# Generalized Parametric Exponential Entropy Measure of degree $\alpha$ and $\beta$ on Fuzzy Set

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**Abstract:** There exist many well known parametric and non-parametric measures with their properties and application on the basis of literature survey of fuzzy information measure. For characterization and quantification of fuzzy uncertainty and vagueness, fuzzy entropy measure is used very frequently. For this we propose a new parametric exponential entropy measure on fuzzy set. After showing the validity of proposed entropy measure some properties are also given.

**Keyword:** Fuzzy set, entropy measure, parametric measure, exponential measure, uncertainty.

## 1. Introduction:

Fuzzy uncertainty and mathematical ambiguity was major problem for mathematician in the starting of $19^{th}$ century. Shannon [1] introduced the entropy to measure the uncertainty of a probability distribution. The information contained in any experiment having probability distribution $P=(p_1, p_2, p_3, ........., p_n)$ of discrete random variable $X$ is given by

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i$$

which is the well known Shannon [1] entropy. Fuzzy entropy is used to express the mathematical value of the fuzziness of fuzzy set. The amount of probabilistic uncertainty removed in an experiment is called measure of information and the amount of measure of vagueness and ambiguity of uncertainty is measure of fuzziness. The degree of uncertainty is usually used in different areas e.g. image processing, pattern recognition, statistical mechanics, finance, decision making, clustering etc. The basic subject of information theory, entropy is firstly used by Zadeh [2]. The degree of fuzziness in fuzzy set is used as rule of logic in examination of system containing ambiguity and vagueness. Shannon's [1] entropy give direction to De Luca and Termini [3] to introduced a framework of axioms of fuzzy entropy measure. Using these axioms many researcher develop new entropy measure on fuzzy set. De Luca and Termini [3] changed the Shannon's variable into membership degree of element and proposed fuzzy entropy measure. Kaufmann [4] proposed a fuzzy entropy measure by a metric distance between its membership function and the characteristic function of its nearest crisp set. Bhandari and Pal [5] presented two fuzzy entropy measures, out of which one is exponential entropy. Kapur [6], Verma and Sharma [7], Hooda [8] and Fan and Ma [9] defined some new fuzzy entropy measure corresponding to probabilistic measure. Tomar and Anshu [10-13] proposed some entropy measure on fuzzy set. Using the concept of above defined entropies we propose an exponential entropy measure on fuzzy set. Some properties are given to show that defined measure is valid.

## 2. Preliminaries

In this section we present some basic concepts of fuzzy set and fuzzy entropy.

**Defination 1**.

Let $X=(x_1,x_2,.........,x_n)$ be a discrete universe of discourse. A fuzzy set $A$ is given as

$$A=\{<x_i,\mu_A(x_i)>/x_i\in X\}$$

where, $\mu_A(x_i):X\to[0,1]$ is the membership function of $A$. The membership value $\mu_A(x_i)$ describes the degree of belongingness of $x_i\in X$ .

**Definition 2.**

Let $K$ be a real function defined on family of fuzzy set $(FS(X))$ such that $K:FS(X)\to R^+$ and if $K$ satisfies following properties then it is called entropy measure on fuzzy set

$(\boldsymbol{B_1})$ $K(A)=0$ if $A$ is a crisp set

$(\boldsymbol{B_2})$ $K(A)$ assumes a unique maximum iff $\mu_A(x_i)=0.5$

$(\boldsymbol{B_3})$ $K(A^*)\le K(A)$, where $A^*$ is the sharpened version of $A$

$(\boldsymbol{B_4})$ $K(A)=K(A^C)$

De Luca and Termini [3] proposed an entropy measure on fuzzy set using the concept of same degree of fuzziness of $\mu_A(x_i)$ and $1-\mu_A(x_i)$ as

$$H(A)=-\sum_{i=1}^{n}\left[\mu_A(x_i)\log\mu_A(x_i)+(1-\mu_A(x_i))\log(1-\mu_A(x_i))\right] \tag{1}$$

Later on Bhandari and Pal [5] introduced two entropy measures on fuzzy set as

$$H_\alpha(A)=\frac{1}{1-\alpha}\sum_{i=1}^{n}\left[\log\mu_A^\alpha(x_i)+(1-\mu_A(x_i))^\alpha\right] \tag{2}$$

$$H_e(A)=\frac{1}{n(\sqrt{e}-1)}\sum_{i=1}^{n}\left[\mu_A(x_i)e^{1-\mu_A(x_i)}+(1-\mu_A(x_i))e^{\mu_A(x_i)}-1\right] \tag{3}$$

