Modal Analysis Of Carbon/Epoxy Plate By Varying Fibre Orientation

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ABSTRACT

Composite laminated plates are widely used in the field of aerospace and navy for some advantages such as higher ratio of stiffness and strength to weight. A variety of laminated plate theories have been developed and applied to engineering practice. Mode shapes do describe the configurations into which a structure will naturally get displaced. Typically, lateral displacement patterns are of primary interest. Based on the load carrying capacity, Structures cannot be regarded safe but should be safe considering the structural dynamic aspects as well. Modal analysis is used to find the offending frequencies and eliminate them by varying the stiffness or mass to ensure the structure is safe from the natural frequency problems.In this paper modal analysis of a SS rectangular plate made of carbon epoxy is carried out using FEA and the results are compared with analytical solution.The fibre angle is changed systematically to see the effect of fibre orientation on natural frequencies and the corresponding mode shapes. The first six natural frequencies and mode shapes of composite laminated plate are obtained. Presented results showed that the properly chosen fibre angle contribute to better dynamic performance, which provides greater flexibility in designing composite structures to suit the engineering need.

Keywords: Carbon/epoxy Composite Laminated Plate, Numerical Model, Fibre Orientation, Natural Frequencies.

1.INTRODUCTION

Recently many techniques have been developed for optimizing laminated composite plates. Numericalsimulation is a modern design method which allows for more complex composite laminated structures to be designed.The research on vibration characteristics of laminatedcomposite plate with various fibre orientation becomes more significant now a days due to their applications in various fields of engineering [1,2]. To avoid resonance for dynamic structures in aerospace, naval, civil, and mechanical structures it is possible to decrease or increase natural frequency by changing fibre orientation. It is evident from the literature. So it was understood that it is possible to optimize the natural frequency of a structure by varying fibre angle in each ply of a laminate. Therefore, many researchers have begun to carry out related research.Martin[3]reportedby changing fibre volume fraction of a plies of laminate it is possible to decrease or increase laminated composite structure natural frequency. People also played with stacking sequence and preparing cross-ply laminates, angle ply laminates, Quasi-isotropic laminates to vary the natural frequency. It is also possible change natural frequencies of structures by varying boundary conditions. Wu[4] found that theshells exhibit different natural frequencies by varying boundary conditions. Research also revealed that with varying fibre orientation in ply leads to change in buckling characteristics of laminated composite structures [5]. For laminated composite plates [6-8] found a FEM model to analyse dynamic response of system by changing fibre orientation which come up with development in design composite materials forstructural applications. However, more study has to be conducted to understand the vibration modes of laminated composite plates with variable fibre orientation. This paper aims to study vibration characteristic of laminatedcomposite plates with various fibre orientation, and to show that the fibre orientation variation as a key parameter may be used to achieve required vibration mode shapes and specific frequencies. Laminated compositefinite element model is constructed based on CLT and its accuracy is investigated. The effects of fibre orientation on the natural frequencies and mode shapes were obtained. The results show that for structural design of composite laminated plates the fibre angle plays a significant role for achieving desired free vibration characteristics.

2. MATERIALS AND METHODS

For the investigation of fibre orientation on natural frequency the carbon/epoxy composite plate of p=2m length,q=1m width and t=10 mm thickness is taken.

The material properties of Carbon/epoxy are given in Table.1.The meaning of the symbols in Table 1. The elasticity modulus in X directionisrepresented asE_X;The elasticity modulus in Y directionisrepresentedE_Y;The elasticity modulus in Z directionisrepresentedas E_Z; The shear moduli are represented as G_{XY}, G_{YZ}, and G_{XZ}; The Poisson ratioin different planes are represented as ν_{XY}, ν_{Yz}, ν_{Xz}, respectively , and ρ represent density.

