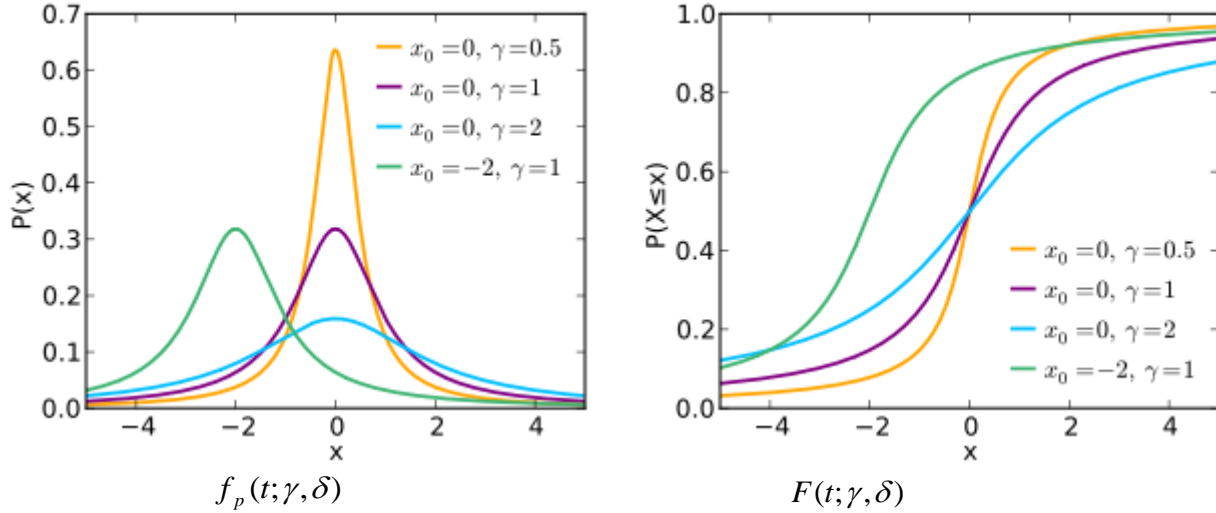
δ : “is location parameter”

“The cumulative distribution function for this distribution is”: $F(t; \gamma, \delta) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1} \left(\frac{t - \gamma}{\delta} \right) \dots(2)$;

for $(-\infty < t < \infty ; -\infty < \gamma < \infty ; \delta > 0)$

“It's survival function is given by” : $s(t; \gamma, \delta) = 1 - \left[\frac{1}{2} + \frac{1}{\pi} \tan^{-1} \left(\frac{t - \gamma}{\delta} \right) \right]$... (3)

“The hazard rate function is given by” : $h(t; \gamma, \delta) = \frac{\gamma}{\pi[\gamma^2 + (t - \delta)^2]} \left[\frac{1}{2} + \frac{1}{\pi} \tan^{-1} \left(\frac{t - \gamma}{\delta} \right) \right]$... (4)



Maximum likelihood estimator method(MLEM):

“The MLM is the most common procedure to estimate the parameter ϕ which specifies a p.d.f. $f(t : \phi)$, based on the observations t_1, t_2, \dots, t_n which were independent. sample from the distribution”.

$$L = \left[\prod_{i=1}^n f(t_i; \gamma, \delta) \right] = \prod_{i=1}^n \left[\frac{\gamma}{\pi[\gamma^2 + (t_i - \delta)^2]} \right] \dots (5)$$

$$\ln L = n \ln \gamma - n \ln \pi - \sum_{i=1}^n \ln[\gamma^2 + (t_i - \delta)^2] \dots (6)$$

$$\frac{\partial \ln L}{\partial \gamma} = \frac{n}{\gamma} - \sum_{i=1}^n \frac{2\gamma}{\gamma^2 + (t_i - \delta)^2} \dots (7) \quad ; \quad \frac{\partial \ln L}{\partial \gamma} = 0 \quad ; \quad \frac{n}{\gamma} - \sum_{i=1}^n \frac{2\gamma}{\gamma^2 + (t_i - \delta)^2} \dots (8)$$

$$\frac{\partial \ln L}{\partial \delta} = \sum_{i=1}^n \frac{2(t_i - \delta)}{\gamma^2 + (t_i - \delta)^2} \dots (9) \quad ; \quad \frac{\partial \ln L}{\partial \delta} = 0 \quad ; \quad \sum_{i=1}^n \frac{2(t_i - \delta)}{\gamma^2 + (t_i - \delta)^2} \dots (10)$$

“There is no chance to find the estimators for the parameters (γ, δ) , and it is kind of difficulty to process the nonlinear equations thus, it is better to make use of iterative methods in numerical analysis as Newton–Raphson method which is the best way to get the estimate values and number of iteration”.

“The Newton–Raphson method requires an initial value of each unknown parameters” (γ, δ) .

