Research Article

On The Question Of Calculation Of The Flow Velocity Distribution When Liquid Flowing Cylindrical Bodies

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Abstract

This article discusses the problem of the influence of the interaction between a circular cylindrical body and a water stream. A possible method for calculating the velocity distribution of cylindrical bodies during flow is proposed.

Keywords: flow around, cylindrical body vector potential, Mathieu functions, potential flow.

Introduction

The question of the magnitude of the resistance experienced by a stationary body in a moving fluid or, conversely, by a moving body in a stationary fluid, belongs to the most ancient problems of hydrodynamics [1, 4, 7, 9].

Despite this, until now it has not been possible to achieve its general theoretical solution, which led to attempts to directly experimentally determine the resistance, thereby giving impetus to the development of experimental hydro - and aerodynamics [2, 3, 10].

But even now, in the presence of enormous experimental material, the need for its general theoretical substantiation and illumination has not diminished, without which it will always represent only a collection of observations on individual random phenomena of nature. The calculation of the velocity field in the flow around cylindrical bodies can be based on the method of expansion of the vector potential of this field

($\vec{V} = rot\vec{A}$) by harmonics determined by the symmetry of the given body. This method was applied by Smythe in problems of potential flow around spherical and ellipsoidal bodies [6, 7], using the results known in electrodynamics. The vector potential outside the streamlined body can be represented as

$$A = A_0 + A_1, \qquad (1)$$

where A_0 – uniform flow potential away from the body $\left(A_0 = \frac{1}{2} pV_0 \right)$;

A_1 – potential created by the charge on the surface of the body in question in the area outside the body.

The potential of the field inside the body $A_0 = A_1 = 0$, (2)

where A_2 – field potential inside the body.

Decomposition of the field A_1 and A_2 for the harmonics mentioned, it reduces Eqs. (1) and (2) to a system of algebraic equations for the expansion coefficients, which completely solves the problem. This method makes it possible to calculate the distribution of velocities both far from the streamlined body and on its surface. To describe this case, it is convenient to introduce an elliptic coordinate system

$$\left.\begin{array}{l} x = ach\xi cos\varphi \\ y = ash\xi \sin\varphi \\ z = z, \end{array}\right\} \qquad (3)$$

where the semiaxes of the ellipse are the quantities $R_x = ach\xi$ и $R_y = ach\xi$

$$\frac{x^2}{R_x^2} + \frac{y^2}{R_y^2} = 1 \qquad (4)$$

and

$$a^2 = R_x^2 - R_y^2$$

The harmonic system for the case under consideration is a solution of the Laplace equation, which in the coordinate system (3) has the form [6]

$$\Delta A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} = \frac{1}{a^2}\frac{1}{ch^2\xi - \cos^2\varphi}\left(\frac{\partial^2 A}{\partial \xi^2} + \frac{\partial^2 A}{\partial \varphi^2}\right) + \frac{\partial^2 A}{\partial z^2} \qquad (5)$$

This equation is separable and can be solved as follows:

$$A = \psi(\xi)\phi(\varphi)Z(z). \qquad (6)$$

As can be seen from (6), for Z (z) there are two solutions

$$\left.\begin{array}{l} Z_m(z) = e^{imz} \\ Z_m(z) = e^{-imz} \end{array}\right\} \qquad (7)$$

For functionΦ (φ) and(5) with (6) the Mathieu equation is obtained [1, 7]

$$\frac{\partial^2\phi}{\partial\varphi^2}-\left(\mu-m^2a^2\cos^2\varphi\right)\phi=0 \qquad (8)$$

or

$$\frac{\partial^2\phi}{\partial\varphi^2}+\left(h+2\Theta\cos2\varphi\right)\phi=0 \qquad (8^1)$$

where

$$h=-\mu+\frac{m^2a^2}{2},\ 2\Theta_m=-\frac{m^2a^2}{2},$$

else μ- an arbitrary constant (whose value is determined by the boundary conditions imposed on the properties of the solution to this equation), while for the function $\psi(\xi)$ an arbitrary constant (whose value is determined by the boundary conditions imposed on the properties of the solution to this equation), while for the function [1, 7]

$$\frac{\partial^2\psi}{\partial\xi}+\left(\mu+m^2a^2ch^2\xi\right)\psi=0. \qquad (9)$$

Solutions to equation (9) can be obtained directly from solutions to equation (8) by replacing $\varphi\to i\xi$.

If we accept as conditions

$$\frac{d\phi}{d\varphi}\left(-\frac{\pi}{2}\right)=\frac{d\phi}{d\varphi}\left(\frac{\pi}{2}\right)=0 \qquad (10)$$

(which is equivalent to the assumption that at these points the direction of the velocity is tangent to the surface of the streamlined body), then solutions (8) will be Mathieu functions of order 2*n:*

$$ce_{2n}\left(\varphi,\Theta\right),n=0,1,\ 2,\ 3,\ \ldots, \quad (11)$$

and the solutions of equation (9) will be

$$ceh_{2n}\left(\xi,\Theta\right)=ce_{2n}\left(i\xi,\Theta\right). \qquad (12)$$

Thus, the expansion of the vector potential can be as follows:

$$A(\xi,\varphi)=\sum_{mn} A_{mn} ce_{2n}\left(\varphi,\Theta_m\right) ceh_{2n}\left(\xi,\Theta_m\right) Z_m(z). \qquad (13)$$

in practice, expansions of the Mathieu function in terms of Bessel series or in a Fourier series can be used

$$ce_{2n}(\varphi,\Theta)=\sum_{r=0}^{\infty} A_{2r}\cos(2r\varphi). \qquad (14)$$

For small Θ, those at small $\frac{m^2a^2}{2}$, as known,

$$ce_n(\varphi,0)=\cos(n\varphi), \qquad (15)$$

which corresponds to the reduction of the problem with an elliptic cylinder to a circular cylinder, and then expansion (13) takes the well-known form [6] for

$r > r_0$

$$A_1=\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r_0}{r}\right)^n \cos n\Theta, \qquad (16)$$

And for $r < r_0$

$$A_2=\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r}{r_0}\right)^n \cos n\Theta, \qquad (17)$$

which makes it possible to directly obtain the required system of algebraic equations.

Smallness Θ, the smallness α^2, is determined by the ratio of the distance from the focus of the ellipse to the point under consideration, so that the resulting system (16) and (17) can describe the distribution of velocities at a distance from the cylinder. The problem of flow past two or more adjacent cylindrical bodies is reduced to the same case (16) and (17).

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