Table1: Material properties of carbon/epoxy composite Lamina

Elastic constants	value
E_X	121Gpa
E_Y	8.6GPa
E_Z	8.6GPa
ν_{XY}	0.27
ν_{Yz}	0.4
ν_{Xz}	0.27
G_{XY}	4.7GPa
G_{YZ}	3.1GPa
G_{XZ}	4.7GPa
Density(ρ)	1490 kg/m^3

3.ESTIMATION NATURAL FREQUENCY FROM CLT

When all edges of plate are simply supported the natural frequency of laminate is found by

$$\omega_{mn} = \frac{\pi^2}{\sqrt{\rho_m h}}\left[D_1\left(\frac{m}{p}\right)^4 + 2\,D_3\left(\frac{m}{p}\right)^2\left(\frac{n}{q}\right)^2 + D_2\left(\frac{n}{q}\right)^4\right]^{\frac{1}{2}}$$

$$f_{mn} = \frac{\omega_{mn}}{2\pi}$$

The natural frequency of carbon/epoxy plate is estimated for (0°/0°/0°/0°) by taking m=1, n=1, p=2m, q=1m.

D_1=D_{11}=10135.85MPa

D_2=D_{22}=720.4 MPa

D_3=D_{12}+2(D_{33})= 194.5 + 2(391.67)= 977.84 MPa

$$\omega_{11} = \ = \frac{\pi^2}{\sqrt{1490*0.01}}\left[10135.85\left(\frac{1}{2}\right)^4 + 2*977.84\left(\frac{1}{2}\right)^2\left(\frac{1}{1}\right)^2 + 720.4\ \left(\frac{1}{1}\right)^4\right]^{\frac{1}{2}}$$

ω_{11}= 109.89 rad/s

f_{mn}= $\frac{109.89}{2\pi}$ = 17.47Hz.

So, for a carbon/epoxy plate with fibre orientations (0°/0°/0°/0°) the 1st mode of natural frequency is occurs at 17.47 Hz.

4. FINITE ELEMENT FORMULATIONS OF RECTANGULAR PLATE

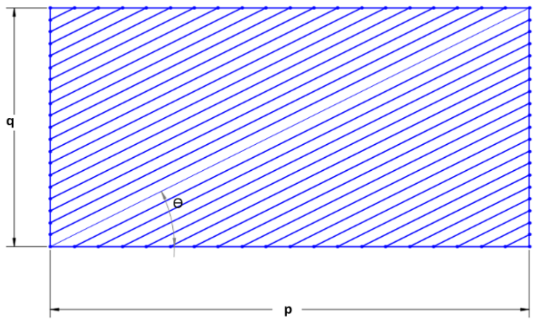

Fig.1. Rectangular plate

It is assumed that under load the plate elements deform according to simple polynomial deflection expression in terms of orthogonally coordinates x&y.

$w = A_1 + A_2\frac{x}{p} + A_3\frac{y}{q} + A_4\frac{x^2}{p^2} + A_5\frac{xy}{pq} + A_6\frac{y^2}{q^2} + A_7\frac{x^3}{p^3} + A_8\frac{x^2\,y}{p^2 q} + A_9\frac{xy^2}{pq^2} + A_{10}\frac{y^3}{q^3} + A_{11}\frac{x^3\,y}{p^3\,q} + A_{12}\frac{xy^3}{pq^3}$

$w=[m]\{A\}$, (1)

where {A} is the column matrix of the constants Ai. The generalized displacements at a node are the lateral deflection w, and two slopes.

$\chi=\frac{\partial w}{\partial x}\psi=\frac{\partial w}{\partial y}$

The constants $A_i$of the deflection expression are evaluated by satisfying the boundary conditions at the node points 1, 2, 3 and 4 to give

$\{A\} = \{B^{-1}\}\{v\}$, (2)

Where

$\{v\}=\{w_1, \psi_1, \chi_1, w_2, \ldots, \chi_4\}$.