This method follows :

$$\begin{bmatrix} \gamma_{i+1} \\ \delta_{i+1} \end{bmatrix} = \begin{bmatrix} \gamma_i \\ \delta_i \end{bmatrix} - J_i^{-1} \begin{bmatrix} g_1(\gamma) \\ g_2(\delta) \end{bmatrix} \dots (11)$$

$$g_1(\gamma) = \frac{n}{\gamma} - \sum_{i=1}^n \frac{2\gamma}{\gamma^2 + (t_i - \delta)^2} \dots (12)$$

$$; \quad g_2(\delta) = \sum_{i=1}^n \frac{2(t_i - \delta)}{\gamma^2 + (t_i - \delta)^2} \dots (13)$$

$$J_i^{-1} = \begin{bmatrix} \frac{\partial g_1(\gamma)}{\partial \gamma} & \frac{\partial g_1(\gamma)}{\partial \delta} \\ \frac{\partial g_2(\delta)}{\partial \gamma} & \frac{\partial g_2(\delta)}{\partial \delta} \end{bmatrix} \dots(14)$$

$$\frac{\partial g_1(\gamma)}{\partial \gamma} = -\frac{n}{\gamma^2} - \sum_{i=1}^n \frac{2(t_i - \delta)^2 - 2\gamma^2}{[\gamma^2 + (t_i - \delta)^2]^2} \dots(15) \quad ; \quad \frac{\partial g_1(\gamma)}{\partial \delta} = -\sum_{i=1}^n \frac{4\gamma(t_i - \delta)}{[\gamma^2 + (t_i - \delta)^2]^2} \dots(16)$$

$$\frac{\partial g_2(\delta)}{\partial \gamma} = -\sum_{i=1}^n \frac{4\gamma(t_i - \delta)}{[\gamma^2 + (t_i - \delta)^2]^2} \dots(17) \quad ; \quad \frac{\partial g_2(\delta)}{\partial \delta} = \sum_{i=1}^n \frac{2(t_i - \delta)^2 - 2\gamma^2}{[\gamma^2 + (t_i - \delta)^2]^2} \dots(18)$$

$$\begin{bmatrix} \varepsilon_{i+1}(\gamma) \\ \varepsilon_{i+1}(\delta) \end{bmatrix} = \begin{bmatrix} \gamma_{i+1} \\ \delta_{i+1} \end{bmatrix} - \begin{bmatrix} \gamma_i \\ \delta_i \end{bmatrix} \dots(19)$$

“Lindley approximation estimator method (LAEM)”:

“Lindley procedure was presented in (1980) first time to approximate the ratio of the integrals of the form(3):

$$\frac{\int w(\lambda)e^{L(\lambda)} d\lambda}{\int v(\lambda)e^{L(\lambda)} d\lambda} \dots(20)$$

“Where $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$ are parameters , $L(\lambda)$ is the logarithm of the likelihood function , $w(\lambda)$ and $v(\lambda)$ are any random functions for parameters”.

Let $v(\lambda)$ be the prior distribution of λ and $w(\lambda) = u(\lambda)v(\lambda)$. From (20) we can get posterior expectation which is as follow :

$$E[u(\lambda) | \underline{x}] = \frac{\int u(\lambda)e^{L(\lambda)+P(\lambda)} d\lambda}{\int e^{L(\lambda)+P(\lambda)} d\lambda} \dots(21)$$

Where $P(\lambda) = \log[v(\lambda)]$

$$\hat{R} = R(\gamma, \delta) + \frac{1}{2} [B + l_{30}C_{12} + l_{03}C_{21} + l_{21}D_{12} + l_{12}D_{21}] + p_1B_{12} + p_2B_{12} \dots(22)$$

$$B = \sum_{i=1}^2 \sum_{j=1}^2 Q_{ij}T_{ij} \quad ; \quad i, j = 1, 2 \dots\dots\dots(23) \quad ; \quad l_{ij} = \frac{\partial^{i+j}L(\gamma, \delta)}{\partial \gamma^i \partial \delta^j} \dots(24)$$

$$p_i = \frac{\partial p}{\partial \delta_i} \quad , \quad p = \ln \Pi(\gamma, \delta) \dots(25)$$

$$Q_1 = \frac{\partial R}{\partial \gamma} \quad , \quad Q_2 = \frac{\partial R}{\partial \delta} \quad , \quad Q_{ij} = \frac{\partial^2 R}{\partial \gamma_i \partial \delta_j} \dots(26)$$

$$B_{ij} = Q_i T_{ij} + Q_j T_{ji} \dots(27) \quad ; \quad C_{ij} = (Q_i T_{ij} + Q_j T_{ji}) T_{ii} \dots(28) \quad ; \quad D_{ij} = 3Q_i T_{ii} T_{ij} + Q_i (T_{ii} T_{jj} + 2T_{ij}^2) \dots(29)$$

