# The Upper Total Triangle Free Detour Number of a Graph

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**Abstract:** For a connected graph G = (V,E) of order at least two, a total triangle free detour set of a graph G is a triangle free detour set S such that the subgraph G[S] induced by S has no isolated vertices. The minimum cardinality of a total triangle free detour set of G is the total triangle free detour number of G. It is denoted by 〚tdn〛_∆f (G). A total triangle free detour set of cardinality 〚tdn〛_∆f (G) is called 〚tdn〛_∆f- set of G. In this article, the concept of upper total triangle free detour number of a graph G is introduced. It is found that the upper total triangle free detour number differs from total triangle free detour number. The upper total triangle free detour number is found for some standard graphs. Their bounds are determined. Certain general properties satisfied by them are studied.

**Keywords:** total triangle free detour set, total triangle free detour number, upper total triangle free detour set, upper total triangle free detour number.
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## 1. Introduction

For a graph $G = (V, E)$, we mean a finite undirected connected simple graph. The order of G is represented by $n$. We consider graphs with at least two vertices. For basic definitions we refer [3]. For vertices $u$ and $v$ in a connected graph $G$, the detour distance $D(u, v)$ is the length of the longest $u - v$ path in $G$. A $u - v$ path of length $D(u, v)$ is called a $u - v$ detour. This concept was studied by Chartrand et.al [1].

A chord of a path $P$ is an edge joining two non-adjacent vertices of $P$. A path $P$ is called a monophonic path if it is a chordless path. A longest $x - y$ monophonic path is called an $x - y$ detour monophonic path. A set $S$ of vertices of $G$ is a detour monophonic set of $G$ if each vertex $v$ of $G$ lies on an $x-y$ detour monophonic path for some $x$ and $y$ in $S$. The minimum cardinality of a detour monophonic set of $G$ is the detour monophonic number of $G$ and is denoted by $dm(G)$. The detour monophonic number of a graph was introduced in [8] and further studied in [7].

A total detour monophonic set of a graph $G$ is a detour monophonic set $S$ such that the subgraph $G[S]$ induced by $S$ has no isolated vertices. The minimum cardinality of a total detour monophonic set of $G$ is the total detour monophonic number of $G$ and is denoted by $dm_t(G)$. A total detour monophonic set of cardinality $dm_t(G)$ is called a $dm_t$-set of $G$. These concepts were studied by A. P. Santhakumaran et. al[6].

The concept of triangle free detour distance was introduced by Keerthi Asir and Athisayanathan [4]. A path $P$ is called a triangle free path if no three vertices of $P$ induce a triangle. For vertices $u$ and $v$ in a connected graph $G$, the triangle free detour distance $D_{\Delta f}(u, v)$ is the length of a longest $u - v$ triangle free path in $G$. A $u - v$ path of length $D_{\Delta f}(u, v)$ is called a $u - v$ triangle free detour. For any two vertices $u$ and $v$ in a connected graph $G$, $0 \leq d(u, v) \leq dm(u, v) \leq D_{\Delta f}(u, v) \leq D(u, v) \leq n - 1$.

The triangle free detour eccentricity of a vertex $v$ in a connected graph $G$ is defined by $e_{\Delta f}(v) = \max\{D_{\Delta f}(u, v): u, v \in V\}$. The triangle free detour radius of $G$is defined by $rad_{\Delta f}(G) = \min\{e_{\Delta f}(v):\ v \in V\}$ and The triangle free detour diameter of $G$is defined by $diam_{\Delta f}(G) = \max\{e_{\Delta f}(v):\ v \in V\}$

A total triangle free detour set of a graph $G$ is a triangle free detour set $S$ such that the subgraph $G[S]$ induced by $S$ has no isolated vertices. The minimum cardinality of a total triangle free detour set of $G$ is the total triangle free detour number of $G$. It is denoted by $tdn_{\Delta f}(G)$. A total triangle free detour set of cardinality $tdn_{\Delta f}(G)$ is called $tdn_{\Delta f}$- set of G.

A vertex v of a connected graph G is called a support vertex of G if it is adjacent to an end vertex of G. Two adjacent vertices are referred to as neighbors of each other. The set N(v) of neighbors of a vertex v is called the neighborhood of v. A vertex v of a graph G is called extreme vertex if the subgraph induced by its neighbourhood is complete.The following theorems will be used in the sequel.

**Theorem 1.1:** Let G be a connected graph of order n, then $2 \leq dn_{\Delta f}(G) \leq tdn_{\Delta f}(G) \leq cdn_{\Delta f}(G) \leq n$.

**Theorem 1.2**. If the set of all extreme vertices and support vertices form a total

triangle free detour set, then it is the unique minimum total triangle free detour set of G.