After this Kapur [6] defined entropy measure on fuzzy set as

$$H_{k\alpha}(A)=\frac{1}{1-\alpha}\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}-1\right]$$

$$H_{\alpha,\beta}(A)=\frac{1}{\alpha+\beta-2}\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}+\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}-2\right]$$

$$H_{\alpha}^{\beta}(A)=\frac{1}{\beta-\alpha}\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}-\mu_A^{\beta}(x_i)-(1-\mu_A(x_i))^{\beta}\right]$$

Later on exponential entropy for fuzzy set is also proposed by Verma and Sharma [7] as

$$H_e(A)=\frac{1}{n\left(e^{1-0.5^{\alpha}}-1\right)}\sum_{i=1}^{n}\left[\mu_A(x_i)e^{1-\mu_A^{\alpha}(x_i)}+(1-\mu_A(x_i))e^{1-(1-\mu_A(x_i))^{\alpha}}-1\right] \tag{4}$$

### 3. New proposed exponential entropy measure

On the survey of above concepts we propose exponential fuzzy entropy measure as

$$B_e(A)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\sum_{i=1}^{n}\left[\begin{array}{l}\mu_A(x_i)e^{1-\mu_A^{\alpha}(x_i)}+(1-\mu_A(x_i))e^{1-(1-\mu_A(x_i))^{\alpha}}\\-\mu_A(x_i)e^{1-\mu_A^{\beta}(x_i)}-(1-\mu_A(x_i))e^{1-(1-\mu_A(x_i))^{\beta}}\end{array}\right]\ \alpha>0,\ \beta>0 \tag{5}$$

**Theorem1:** The measure defines $B_e(A)$ is a valid entropy measure on fuzzy set $A$.

Proof. We have to satisfy four properties $B_1$ *to* $B_4$ for validity of measure $B_e(A)$. ***$B_1$(Sharpness):*** To prove $B_e(A)=0$ iff $\mu_A(x_i)=0$ *or* $\mu_A(x_i)=1$

First we suppose that $B_e(A)=0$

$$\Rightarrow \frac{1}{ne\left(e^{2^{-\alpha}}-e^{2^{-\beta}}\right)}\sum_{i=1}^{n}\left[\begin{array}{l}\mu_A(x_i)e^{1-\mu_A^{\alpha}(x_i)}+(1-\mu_A(x_i))e^{1-(1-\mu_A(x_i))^{\alpha}}\\-\mu_A(x_i)e^{1-\mu_A^{\beta}(x_i)}-(1-\mu_A(x_i))e^{1-(1-\mu_A(x_i))^{\beta}}\end{array}\right]=0$$

$$\Rightarrow \mu_A(x_i)e^{1-\mu_A^{\alpha}(x_i)}+(1-\mu_A(x_i))e^{1-(1-\mu_A(x_i))^{\alpha}}=\mu_A(x_i)e^{1-\mu_A^{\beta}(x_i)}+(1-\mu_A(x_i))e^{1-(1-\mu_A(x_i))^{\beta}}$$

This is true if $\mu_A(x_i)=0$ *or* $\mu_A(x_i)=1$.

Conversely: Now we assume that $\mu_A(x_i)=0$ *or* $\mu_A(x_i)=1$

$$\Rightarrow \mu_A(x_i)e^{1-\mu_A^{\alpha}(x_i)}+(1-\mu_A(x_i))e^{1-(1-\mu_A(x_i))^{\alpha}}=0$$

$$\Rightarrow \mu_A(x_i)e^{1-\mu_A^{\beta}(x_i)}+(1-\mu_A(x_i))e^{1-(1-\mu_A(x_i))^{\beta}}=0$$

$\Rightarrow B_e(A) = 0$.

Hence we can say that $B_e(A) = 0$ iff $\mu_A(x_i) = 0$ *or* $\mu_A(x_i) = 1$.