$$f_p(t; \gamma, \delta) = \frac{\gamma}{\pi[\gamma^2 + (t - \delta)^2]}$$

We assumed that γ , δ have the following Gamma conjugate prior distribution such that : $\gamma \sim \overline{)(n, a)}$; $\delta \sim \overline{)(n, b)}$

$$f(\gamma) = \begin{cases} \frac{a^n}{\overline{)(n)}} \gamma^{n-1} e^{-a\gamma} & a > 0 , n > 0 , \gamma > 0 \\ 0 & o.w \end{cases} \dots(30)$$

$$f(\delta) = \begin{cases} \frac{b^n}{\overline{)(n)}} \delta^{n-1} e^{-b\delta} & b > 0 , n > 0 , \delta > 0 \\ 0 & o.w \end{cases} \dots(31)$$

$$p(\gamma, \delta | t_1, t_2, \dots, t_n) = \frac{L(t_1, t_2, \dots, t_n; \gamma, \delta) f(\gamma) f(\delta)}{\int_0^\infty \int_0^\infty L(t_1, t_2, \dots, t_n; \gamma, \delta) f(\gamma) f(\delta) d\gamma d\delta} \dots(32)$$

$$\hat{R} = E[R(\gamma, \delta)] = \frac{\int_0^\infty \int_0^\infty R(\gamma, \delta) L(t_1, t_2, \dots, t_n; \gamma, \delta) f(\gamma) f(\delta) d\gamma d\delta}{\int_0^\infty \int_0^\infty L(t_1, t_2, \dots, t_n; \gamma, \delta) f(\gamma) f(\delta) d\gamma d\delta} \dots(33)$$

$$\hat{R} = \frac{\int_0^\infty \int_0^\infty R(\gamma, \delta) \prod_{i=1}^n \left[\frac{\gamma}{\pi[\gamma^2 + (t_i - \delta)^2]} \right] \frac{a^n}{\overline{)(n)}} \gamma^{n-1} e^{-a\gamma} \frac{b^n}{\overline{)(n)}} \delta^{n-1} e^{-b\delta} d\gamma d\delta}{\int_0^\infty \int_0^\infty \prod_{i=1}^n \left[\frac{\gamma}{\pi[\gamma^2 + (t_i - \delta)^2]} \right] \frac{a^n}{\overline{)(n)}} \gamma^{n-1} e^{-a\gamma} \frac{b^n}{\overline{)(n)}} \delta^{n-1} e^{-b\delta} d\gamma d\delta} \dots(34)$$

“We make used of Lindley's approximate \hat{R} which approximate the ratio of the two integrals to obtain Bayes estimators approximation that can be resulted as follows” :

Using equation (6) we get the following:

$$l_{12} = \frac{\partial^3 \ln L(\gamma, \delta)}{\partial \gamma \partial \delta^2} = \sum_{i=1}^n \frac{4\gamma[\gamma^2 + (t_i - \delta)^2 (1 - 4\gamma)]}{[\gamma^2 + (t_i - \delta)^2]^3} \dots(35)$$

$$l_{21} = \frac{\partial^3 \ln L(\gamma, \delta)}{\partial \gamma^2 \partial \delta} = \sum_{i=1}^n \frac{4(t_i - \delta)[3\gamma^2 - (t_i - \delta)^2]}{[\gamma^2 + (t_i - \delta)^2]^3} \dots(36)$$

$$l_{03} = \frac{\partial^3 \ln L(\gamma, \delta)}{\partial \delta^3} = \sum_{i=1}^n \frac{4(t_i - \delta)[(t_i - \delta)^2 - 3\gamma^2]}{[\gamma^2 + (t_i - \delta)^2]^3} \dots(37)$$

$$l_{30} = \frac{\partial^3 \ln L(\gamma, \delta)}{\partial \gamma^3} = \frac{2n}{\gamma^3} + \sum_{i=1}^n \frac{4\gamma[3(t_i - \delta)^2 - \gamma^2]}{[\gamma^2 + (t_i - \delta)^2]^3} \dots(38)$$

$$\text{When } R(\gamma, \delta) = \gamma \quad ; \quad Q_1 = \frac{\partial R}{\partial \gamma} = 1 \quad , \quad Q_2 = \frac{\partial R}{\partial \delta} = 0 \quad , \quad Q_{12} = Q_{21} = \frac{\partial^2 R}{\partial \gamma_i \partial \delta_j} = 0 \quad \dots(39)$$

$$E = \frac{\partial^2 \ln L(\gamma, \delta)}{\partial \gamma^2} = -\frac{n}{\lambda^2} - \sum_{i=1}^n \frac{2(t_i - \delta)^2 - 2\gamma^2}{[\gamma^2 + (t_i - \delta)^2]^2} \quad \dots(40)$$

$$F = \frac{\partial^2 \ln L(\gamma, \delta)}{\partial \delta^2} = \sum_{i=1}^n \frac{2(t_i - \delta)^2 - 2\gamma^2}{[\gamma^2 + (t_i - \delta)^2]^2} \quad \dots(41)$$