**2. The Upper Total Triangle Free Detour Number**

**Definition 2. 1.** A total triangle free detour set in a connected graph *G* is called a minimal total triangle free detour set of *G* if no proper subset of *S* is a total triangle free detour set of *G*. The upper total triangle free detour number

$\text{tdn}_{\Delta f}{}^{+}(G)$ of *G* is the maximum cardinality of a minimal total triangle free detour set of *G*.

**Example 2.1**. For the graph G given in Figure:2.1, $S_1 = \{u_5, u_6, u_4\}$, $S_2 = \{u_5, u_6, u_3\}$ are the minimum total triangle free detour sets of G and $S_3 = \{u_5, u_6, u_1, u_2\}$ is a minimal total triangle free detour set of *G*. Clearly $S_3$ is minimal total triangle free detour set of G with maximum cardinality. Thus

$\text{tdn}_{\Delta f}{}^{+}(G) = 4$.

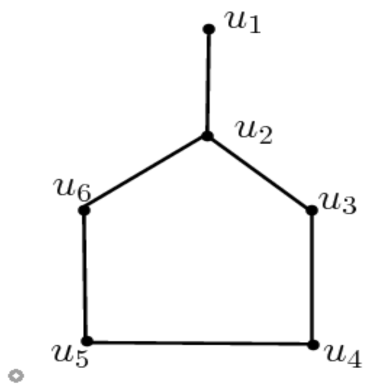


**Figure 2.1**

**Note 2.1:** Every minimal total triangle free detour set is a total triangle free detour set. But the total triangle free detour number and the upper total triangle free detour number need not be same.

**Theorem 2.1:** Every minimum total triangle free detour set is a minimal total triangle free detour set.

**Proof:**

Let *S* be a minimal total triangle free detour set. Then no proper subset of *S* is a total triangle free detour set of *G*. Thus *S* is a minimal total triangle free detour set.

**Remark 2.1:** Converse of the above theorem need not be true. For the graph G given in Figure:2.1, $S_3 = \{u_5, u_6, u_1, u_2\}$ is a minimal total triangle free detour set of *G*. But $S_3$ is not a minimum total triangle free detour set.

**Theorem 2.2:** Let *n* be the order of a connected graph *G*, then $2 \leq tdn_{\Delta f}(G) \leq tdn_{\Delta f}^{+}(G) \leq n$.

**Proof:** By theorem 1.1, we can conclude that $2 \leq tdn_{\Delta f}(G)$. Since the order of the given graph is *n*. The upper total triangle free detour number cannot exclude *n*. Thus $tdn_{\Delta f}^{+}(G) \leq n$. By theorem 2.1, every minimum total triangle free detour set is a minimal total triangle free detour set, $tdn_{\Delta f}(G) \leq tdn_{\Delta f}^{+}(G)$. Hence $2 \leq tdn_{\Delta f}(G) \leq tdn_{\Delta f}^{+}(G) \leq n$.

**Remark 2.2:** The bound in the theorem 2.2 is sharp. For a cycle $C_n$, $tdn_{\Delta f}(G) = tdn_{\Delta f}^{+}(G) = 2$ and for a path $P_n$ $(n \geq 4)$, $tdn_{\Delta f}(G) = tdn_{\Delta f}^{+}(G) = 4$. For the graph given in the figure 2.1 $tdn_{\Delta f}(G) = 3$, $tdn_{\Delta f}^{+}(G) = 4$. Thus $2 < tdn_{\Delta f}(G) < tdn_{\Delta f}^{+}(G) < n$.

**Theorem 2.3:** Let *n* be the order of a connected graph *G*, $tdn_{\Delta f}(G) = n$ if and only if $tdn_{\Delta f}^{+}(G) = n$.

**Proof:** Let $tdn^{+}_{\Delta f}(G) = n$. Then by theorem 2.2, $tdn_{\Delta f}(G) \leq n$ . If $tdn_{\Delta f}(G) < n$, then there exist a total triangle free detour set with cardinality less than $n$, which is a subset of minimal total triangle free detour set. This is impossible. Hence $tdn_{\Delta f}(G) = n$. Conversely, let $tdn_{\Delta f}(G) = n$. Then by theorem 2.2 $tdn^{+}_{\Delta f}(G) = n$.

**Theorem 2.4:** Let $G = K_n$. Then $tdn^{+}_{\Delta f}(G) = n$.

**Proof:** Every vertex of a complete graph is an extreme vertex. Then by the theorem 1.2, $tdn_{\Delta f}(G) = n$. Then by theorem 2.3, $tdn^{+}_{\Delta f}(G) = n$..

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