Strategic Management in Marketing: A Game Theoretic Approach

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Abstract: In marketing, a real-world dilemma emerging between two rivals, McDonald's and Burger King, is investigated. Both firms use three strategies: discounted pricing, status quo, and aggressive commercial. In such cases, ambiguity is a determining factor. To deal with confusion in payoffs, octagonal fuzzy numbers are used. To rank fuzzy numbers, the average of odd positions, average of even positions, and quartile deviations are used. To solve the reduced modelled two competitors zero sum fuzzy matrix games, the proposed ranking methods are used. Finally, the findings are compared to current approaches that are quite similar to the proposed approach.

Keywords: Marketing Management, Fuzzy Matrix Games, Octagonal Fuzzy Numbers, Ranking

1. Introduction

Uncertain parameters are known as fuzzy numerical data, and they can be expressed by a variety of fuzzy numbers. Zadeh (1965) developed this principle and applied it to real-world problems. McGvire and Staelin examined and established chain equilibrium principles (1983). Milgroom and Roberts (1986) conducted extensive research on pricing and advertisement signals of product quality. Mitchell and Hustad pioneered product screening techniques (1981). Nadia and Pishkoohi investigated traffic management issues in a fuzzy environment (2018). Bortolan and Degani (1985) developed a variety of methods for fuzzy set applications that can be used to rank different preferences developed by Watson, Yager, Chang, Adamo, Bass, Skwakernaak, Baldwins Giuld, Jain Dubois, and Prade. Butnariu (1978) defined N-person fuzzy games. Using a theoretical approach to fuzzy mathematics, Cevikel and Ahlataiuglu (2009) published some new solution procedures for fuzzy matrix games. They spoke about the case of linearity in fuzzy payoffs. Following the formulation of the linear programming problem, Sakawa's method was used to find the solution. Christi and Kalpana (2016) used trapezoidal fuzzy numbers as payoffs in fuzzy matrix games and obtained optimal value results using an average weighted approach. Delgado and Verdegay developed fuzzy ordered relations within fuzzy numbers (1988). In a fuzzy setting, Liou and Wang (1992) established some alternative approaches. Yuan (1991) developed a ranking system with integral value and evaluated it using four evolutionary criteria. These four requirements are fuzzy ordering rationality, robustness, fuzzy choice, representation, and distinguishability. Sharma and Kumar (2016) used the idea of fuzzy numbers in game theory to predict election outcomes. Malini and Kennedy also addressed OFN and their activities (2013). Malini and Anthanarajan (2016) used OFN to develop fuzzy transportation problems.

The current paper is structured as follows: section 2 contains some simple preliminaries. Section 3 discusses basic terminology for two-person zero-sum games in a fuzzy environment. Section 4 investigates current methods for ranking fuzzy numbers. Section 5 presents our proposed process. In section 6, suggested methods are applied to find optimal strategies for two firms, and numerical examples are given. Section 7 discusses graphical representations of performance, and Section 8 provides conclusions.

2.Preliminaries

Definition 2.1 "Let X be the universe whose generic elements are denoted by x. A fuzzy set $\widetilde{A}$ in X is characterized by its membership function $\mu_{\widetilde{A}} : X \to [0,1]$ and $\mu_{\widetilde{A}}(x)$ is interpreted as the degree of membership of element x in the fuzzy set $\widetilde{A}$ for each $x \in X$. Thus a fuzzy set $\widetilde{A}$ in X can also be represented as $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))\}$. The value zero is used to represent complete non-membership, the value one is used to represent complete membership and the values between zero and one are used to represent intermediate degrees of membership."

Definition 2.2"A fuzzy set $\widetilde{A}$ in R is called a fuzzy number if it satisfies the following axioms:

(i) There exist at least one $x_0 \in R$ with $\mu_{\widetilde{A}}(x_0) = 1$

(ii) $\mu_{\widetilde{A}}(x)$ is piecewise continuous.

(iii) $\widetilde{A}$ must be normal and convex."