$$G = \frac{\partial^2 \ln L(\gamma, \delta)}{\partial \delta \partial \gamma} = \frac{\partial^2 \ln L(\gamma, \delta)}{\partial \gamma \partial \delta} = -\sum_{i=1}^n \frac{4\gamma(t_i - \delta)}{[\gamma^2 + (t_i - \delta)^2]^2} \quad \dots(42)$$

$$T_{11} = \frac{F}{EF - G^2} \quad \dots(43) \quad ; \quad T_{22} = \frac{E}{EF - G^2} = \quad \dots(44) \quad ; \quad T_{12} = T_{21} = \frac{-G}{EF - G^2} \quad \dots(45)$$

$$p = \ln f(\gamma, \delta) = n \ln a - \ln \sqrt{(n)} + (n-1) \ln \gamma - a\gamma + n \ln \delta - \ln \sqrt{(n)} + (n-1) \ln \delta - b\delta \quad \dots(46)$$

$$p_1 = \frac{\partial p}{\partial \gamma} = \frac{n-1}{\gamma} - a \quad \dots(47) \quad ; \quad p_2 = \frac{\partial p}{\partial \delta} = \frac{n-1}{\delta} - b \quad \dots(48)$$

$$B = \sum_{i=1}^2 \sum_{j=1}^2 Q_{ij} T_{ij} = Q_{12} T_{12} + Q_{21} T_{21} = 0 \quad \dots(49)$$

$$C_{12} = T_{12} T_{11} \quad ; \quad C_{21} = T_{21} T_{22} \quad \dots(50)$$

$$D_{12} = 3T_{11} T_{12} + T_{11} T_{22} + 2T_{12}^2 \quad ; \quad D_{21} = 0 \quad \dots(51)$$

$$B_{12} = T_{12} \quad ; \quad B_{21} = T_{21} \quad \dots(52)$$

$$\hat{\gamma} = \hat{\gamma}_{MLE} + \frac{1}{2} [l_{30} T_{12} T_{11} + l_{03} T_{21} T_{22} + l_{21} (3T_{11} T_{12} + T_{11} T_{22} + 2T_{12}^2)] + \left(\frac{n-1}{\gamma} - a\right) T_{12} + \left(\frac{n-1}{\delta} - b\right) T_{12} \quad \dots(53)$$

$$\text{When } R(\gamma, \delta) = \delta \Rightarrow Q_1 = \frac{\partial R}{\partial \gamma} = 0 \quad , \quad Q_2 = \frac{\partial R}{\partial \delta} = 1 \quad , \quad Q_{12} = Q_{21} = \frac{\partial^2 R}{\partial \gamma_i \partial \delta_j} = 0 \quad \dots(54)$$

$$\text{From equation (49), (50) and } D_{12} = 0 \quad ; \quad D_{21} = 3T_{22} T_{21} + T_{22} T_{11} + 2T_{21}^2 \quad \dots(55)$$

$$B_{12} = T_{21} \quad ; \quad B_{21} = T_{21} \quad \dots(56)$$

$$\hat{\delta} = \hat{\delta}_{MLE} + \frac{1}{2} [l_{30} T_{12} T_{11} + l_{03} T_{21} T_{22} + l_{12} (3T_{22} T_{21} + T_{22} T_{11} + 2T_{21}^2)] + \left(\frac{n-1}{\gamma} - a\right) T_{21} + \left(\frac{n-1}{\delta} - b\right) T_{21} \quad \dots(57)$$

Ordinary Least Squares Estimator Method (OLSEM):

“The OLSEM is the most used way to estimate parameters in linear or nonlinear model. Researchers make use of this method to lessen the sum squares differences concerning observed sample values and expected estimated values by linear approximation”. (4,5)

$$Y = \beta_0 + \beta_1 x + \varepsilon \quad \dots(58) \quad ; \quad \varepsilon = Y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i \quad \dots(59)$$

$$\sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n [y_i - \hat{y}_i]^2 \quad \dots(60) \quad ; \quad \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n [y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i]^2 \quad \dots(61)$$

$$F(t_i) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{t_i - \gamma}{\delta}\right) \dots(62) \quad ; \quad \varepsilon = F(t_i) - \frac{1}{2} - \frac{1}{\pi} \tan^{-1}\left(\frac{t_i - \gamma}{\delta}\right) \dots(63)$$

$$\sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n \left[F(t_i) - \frac{1}{2} - \frac{1}{\pi} \tan^{-1}\left(\frac{t_i - \gamma}{\delta}\right) \right] \dots(64) \quad ; \quad \text{Let } \sum_{i=1}^n \varepsilon_i^2 = H(\gamma, \delta)$$

$$H(\gamma, \delta) = \sum_{i=1}^n \left[F(t_i) - \frac{1}{2} - \frac{1}{\pi} \tan^{-1}\left(\frac{t_i - \gamma}{\delta}\right) \right] \dots(65)$$