The bending strain energy of the element is $U=\frac{1}{2}\iint [C]^T[D][C]dx\,dy$

where$\{C\} = \left\{\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, \frac{\partial^2 w}{\partial xy}\right\}^T$

$$[D] = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix}$$

$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} \overline{Q}_{ij\ k}\,(z_k^3 - z_{k-1}^3)$ (i=1,2,6, j=1,2,6, N= No. of layers)

The reduced transformed stiffnesses $\overline{Q}$are given by,

$\overline{Q}_{11} = Q_{11}cos^4\theta + 2(Q_{12} + 2\,Q_{66})sin^2\theta cos^2\theta + Q_{22}sin^4\theta$

$\overline{Q}_{22} = Q_{11}sin^4\theta + 2(Q_{12} + 2\,Q_{66})sin^2\theta cos^2\theta + Q_{22}cos^4\theta$

$\overline{Q}_{12} = (Q_{11}+Q_{22} - 4\,Q_{66})sin^2\theta cos^2\theta + Q_{12}(cos^4\theta + sin^4\theta)$

$\overline{Q}_{16} = (Q_{11} - Q_{12} - 2\,Q_{66})\,sin\,\theta cos^3\theta + (Q_{12} - Q_{22} + 2\,Q_{66})\,sin^3\theta cos\theta$

$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2\,Q_{66})\,sin^3\theta cos\theta + (Q_{12} - Q_{22} + 2\,Q_{66})\,sin\,\theta cos^3\theta$

$\overline{Q}_{66} = (Q_{11}+Q_{22} - 2\,Q_{12} - 2\,Q_{66})sin^2\theta cos^2\theta + Q_{66}(cos^4\theta + sin^4\theta)$

Where,

$Q_{11} = \frac{E_1}{1-\vartheta_{12}\vartheta_{21}}$

$Q_{12} = \frac{\vartheta_{12}E_2}{1-\vartheta_{12}\vartheta_{21}}$

$Q_{22} = \frac{E_2}{1-\vartheta_{12}\vartheta_{21}}$

$Q_{66} = G_{12}$

$\vartheta_{12}E_2=\vartheta_{21}E_1$

The curvatures can be obtained by differentiating equation(1):

$\{C\} = [E]\{A\} = [E]\{B^{-1}\}\{v\}$, (3)

$U= \frac{1}{2}\iint [C]^T[D][C]dx\,dy= \frac{1}{2}[v]^T[B^{-1}]^T(\int_{y=0}^{b}\int_{x=0}^{a}[E]^T[D][E]\,dx\,dy)[B^{-1}]\,\{v\}$ (4)

$U= \frac{1}{2}[v]^T[k][v]$

Using Castigliano's theorem,

$\frac{\partial U}{\partial v_i}= F_i$

Gives

$\{F\}=[k]\{v\}$ (5)

Where[k] is the stiffness matrix of the element. The inertia matrix is given as

$\{F_{in}\} = \rho\, P^2[B^{-1}]^T(\int_{y=0}^{q}\int_{x=0}^{p}[m]^T[m]\,dx\,dy)\,[B^{-1}]\,\{v\} = \lambda\,[M_e]\,\{v\}$

Where λ is proportional to p^2 and $[M_e]$ is inertia matrix of a rectangular plate element. The governing equation of vibration in matrix form is

$\{k\}\{v\} - \lambda\,[M_e]\{v\} =0$ (6)

where [K] is the assembled stiffness matrix and [Me] is the assembled inertia matrix, λ is the eigen value and {v} is the eigen vector. Equation (6) is solved using a standard algorithm for obtaining eigen values and eigen vectors.The equation(6) finite element formulation is used to study the effect of fibre orientation and boundary conditions on the frequencies of rectangular plates.

5.NUMERICAL RESULTS

Numerical simulations were performed for four-layer composite laminated plate with various fibre orientation angles, natural frequencies and mode shapes of composite laminated plates are obtained.

The comparison of first natural frequency of(0°/0°/0°/0°) from different calculation methods is represented in Table.2.