Definition 2.3 "A fuzzy number $\widetilde{A}$ is a normal octagonal fuzzy number denoted by $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ are real numbers and its membership function $\mu_{\widetilde{A}}(x)$ is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; x < a_1 \\ k\left(\dfrac{x - a_1}{a_2 - a_1}\right) & ; a_1 \le x \le a_2 \\ k & ; a_2 \le x \le a_3 \\ \left\{k + (1-k)\left(\dfrac{x - a_3}{a_4 - a_3}\right)\right\} & ; a_3 \le x \le a_4 \\ 1 & ; a_4 \le x \le a_5 \\ \left\{k + (1-k)\left(\dfrac{a_6 - x}{a_6 - a_5}\right)\right\} & ; a_5 \le x \le a_6 \\ k & ; a_6 \le x \le a_7 \\ k\left(\dfrac{a_8 - x}{a_8 - a_7}\right) & ; a_7 \le x \le a_8 \\ 0 & ; a_8 < x \end{cases}$$

Definition 2.4 "Let $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ be an octagonal fuzzy number, then the α-cut is defined as

$$\widetilde{A}_\alpha = \begin{cases} \left[a_1 + \dfrac{\alpha}{k}(a_2 - a_1),\ a_8 - \dfrac{\alpha}{k}(a_8 - a_7)\right] & ; \alpha \in [0, k] \\ \left[a_3 + \left(\dfrac{\alpha - k}{1-k}\right)(a_4 - a_3),\ a_6 - \left(\dfrac{\alpha - k}{1-k}\right)(a_6 - a_5)\right] & ; \alpha \in (k, 1] \end{cases}$$

3.Two Person Zero Sum Fuzzy Game

- Two person (player) zero sum fuzzy game is denoted by FG $\approx (\tilde{S}_1, \tilde{S}_2, \widetilde{K}, \widetilde{A})$ where
- $\tilde{S}_1 \approx \{\widetilde{X} \approx (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_m) \mid \tilde{x}_i \ge \tilde{0}, \forall\, i = 1,2,3, \ldots m,\ \sum_{i=1}^{m} \tilde{x}_i \approx \tilde{I}\}$
- $\tilde{S}_2 \approx \{\widetilde{Y} \approx (\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \ldots, \tilde{y}_n) \mid \tilde{y}_j \ge \tilde{0}, \forall\, j = 1,2,3, \ldots n,\ \sum_{j=1}^{n} \tilde{y}_j \approx \tilde{I}\}$
- Where $\tilde{S}_1$ and $\tilde{S}_2$ are strategic spaces available for both player I and II respectively. Then the fuzzy payoffs gained by maximizing player is defined as,
- $\widetilde{K}(\widetilde{X}, \widetilde{Y}) = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{a}_{ij} \cdot \tilde{x}_i \cdot \tilde{y}_j \approx \widetilde{X}^T \cdot \widetilde{A} \cdot \widetilde{Y}$

4. **Existing Ranking Methods to Convert OFN into Crisp Value**

Let $\tilde{A}$ be an octagonal fuzzy number then different kinds of ranking methods or techniques are used for defuzzification are tabled as:

Table: 4.1 Existing Ranking Methods

Ranking Method	Formula to Obtain Crisp Value of OFN
Measure of an OFN (Magnitude ranking method)	$M_o^{oct}(\widetilde{A}) = \frac{1}{4}[k(a_1 + a_2 + a_7 + a_8) + (1-k)(a_3 + a_4 + a_5 + a6\ ; k \in [0,1]$
Pascal´s triangular graded mean method	$P(\widetilde{A}) = \frac{(a_1+a_8)+7(a_2+a_7)+21(a_3+a_6)+35(a_4+a_5)}{128}$
Simple average method	$P(\widetilde{A}) = \frac{a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8}{8}$
Mean-max membership principle	$P(\widetilde{A}) = \frac{a_4+a_5}{2}$
Centroid method	$P(\widetilde{A}) = \frac{\int_{a_1}^{a_8} x\mu_{\widetilde{A}}(x)dx}{\int_{a_1}^{a_8} \mu_{\widetilde{A}}(x)dx}$

First of maxima (FOM) method	$P(\widetilde{A}) = a_4$
Last of maxima (LOM) method	$P(\widetilde{A}) = a_5$

5. Proposed Ranking Methods to Convert an OFN into Crisp Value

Let $\widetilde{A}$ be an octagonal fuzzy number then we propose some defuzzified ranking methods namely average of odd positional values in OFN, average of even positional values in OFN, Quartile deviations methods, average of the smallest possible α-cut approach for an OFN and average of the largest possible α-cut approach for an OFN to obtain a crisp value as defined as follows

5.1 Average of Odd Positional Values in OFN
This defuzzification can be expressed as $P(\widetilde{A}) = \frac{(a_1+a_3+a_5+a_7)}{4}$