$$\frac{\partial H}{\partial \gamma} = -\frac{1}{\pi} \sum_{i=1}^n \left(\frac{-1}{\delta} \right) \dots(66) \quad ; \quad \frac{\partial H}{\partial \gamma} = 0 \quad ; \quad -\frac{1}{\pi} \sum_{i=1}^n \left(\frac{-1}{\delta} \right) \dots(67)$$

$$\frac{\partial H}{\partial \delta} = -\frac{1}{\pi} \sum_{i=1}^n \left(\frac{-1(t_i - \gamma)}{\delta^2} \right) \dots(68) \quad ; \quad \frac{\partial H}{\partial \delta} = 0 \quad ; \quad -\frac{1}{\pi} \sum_{i=1}^n \left(\frac{-1(t_i - \gamma)}{\delta^2} \right) \dots(69)$$

There is no chance to find the estimators for the parameters (γ, δ) , and it is kind of difficulty to process the nonlinear equations thus, it is better to make use of iterative methods in numerical analysis as Newton–Raphson method which is the best way to get the estimate values and number of iteration.

The Newton–Raphson method requires an initial value of each unknown parameters (γ, δ) .

$$\begin{bmatrix} \gamma_{i+1} \\ \delta_{i+1} \end{bmatrix} = \begin{bmatrix} \gamma_i \\ \delta_i \end{bmatrix} - J_i^{-1} \begin{bmatrix} z_1(\gamma) \\ z_2(\delta) \end{bmatrix} \dots(70)$$

$$z_1(\gamma) = -\frac{1}{\pi} \sum_{i=1}^n \left(\frac{-1}{\delta} \right) \dots(71) \quad ; \quad z_2(\delta) = -\frac{1}{\pi} \sum_{i=1}^n \left(\frac{-1(t_i - \gamma)}{\delta^2} \right) \dots(72)$$

$$J_i^{-1} = \begin{bmatrix} \frac{\partial z_1(\gamma)}{\partial \gamma} & \frac{\partial z_1(\gamma)}{\partial \delta} \\ \frac{\partial z_2(\delta)}{\partial \gamma} & \frac{\partial z_2(\delta)}{\partial \delta} \end{bmatrix} \dots(73)$$

$$\frac{\partial z_1(\gamma)}{\partial \gamma} = \frac{1}{\pi} \sum_{i=1}^n \left(\frac{2(t_i - \lambda)}{\delta^3} \right) \dots(74) \quad ; \quad \frac{\partial z_1(\gamma)}{\partial \delta} = -\frac{1}{\pi} \sum_{i=1}^n \left(\frac{1}{\delta^2} \left[1 - \left(\frac{t_i - \lambda}{\delta} \right)^2 \right] \right) \dots(75)$$

$$\frac{\partial z_2(\delta)}{\partial \gamma} = -\frac{1}{\pi} \sum_{i=1}^n \left(\frac{1}{\delta^2} \left[1 - \left(\frac{t_i - \lambda}{\delta} \right)^2 \right] \right) \dots(76) \quad ; \quad \frac{\partial z_2(\delta)}{\partial \delta} = -\frac{1}{\pi} \sum_{i=1}^n \left(\frac{2(t_i - \lambda)}{\delta^3} \right) \dots(77)$$

$$\begin{bmatrix} \varepsilon_{i+1}(\gamma) \\ \varepsilon_{i+1}(\delta) \end{bmatrix} = \begin{bmatrix} \gamma_{i+1} \\ \delta_{i+1} \end{bmatrix} - \begin{bmatrix} \gamma_i \\ \delta_i \end{bmatrix} \dots(78)$$

Results and Discussion:

The Educational Hospital in Diwaniyah province was the place from which the data was gathered.

Keeping in mind that this work relies on data taken from real life, it is reached to select this kind of cancer (Breast cancer) because it is remarkable widespread and deadly in Iraq; this disease has failure time (death time) which is phenomenon in this paper.