Table.2. Natural frequency of (0°/0°/0°/0°) from different calculation methods

Mode shape (m,n)	**Numerical solution [Hz]**	**Theoretical solution (CLT)[Hz]**	**Relative difference (%)**
(1,1)	**17.46**	**17.47**	**0.05**
(2,1)	**45.94**	**46.07**	**0.28**
(1,2)	**48.23**	**48.36**	**0.26**
(2,2)	**69.79**	**69.9**	**0.16**
(3,1)	**96.41**	**96.7**	**0.29**
(1,3)	**102.39**	**102.76**	**0.36**

To study vibration characteristic of composite laminated plates with various fibre orientation, a four-layer composite laminate plates with various fibre orientation angles varying as(0°/15°/15°/0°), (0°/30°/30°/0°) , (0°/45°/45°/0°) , (0°/60°/60°/0°) , (0°/75°/75°/0°) (0°/90°/90°/0°) are studied. The first six natural frequencies are shown in Table.3 and the first six mode shapes are illustrated in Fig.2.to7 It is found that the change of fibre orientation can lead to a significant decrease or increase in the natural frequencies of composite laminated plates, therefore, in engineering application, to avoid resonance, designers can change the natural frequencies to higher or lower values by changing fibre orientation of composite laminated plates.

Table.3. Natural frequencies of carbon/epoxy composite laminated plate

Fibre orientation	**Natural frequency (Hz)**					
	1	**2**	**3**	**4**	**5**	**6**

(0°/0°/0°/0°)	**17.46**	**45.94**	**48.23**	**69.79**	**96.41**	**102.3**
(0°/15°/15°/0°)	**17.86**	**46.47**	**49.21**	**71.85**	**96.77**	**117.6**
(0°/30°/30°/0°)	**18.99**	**46.83**	**52.56**	**75.35**	**96.67**	**109.8**
(0°/45°/45°/0°)	**20.10**	**47.06**	**58.16**	**79.21**	**96.17**	**120.8**
(0°/60°/60°/0°)	**21.06**	**46.72**	**65.13**	**82.92**	**94.89**	**122.3**
(0°/75°/75°/0°)	**21.90**	**46.57**	**71.48**	**87.73**	**93.26**	**125.3**
(0°/90°/90°/0°)	**22.08**	**45.93**	**73.53**	**88.21**	**92.19**	**123.5**