5.2 Average of Even Positional Values in OFN
This defuzzification can be expressed as $P(\widetilde{A}) = \frac{(a_2+a_4+a_6+a_8)}{4}$

5.3 Quartile Deviations
First quartile deviation $P_{Q_1}(\widetilde{A}) = \left(\frac{n+1}{4}\right)^{th}$ observational value in OFN
Second quartile deviation $P_{Q_2}(\widetilde{A}) = 2\left(\frac{n+1}{4}\right)^{th}$ observational value in OFN
Third quartile deviation $P_{Q_3}(\widetilde{A}) = 3\left(\frac{n+1}{4}\right)^{th}$ observational value in OFN

5.4 Average of the Smallest Possible α-Cut Approach for an OFN
The crisp value can be approximated by the average of α-Cut of bounded left continuous non-decreasing and non-increasing functions over the interval $[0, k]$i.e.
$P(\widetilde{A}) = \frac{1}{2}\left\{\left(a_1 + \frac{\alpha}{k}(a_2 - a_1)\right) + \left(a_8 - \frac{\alpha}{k}(a_8 - a_7)\right)\right\}$over $\alpha \in [0, k]$.

5.5 Average of the Largest Possible α-Cut Approach for an OFN
The crisp value can be approximated by the average of α-Cut of bounded left continuous non-decreasing and non-increasing functions over the interval$[k, 1]$ i.e.
$P(\widetilde{A}) = \frac{1}{2}\left\{\left(a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3)\right) + \left(a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)\right)\right\}$over $\alpha \in [k, 1]$.

6. Application in Marketing

There are two companies namely, McDonald's and Burger King in market. Both the companies have three strategies viz. Discounted price, status quo and Aggressive commercial. In such situations uncertainty is measure factor. These situations can be modeled efficiently by using ranking functions. In this particular problem we are taking octagonal fuzzy numbers as payoffs of both the companies. Verbal phrases for octagonal fuzzy numbers in terms of profit are,
VVL-Very Very Low,
VL-Very Low, L-Low,
LM-Low Mean,
HM-High Mean,
H-High,
VH-Very High,
VVH-Very Very High

Table: 6.1 Verbal Phrases for Octagonal Fuzzy Numbers

	Discounted price	status quo	Aggressive commercial
Discounted price	(VVL,VL,L,LM,HM,H,VH,VVH)	(VVL,VL,L,LM,HM,H,VH,VVH)	(VVL,VL,L,LM,HM,H,VH,VVH)
status quo	(VVL,VL,L,LM,HM,H,VH,VVH)	(VVL,VL,L,LM,HM,H,VH,VVH)	(VVL,VL,L,LM,HM,H,VH,VVH)
Aggressivec ommercial	(VVL,VL,L,LM,HM,H,VH,VVH)	(VVL,VL,L,LM,HM,H,VH,VVH)	(VVL,VL,L,LM,HM,H,VH,VVH)

Table: 6.2 Payoff matrix for McDonald's and Burger King

	Discounted price	status quo	Aggressive commercial
Discounted price	(5,6,7,8,9,10,11,12)	(0,1,2,3,4,5,6,7)	(0,1,2,3,4,5,6,7)
status quo	(3,4,5,6,7,8,9,10)	(4,5,6,7,8,9,10,11)	(4,5,6,7,8,9,10,11)
Aggressive commercial	$(-11,-10,-9,-8,-7,-6,-5,-4)$	(1,2,3,4,5,6,7,8)	$(-2,-1,0,1,2,3,4,5)$

$$\begin{array}{cc} & \text{Burger King} \\ \text{McDonald's} & \begin{bmatrix} (5,6,7,8,9,10,11,12) & (-1,0,1,2,3,4,5,6,) & (0,1,2,3,4,5,6,7) \\ (3,4,5,6,7,8,9,10) & (2,3,4,5,6,7,8,9) & (4,5,6,7,8,9,10,11) \\ (-11,-10,-9,-8,-7,-6,-5,-4) & (1,2,3,4,5,6,7,8) & (-2,-1,0,1,2,3,4,5) \end{bmatrix} \end{array}$$