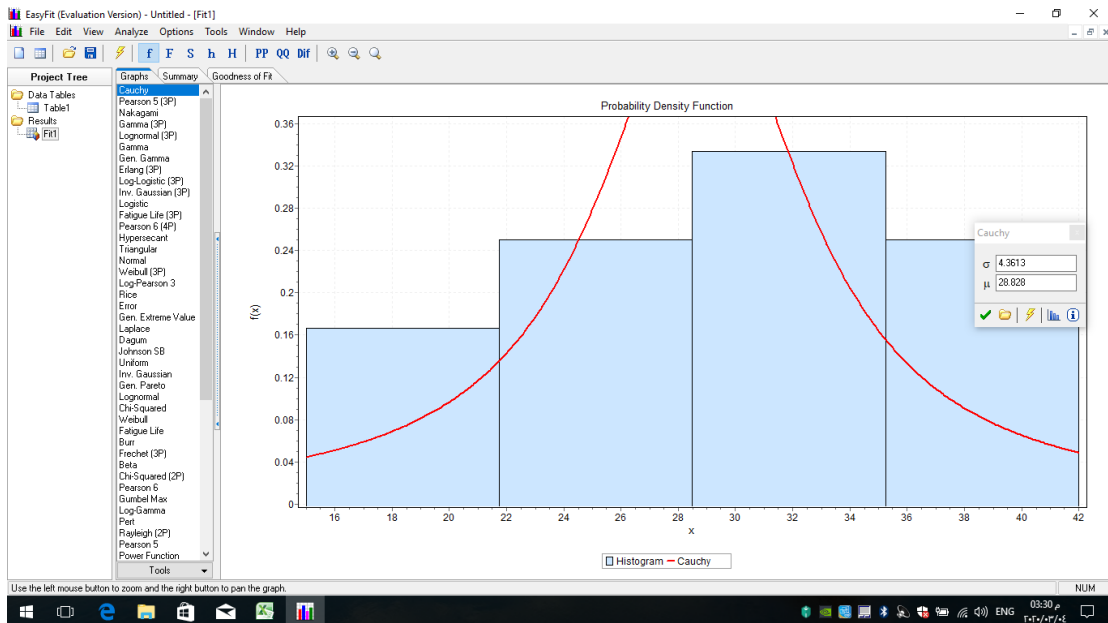
The study of this paper covers a period of six months; it begins from Jun 2019 until December 2019; it is an experiment that includes (14) patients.(12) patients were dead and (2) patients remain alive .

When applying the test statistic (Kolmogorov-Smirnov) depending upon statistical programming (EasyFit 5.5 Professional) in order to fit Cauchy distribution data , it is discovered that the calculated value is (0. 11034) , this means data is distributed according to Cauchy distribution .

The null and alternative hypotheses are as follows :

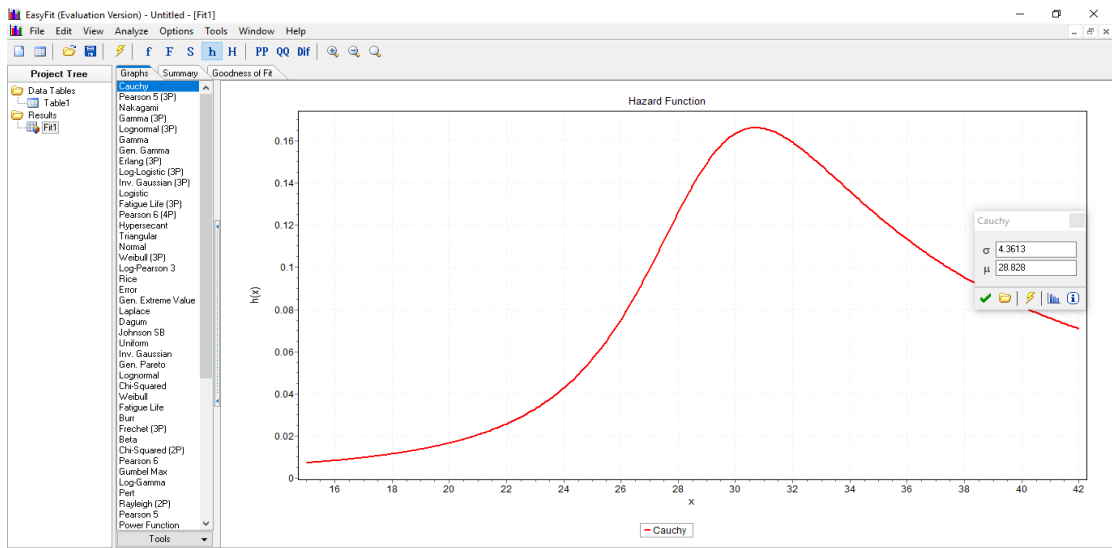
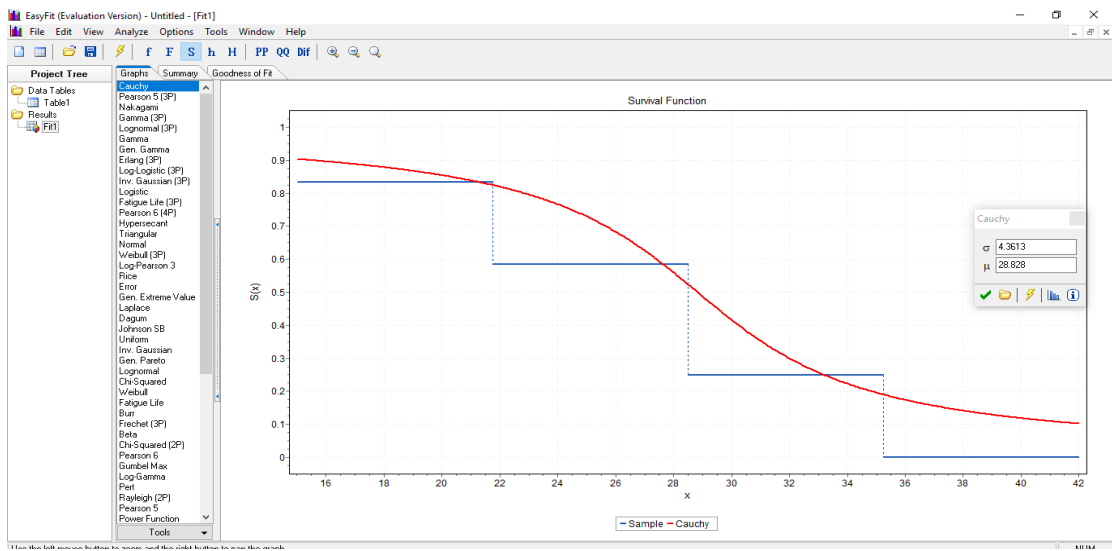
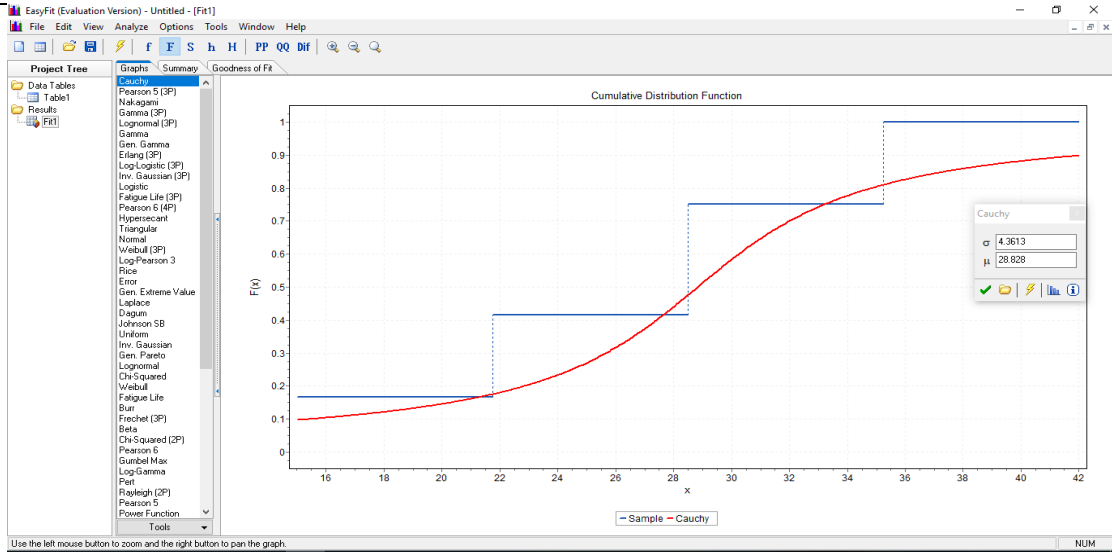
H_0 : The survival time data is distributed as Cauchy.