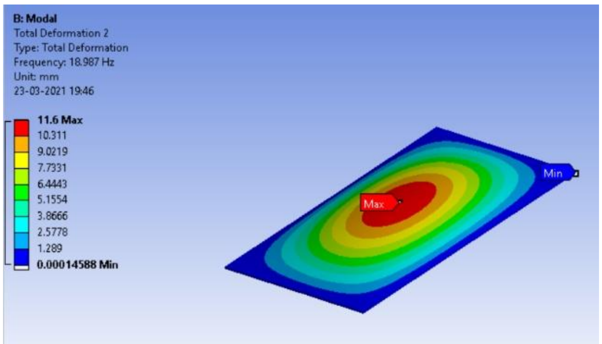

Fig.2. Mode shape(1,1) for (0°/30°/30°/0°) plate

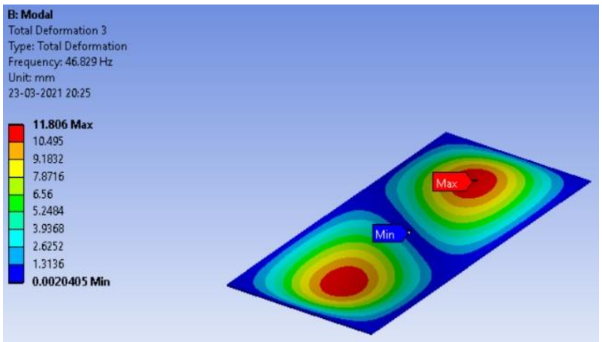

Fig.3. Mode shape(2,1) for (0°/30°/30°/0°) plate

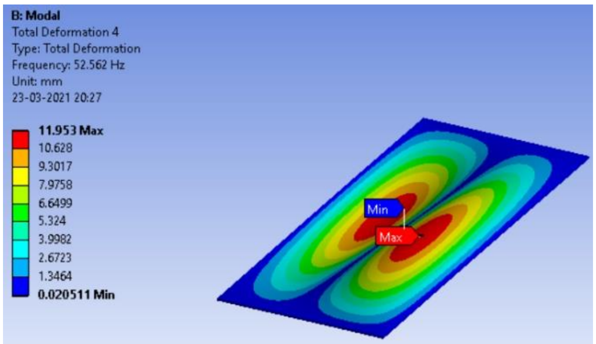

Fig.4. Mode shape(1,2) for (0°/30°/30°/0°) plate

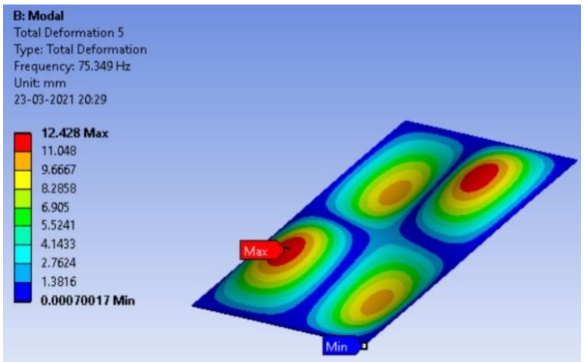

Fig.5. Mode shape(2,2) for (0°/30°/30°/0°) plate

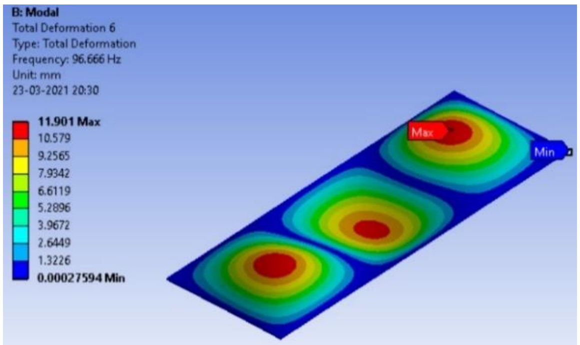

Fig.6. Mode shape(3,1) for (0°/30°/30°/0°) plate

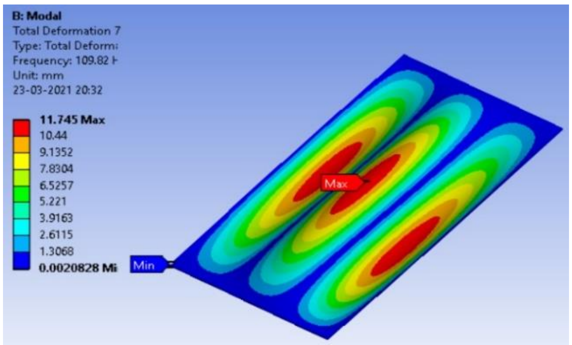

Fig.6. Mode shape(1,3) for (0°/30°/30°/0°) plate

6.CONCLUSIONS

The important conclusions obtained by the above analysis are summarized as follows:

(1)For carbon/epoxy laminated compositeplateswith various fibre orientationsfinite element models were established.Modal analysis was performed to obtain natural frequencies and mode shapes. The accuracy of FEM solutions was verified, the maximum relative differencebetweentheoretical results numerical and does not exceed 0.36%.

(2)The effects of fibre orientation on the mode shapes and natural frequencies of vibration of composite laminated plates were investigated. By increasing the fibre angle for inner layers, the natural frequency increases.The results show that the changes of fibre orientation bring a greater degree of flexibility for structure design of laminated composite plates, which can provide theoretical guidance for the better engineering structure design of composite material.

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