***$B_2$(Maximality):*** To prove $B_e(A)$ is maximum at $\mu_A(x_i) = 0.5$

Now

$$\frac{\partial B_e(A)}{\partial \mu_A(x_i)} = \frac{1}{ne\left(e^{2^{-\alpha}} - e^{2^{-\beta}}\right)} \sum_{i=1}^{n} \begin{bmatrix} e^{1-\mu_A^{\alpha}(x_i)} - \alpha\mu_A^{\alpha}(x_i)e^{1-\mu_A^{\alpha}(x_i)} + \alpha(1-\mu_A(x_i))^{\alpha} e^{1-(1-\mu_A(x_i))^{\alpha}} \\ - e^{1-(1-\mu_A(x_i))^{\alpha}} - e^{1-\mu_A^{\beta}(x_i)} + \beta\mu_A^{\beta}(x_i)e^{1-\mu_A^{\beta}(x_i)} \\ - \beta(1-\mu_A(x_i))^{\beta} e^{1-(1-\mu_A(x_i))^{\beta}} + e^{1-(1-\mu_A(x_i))^{\beta}} \end{bmatrix}$$

Case – 1: Let $0 \le \mu_A(x_i) < 0.5$ then, we have $\frac{\partial B_e(A)}{\partial \mu_A(x_i)} > 0$ *for all* $0 < \alpha$ *and* $0 < \beta$

this give that $B_e(A)$ is increasing function.

Case – 2: Let $0.5 < \mu_A(x_i) \le 1$ then, we have $\frac{\partial B_e(A)}{\partial \mu_A(x_i)} < 0$ *for all* $0 < \alpha$ *and* $0 < \beta$

this give that $B_e(A)$ is decreasing function.

Case - 3: Let $\mu_A(x_i) = 0.5$ then, we have $\frac{\partial B_e(A)}{\partial \mu_A(x_i)} = 0$ *for all* $0 < \alpha$ *and* $0 < \beta$

This clearly shows that $B_e(A)$ is a concave function as shown in graph1.1 which has a global maximum at $\mu_A(x_i) = 0.5$. Hence $B_e(A)$ is maximum iff $A$ is the most fuzzy set. ***$B_3$(Resolution):*** To prove $B_e(A^*) \le B_e(A)$ where $A^*$ is sharpened version of $A$. On the basis of previous axioms we know that $B_e(A)$ is increasing function for $0 \le \mu_A(x_i) < 0.5$ and decreasing function for $0.5 < \mu_A(x_i) \le 1$. So using above concept, we have

$\mu_{A^*}(x_i) \le \mu_A(x_i) \Rightarrow B_e(A^*) \le B_e(A)$ and

$\mu_{A^*}(x_i) \ge \mu_A(x_i) \Rightarrow B_e(A^*) \le B_e(A)$

$\Rightarrow \quad B_e(A^*) \le B_e(A)$ in both cases.

***$B_4$(Symmetry):*** Let $A^C$ is the compliment of $A$ so we can take $\mu_A(x_i) = 1 - \mu_A(x_i)$ due to membership function. Then clearly $\Rightarrow B_e(A^C) = B_e(A)$.

This shows that $B_e(A)$ is valid entropy measure on fuzzy set.

**Limiting and Particular Cases:**

It can be seen, from the proposed measure of order $\alpha$ *and* $\beta$ that some existing measure can deduce from it as follows:

(I) If $\beta \to 0$ then our proposed entropy measure reduces to Verma and Sharma [7] exponential entropy measure.

(II) If $\alpha \to 1, \beta \to 0$ then our proposed entropy measure reduces to Bhandari and Pal [5] exponential entropy measure.

(III) If $\alpha \to 0, \beta \to 0$ then our proposed entropy measure reduces to De Luca and Termini [3] logarithmic entropy measure.

**4. Some properties of entropy measure**

**Definition 4.1:** Assume that the family of all fuzzy set of universe $X$, is denote by $FS(X)$ and $P,Q,R \in FS(X)$ is given

$$P = \left[\prec x, \mu_P(x) \succ / x \in X\right]$$
$$Q = \left[\prec x, \mu_Q(x) \succ / x \in X\right]$$
$$R = \left[\prec x, \mu_R(x) \succ / x \in X\right]$$

then some set operation can be defined as follows:

a. $P \cup Q = \left[\prec x, \max\left(\mu_P(x), \mu_Q(x)\right) \succ / x \in X\right]$

b. $P \cap Q = \left[\prec x, \min\left(\mu_P(x), \mu_Q(x)\right) \succ / x \in X\right]$

c. $(P \cup Q) \cup R = \left[\prec x, \max\left\{\max\left(\mu_P(x), \mu_Q(x)\right), \mu_R(x)\right\} \succ / x \in X\right]$

d. $(P \cap Q) \cap R = \left[\prec x, \min\left\{\min\left(\mu_P(x), \mu_Q(x)\right), \mu_R(x)\right\} \succ / x \in X\right]$

e. $P^C = \left[\prec x, \mu_{P^C}(x) = 1 - \mu_P(x) \succ / x \in X\right]$

The exponential entropy measure has some important properties as follows:

**Theorem 2.** For fuzzy set $P,Q \in FS(X)$ prove that

$(I)$ $B_e(P \cup Q) + B_e(P \cap Q) = B_e(P) + B_e(Q)$

$(II)$ $B_e((P \cup Q) \cup R) = B_e(P \cup (Q \cup R))$

$(III)\quad B_e((P\cap Q)\cap R)=B_e(P\cap(Q\cap R))$

$(IV)\quad B_e(P)=B_e(P\cap P^C)=B_e(P\cup P^C)=B_e(P^C)$

$(V)\quad B_e(P\cap Q)=B_e(P^C\cup Q^C)^C$

$(VI)\quad B_e(P\cup Q)=B_e(P^C\cap Q^C)^C$

Proof: We suppose that

$$X_\sigma=\lfloor x_i / x_i\in X,\mu_P(x_i)\geq\mu_Q(x_i)\geq\mu_R(x_i)\rfloor$$

$$X_\theta=\lfloor x_i / x_i\in X,\mu_P(x_i)<\mu_Q(x_i)<\mu_R(x_i)\rfloor$$

where $\mu_P(x_i)$, $\mu_Q(x_i)$ be the fuzzy membership functions of P and Q respectively.

$(I)$ We know that $B_e(P\cup Q)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\sum_{i=1}^{n}\begin{bmatrix}\mu_{P\cup Q}(x_i)e^{1-\mu_{P\cup Q}^{\alpha}(x_i)}\\+(1-\mu_{P\cup Q}(x_i))e^{1-(1-\mu_{P\cup Q}(x_i))^{\alpha}}\\-\mu_{P\cup Q}(x_i)e^{1-\mu_{P\cup Q}^{\beta}(x_i)}\\-(1-\mu_{P\cup Q}(x_i))e^{1-(1-\mu_{P\cup Q}(x_i))^{\beta}}\end{bmatrix}$

$$\Rightarrow B_e(P\cup Q)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\begin{bmatrix}\sum_{x_i=X_\sigma}\begin{bmatrix}\mu_P(x_i)e^{1-\mu_P^{\alpha}(x_i)}+(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\alpha}}\\-\mu_P(x_i)e^{1-\mu_P^{\beta}(x_i)}-(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\beta}}\end{bmatrix}\\+\sum_{x_i=X_\theta}\begin{bmatrix}\mu_Q(x_i)e^{1-\mu_Q^{\alpha}(x_i)}+(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^{\alpha}}\\-\mu_Q(x_i)e^{1-\mu_Q^{\beta}(x_i)}-(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^{\beta}}\end{bmatrix}\end{bmatrix}\tag{6}$$

$$\Rightarrow B_e(P\cap Q)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\begin{bmatrix}\sum_{x_i=X_\sigma}\begin{bmatrix}\mu_Q(x_i)e^{1-\mu_Q^{\alpha}(x_i)}+(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^{\alpha}}\\-\mu_Q(x_i)e^{1-\mu_Q^{\beta}(x_i)}-(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^{\beta}}\end{bmatrix}\\+\sum_{x_i=X_\theta}\begin{bmatrix}\mu_P(x_i)e^{1-\mu_P^{\alpha}(x_i)}+(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\alpha}}\\-\mu_P(x_i)e^{1-\mu_P^{\beta}(x_i)}-(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\beta}}\end{bmatrix}\end{bmatrix}\tag{7}$$