H_1 : The survival time data is not distributed as Cauchy.



‘Figure(1)’

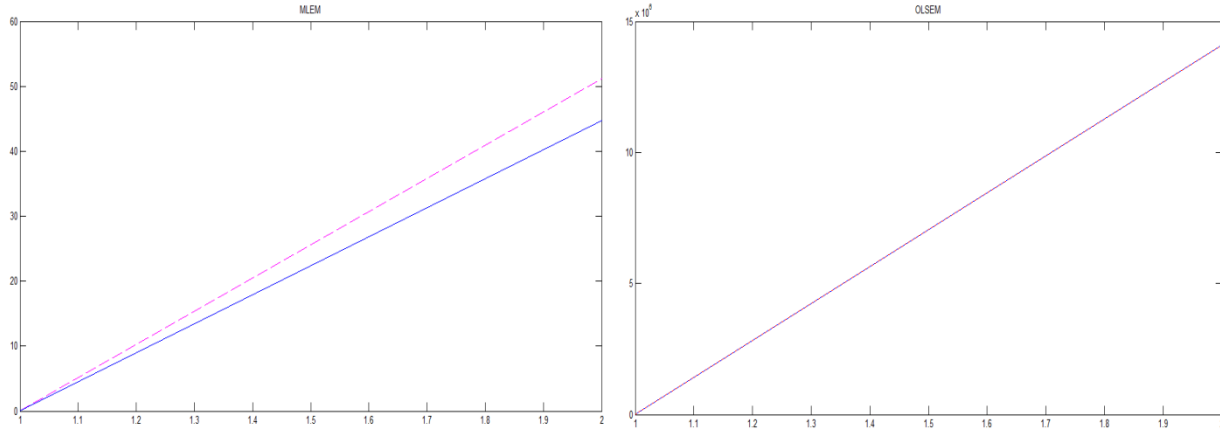
Fit the data for Breast cancer from Educational Hospital Diwaniyah