Where

$\tilde{a}_{11} = (5,6,7,8,9,10,11,12)$;$\tilde{a}_{12} = (-1,0,1,2,3,4,5,6,)$;$\tilde{a}_{13} = (0,1,2,3,4,5,6,7)$;
$\tilde{a}_{21} = (3,4,5,6,7,8,9,10)$;$\tilde{a}_{22} = (2,3,4,5,6,7,8,9)$;$\tilde{a}_{23} = (4,5,6,7,8,9,10,11)$;
$\tilde{a}_{31} = (-11,-10,-9,-8,-7,-6,-5,-4)$;$\tilde{a}_{32} = (1,2,3,4,5,6,7,8)$;$\tilde{a}_{33} = (-2,-1,0,1,2,3,4,5)$

Table: 6.3 Best strategies and value of game

Defuzzification Method	Optimal Strategy McDonald's	Optimal Strategy Burger King	Value of the Game
Measure of an OFN (Magnitude ranking method)	II	II	5.5
Pascal´s triangular graded mean method	II	II	5.5
Simple average method	II	II	5.5
Mean-max membership principle (MOM Method)	II	II	5.5
Centroid method	II	II	5.5
First of maxima (FOM) method	II	II	5.0
Last of maxima (LOM) method	II	II	6.0
Average of Odd Positional Values in OFN	II	II	5.0
Average of Even Positional Values in OFN	II	II	6.0
First Quartile Deviation	II	II	3.25
Second Quartile Deviation	II	II	5.5
Third Quartile Deviation	II	II	7.75
Average of the Smallest Possible α-Cut Approach for an OFN	II	II	5.5
Average of the Largest Possible α-Cut Approach for an OFN	II	II	5.5

If payoff matrix is given by,

$$\begin{array}{cc} & \text{Burger King} \\ \text{McDonald's} & \begin{bmatrix} (-2,-1,0,1,2,3,4,5) & (0,1,2,3,4,5,6,7) & (2,3,4,5,6,7,8,9) \\ (1,2,3,4,5,6,7,8) & (9,10,11,12,13,14,15,16) & (6,7,8,9,10,11,12,13) \\ (8,9,10,11,12,13,14,15) & (-1,0,1,2,3,4,5,6) & (11,12,13,14,15,16,17,18) \end{bmatrix} \end{array}$$

Where

$\tilde{a}_{11} = (-2,-1,0,1,2,3,4,5)$;$\tilde{a}_{12} = (0,1,2,3,4,5,6,7)$;$\tilde{a}_{13} = (2,3,4,5,6,7,8,9)$;$\tilde{a}_{21} = (1,2,3,4,5,6,7,8)$;
$\tilde{a}_{22} = (9,10,11,12,13,14,15,16)$;$\tilde{a}_{23} = (6,7,8,9,10,11,12,13)$;$\tilde{a}_{31} = (8,9,10,11,12,13,14,15)$;
$\tilde{a}_{32} = (-1,0,1,2,3,4,5,6)$;$\tilde{a}_{33} = (11,12,13,14,15,16,17,18)$

Table: 6.4 Best strategies and value of game

Defuzzification Method	Optimal Strategy McDonald's	Optimal Strategy Burger King	Value of the Game
Measure of an OFN (Magnitude ranking method)	(0,0.529,0.470)	(0.588,0.411,0)	7.794
Pascal´s triangular graded mean method	(0,0.529,0.470)	(0.588,0.411,0)	7.794
Simple average method	(0,0.529,0.470)	(0.588,0.411,0)	7.794
Mean-max membership principle (MOM Method)	(0,0.529,0.470)	(0.588,0.411,0)	7.794
Centroid method	(0,0.529,0.470)	(0.588,0.411,0)	7.794
First of maxima (FOM) method	(0,0.529,0.470)	(0.588,0.411,0)	7.294
Last of maxima (LOM) method	(0,0.529,0.470)	(0.588,0.411,0)	8.294

Average of Odd Positional Values in OFN	(0,0.529,0.470)	(0.588,0.411,0)	7.294
Average of Even Positional Values in OFN	(0,0.529,0.470)	(0.588,0.411,0)	8.294
First Quartile Deviation	(0,0.529,0.470)	(0.588,0.411,0)	5.544
Second Quartile Deviation	(0,0.529,0.470)	(0.588,0.411,0)	7.794
Third Quartile Deviation	(0,0.529,0.470)	(0.588,0.411,0)	10.044
Average of the Smallest Possible α -Cut Approach for an OFN	(0,0.529,0.470)	(0.588,0.411,0)	7.794
Average of the Largest Possible α -Cut Approach for an OFN	(0,0.529,0.470)	(0.588,0.411,0)	7.794