Adding (6) & (7) we obtain

$$\Rightarrow B_e(P\cup Q)+B_e(P\cap Q)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\left[\begin{array}{l}\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_P(x_i)e^{1-\mu_P^\alpha(x_i)}+(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^\alpha}\\-\mu_P(x_i)e^{1-\mu_P^\beta(x_i)}-(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^\beta}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_P(x_i)e^{1-\mu_P^\alpha(x_i)}+(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^\alpha}\\-\mu_P(x_i)e^{1-\mu_P^\beta(x_i)}-(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^\beta}\end{array}\right]\\\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_Q(x_i)e^{1-\mu_Q^\alpha(x_i)}+(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^\alpha}\\-\mu_Q(x_i)e^{1-\mu_Q^\beta(x_i)}-(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^\beta}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_Q(x_i)e^{1-\mu_Q^\alpha(x_i)}+(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^\alpha}\\-\mu_Q(x_i)e^{1-\mu_Q^\beta(x_i)}-(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^\beta}\end{array}\right]\end{array}\right]$$

$$\Rightarrow B_e(P\cup Q)+B_e(P\cap Q)=B_e(P)+B_e(Q).$$

Hence first property is satisfied.

$$(II)\ B_e(P\cup Q)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\left[\begin{array}{l}\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_P(x_i)e^{1-\mu_P^\alpha(x_i)}+(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^\alpha}\\-\mu_P(x_i)e^{1-\mu_P^\beta(x_i)}-(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^\beta}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_Q(x_i)e^{1-\mu_Q^\alpha(x_i)}+(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^\alpha}\\-\mu_Q(x_i)e^{1-\mu_Q^\beta(x_i)}-(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^\beta}\end{array}\right]\end{array}\right]$$

$$B_e((P\cup Q)\cup R)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\left[\begin{array}{l}\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_P(x_i)e^{1-\mu_P^\alpha(x_i)}+(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^\alpha}\\-\mu_P(x_i)e^{1-\mu_P^\beta(x_i)}-(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^\beta}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_R(x_i)e^{1-\mu_R^\alpha(x_i)}+(1-\mu_R(x_i))e^{1-(1-\mu_R(x_i))^\alpha}\\-\mu_R(x_i)e^{1-\mu_R^\beta(x_i)}-(1-\mu_R(x_i))e^{1-(1-\mu_R(x_i))^\beta}\end{array}\right]\end{array}\right] \quad (8)$$

Now,

$$B_e(Q\cup R)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\left[\begin{array}{l}\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_Q(x_i)e^{1-\mu_Q^\alpha(x_i)}+(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^\alpha}\\-\mu_Q(x_i)e^{1-\mu_Q^\beta(x_i)}-(1-\mu_Q(x_i))e^{1-(1-\mu_Q(x_i))^\beta}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_R(x_i)e^{1-\mu_R^\alpha(x_i)}+(1-\mu_R(x_i))e^{1-(1-\mu_R(x_i))^\alpha}\\-\mu_R(x_i)e^{1-\mu_R^\beta(x_i)}-(1-\mu_R(x_i))e^{1-(1-\mu_R(x_i))^\beta}\end{array}\right]\end{array}\right]$$

$$B_e(P\cup(Q\cup R))=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\left[\begin{array}{l}\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_P(x_i)e^{1-\mu_P^{\alpha}(x_i)}+(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\alpha}}\\-\mu_P(x_i)e^{1-\mu_P^{\beta}(x_i)}-(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\beta}}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_R(x_i)e^{1-\mu_R^{\alpha}(x_i)}+(1-\mu_R(x_i))e^{1-(1-\mu_R(x_i))^{\alpha}}\\-\mu_R(x_i)e^{1-\mu_R^{\beta}(x_i)}-(1-\mu_R(x_i))e^{1-(1-\mu_R(x_i))^{\beta}}\end{array}\right]\end{array}\right] \tag{9}$$

By equation (8) & (9) we can easily show that

$$B_e((P\cup Q)\cup R)=B_e(P\cup(Q\cup R)).$$

$(III)$ Using above result we can easily show that

$$B_e((P\cap Q)\cap R)=B_e(P\cap(Q\cap R)).$$

$(IV)$ We know that

$$B_e\left(P\cup P^C\right)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\left[\begin{array}{l}\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_P(x_i)e^{1-\mu_P^{\alpha}(x_i)}+(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\alpha}}\\-\mu_P(x_i)e^{1-\mu_P^{\beta}(x_i)}-(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\beta}}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_{P^C}(x_i)e^{1-\mu_{P^C}^{\alpha}(x_i)}+(1-\mu_{P^C}(x_i))e^{1-(1-\mu_{P^C}(x_i))^{\alpha}}\\-\mu_{P^C}(x_i)e^{1-\mu_{P^C}^{\beta}(x_i)}-(1-\mu_{P^C}(x_i))e^{1-(1-\mu_{P^C}(x_i))^{\beta}}\end{array}\right]\end{array}\right] \tag{10}$$