When applying MATLAB (R2014a) , the estimated parameters results are as follows :
 The assumed initial values for two-parameters are as follows:

Table (1)

MLEM	OLSEM	LAEM
Initial values of parameters		
$\gamma_0 = 0.052$; $\delta_0 = 31$	$\gamma_0 = 52$; $\delta_0 = 31$	$\gamma_0 = 0.0019$; $\delta_0 = 51.1635$
Estimated values for the parameters		
$\hat{\gamma} = 0.019$; $\hat{\delta} = 51.1635$	$\hat{\gamma} = 0.0004$; $\hat{\delta} = 1.4100$	$\hat{\gamma} = 0.01900596$; $\hat{\delta} = 51.16350063$



After that , using these estimated values for two-parameters in Cauchy distribution to find the numerical values for $f(t)$, $F(t)$, $s(t)$ and $h(t)$.

Table (2) : Estimated values for functions $f(t)$, $F(t)$, $s(t)$, $h(t)$ by LAEM

Failure Time	$f(t)$	$F(t)$	$s(t)$	$h(t)$
15	4.62E-07	5.698456	0.499992	9.25E-07
21	6.65E-07	7.599852	0.499988	1.33E-06
23	7.62E-07	8.201278	0.499986	1.52E-06
25	8.83E-07	8.785386	0.499984	1.77E-06
28	1.13E-06	9.628227	0.499979	2.25E-06
29	1.23E-06	9.900173	0.499978	2.46E-06
29	1.23E-06	9.900173	0.499978	2.46E-06
30	1.35E-06	10.1676	0.499975	2.70E-06
34	2.05E-06	11.19218	0.499963	4.11E-06
38	3.49E-06	12.14562	0.499936	6.98E-06
39	4.09E-06	12.37313	0.499925	8.18E-06
42	7.20E-06	13.03039	0.499869	1.44E-05

$$MSE[\hat{s}(t_i)] = \frac{1}{n} \sum_{i=1}^n [\hat{s}(t_i) - s(t_i)]^2 = 0.014458476$$

$$MAPE[\hat{s}(t_i)] = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{s}(t_i) - s(t_i)}{s(t_i)} \right| = 0.458202743$$

Table (3) : Estimated values for functions $f(t)$, $F(t)$, $s(t)$, $h(t)$ by OLSEM

Failure Time	$f(t)$	$F(t)$	$s(t)$	$h(t)$
15	7.72E-08	28.52456448	0.499998592	1.55E-07
21	9.01E-08	28.47754203	0.499998358	1.80E-07
23	9.51E-08	28.46022013	0.499998267	1.90E-07
25	1.20E-07	28.38075918	0.499997815	2.40E-07
28	1.56E-07	28.28014998	0.499997161	3.11E-07
29	1.67E-07	28.25067063	0.499996952	3.34E-07
29	1.67E-07	28.25067063	0.499996952	3.34E-07
30	1.80E-07	28.21909072	0.499996718	3.60E-07
34	2.29E-07	28.10923657	0.499995831	4.57E-07
38	2.73E-07	28.0201338	0.499995022	5.46E-07
39	3.32E-07	27.91412867	0.499993954	6.63E-07
42	6.89E-07	27.42767438	0.499987437	1.38E-06

$$MSE[\hat{s}(t_i)] = \frac{1}{n} \sum_{i=1}^n [\hat{s}(t_i) - s(t_i)]^2 = 0.014467495$$

$$MAPE[\hat{s}(t_i)] = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{s}(t_i) - s(t_i)}{s(t_i)} \right| = 0.0458332972$$

Conclusions:

- 1- We notice in both methods that the estimated values of the probability survival function decrease with increasing failure times (an inverse relationship between them).
- 2- We notice in both methods that the estimated values of the potential risk function increase with increasing times of failure (a direct relationship between them).
- 3- It is recommended to use (LAEM) of Cauchy distribution of Breast cancer by employing MSE criterion.

References:

- (1) Gerald Haas , Lee Bain and Gharles Antle 1970 “Inferences for the Cauchy distribution based on maximum likelihood estimators”. *Printed in Great Britain. Biometrika* , 57, 2 , PP.403.
- (2) M. H. Tahir , M. Zubair , Gauss M , Cordeiro , Ayman Alzaatreh and M. Mansoor 2017 “The Weibull-Power Cauchy distribution : model, properties and applications”. *Hacettepe Journal of Mathematics and Statistics* . Vol. 46 (4) , PP. 767 – 789.
- (3) Lindley , D. V. 1980 “ Approximate Bayesian Method ” , *Trabajos de Estadistica* , vol. 31 , PP. 223-237 .
- (4) Douglas C. M. and George C. R. 2003 “ Applied Statistics and Probability for Engineering ” . Third Edition , *John Wiley and Sons* , Inc , PP. 1-976.
- (5) Hutcheson, G. D. and Sofroniou , N. 1999 “ The Multivariate Social Scientist ” , *London : Sage Publications* ,PP.1-228.