7.Graphical Representation

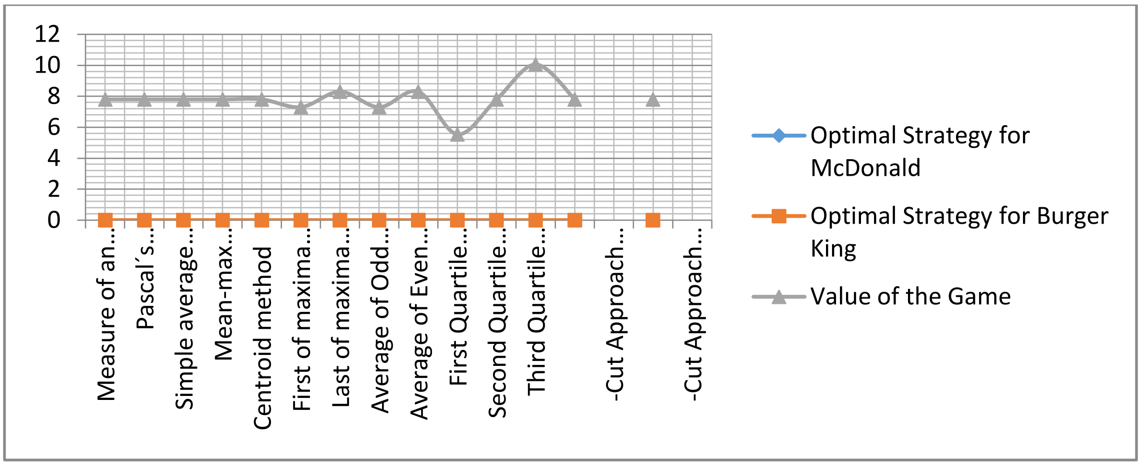

Figure: 7.1 Best strategies and value of game

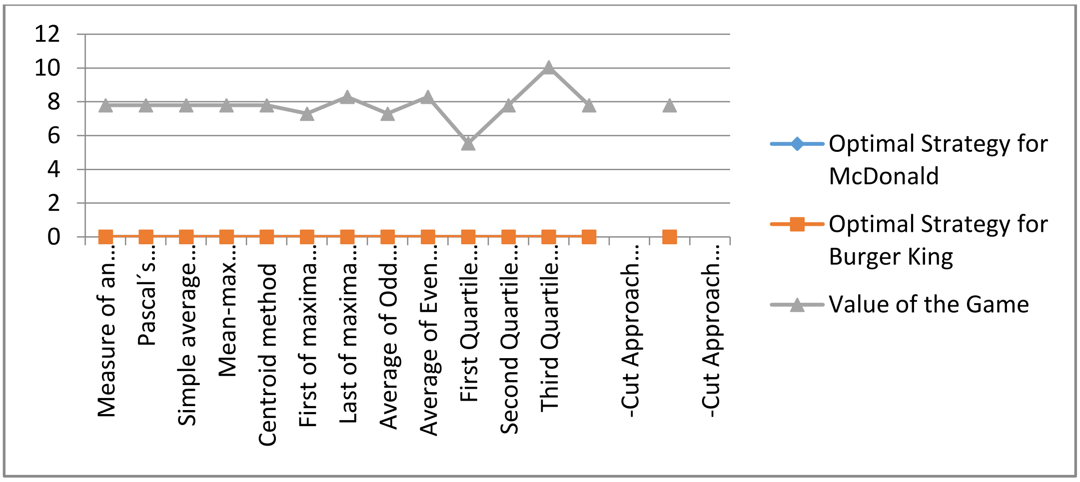

Figure: 7.2 Best strategies and value of game

8.Conclusion

It is established that competitive situations of marketing can be modeled in form of fuzzy matrix games. By solving matrix games, best strategies and optimal value can be obtained. In the present paper, an approach for solving octagonal fuzzy matrix game problem is proposed. It is established that our results are same as by using different kinds of techniques like as ranking for a fuzzy number, Pascal′s triangular graded mean formula, simple average method, centroid method, FOM, LOM, MOM methods, odd, and even positional values methods, quartile deviation methods, and α-cut approach. Decision maker can suitably modify the parameter k to get the desired result. Different fuzzy game values yields for the same fuzzy game corresponding to distinct values of the parameter k.

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