Or

$$B_e\left(P\cup P^C\right)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\left[\begin{array}{l}\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_{P^C}(x_i)e^{1-\mu_{P^C}^{\alpha}(x_i)}+(1-\mu_{P^C}(x_i))e^{1-(1-\mu_{P^C}(x_i))^{\alpha}}\\-\mu_{P^C}(x_i)e^{1-\mu_{P^C}^{\beta}(x_i)}-(1-\mu_{P^C}(x_i))e^{1-(1-\mu_{P^C}(x_i))^{\beta}}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_P(x_i)e^{1-\mu_P^{\alpha}(x_i)}+(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\alpha}}\\-\mu_P(x_i)e^{1-\mu_P^{\beta}(x_i)}-(1-\mu_P(x_i))e^{1-(1-\mu_P(x_i))^{\beta}}\end{array}\right]\end{array}\right] \tag{11}$$

After solving both equation (10) & (11) we obtain same result as:

$$B_e(P)=B_e\left(P\cup P^C\right)$$

Using above result we can show that

$$B_e(P)=B_e\left(P\cap P^C\right)=B_e\left(P\cup P^C\right)=B_e\left(P^C\right)$$

$(V)$ We know that

$$B_e\left(P^C \cup Q^C\right)=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\left[\begin{array}{l}\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_{Q^C}(x_i)e^{1-\mu^{\alpha}_{Q^C}(x_i)}+\left(1-\mu_{Q^C}(x_i)\right)e^{1-\left(1-\mu_{Q^C}(x_i)\right)^{\alpha}}\\-\mu_{Q^C}(x_i)e^{1-\mu^{\beta}_{Q^C}(x_i)}-\left(1-\mu_{Q^C}(x_i)\right)e^{1-\left(1-\mu_{Q^C}(x_i)\right)^{\beta}}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_{P^C}(x_i)e^{1-\mu^{\alpha}_{P^C}(x_i)}+\left(1-\mu_{P^C}(x_i)\right)e^{1-\left(1-\mu_{P^C}(x_i)\right)^{\alpha}}\\-\mu_{P^C}(x_i)e^{1-\mu^{\beta}_{P^C}(x_i)}-\left(1-\mu_{P^C}(x_i)\right)e^{1-\left(1-\mu_{P^C}(x_i)\right)^{\beta}}\end{array}\right]\end{array}\right]$$

$$\begin{aligned}B_e\left(P^C \cup Q^C\right)^C&=\frac{1}{n\left(e^{1-0.5^{-\alpha}}-e^{1-0.5^{-\beta}}\right)}\left[\begin{array}{l}\sum_{x_i=X_\sigma}\left[\begin{array}{l}\mu_{Q}(x_i)e^{1-\mu^{\alpha}_{Q}(x_i)}+\left(1-\mu_{Q}(x_i)\right)e^{1-\left(1-\mu_{Q}(x_i)\right)^{\alpha}}\\-\mu_{Q}(x_i)e^{1-\mu^{\beta}_{Q}(x_i)}-\left(1-\mu_{Q}(x_i)\right)e^{1-\left(1-\mu_{Q}(x_i)\right)^{\beta}}\end{array}\right]\\+\sum_{x_i=X_\theta}\left[\begin{array}{l}\mu_{P}(x_i)e^{1-\mu^{\alpha}_{P}(x_i)}+\left(1-\mu_{P}(x_i)\right)e^{1-\left(1-\mu_{P}(x_i)\right)^{\alpha}}\\-\mu_{P}(x_i)e^{1-\mu^{\beta}_{P}(x_i)}-\left(1-\mu_{P}(x_i)\right)e^{1-\left(1-\mu_{P}(x_i)\right)^{\beta}}\end{array}\right]\end{array}\right]\\&=B_e(P\cap Q)\end{aligned}$$

Hence the result.

$(VI)$ By above result clearly $B_e(P\cup Q)=B_e\left(P^C \cap Q^C\right)^C$.

## 5. Conclusion

In this article we introduce an exponential parametric entropy measure on fuzzy set. The proposed entropy measure is important and valid which reduce to the existing entropy measure after substituting the suitable value of parameters. Some interesting and important properties of these entropy measures have also been studied. Thus, we can say that the defined entropy measure is more flexible measure from the application point of view.

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