Relaxed Skolam Mean Labeling of 5 - Star Graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2}$

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Abstract: To prove that the 5 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2}$ where $\alpha_1 \le \alpha_2 \le \alpha_3$ and $\beta_1 \le \beta_2$ is a relaxed skolam mean graph if $|\beta_1 + \beta_2 - \alpha_1 - \alpha_2 - \alpha_3| = 6$ is the core objective of this article.

Keywords: Relaxed skolam mean graphs, relaxed skolam mean labeling, 5-star graph.

1. Introduction

Relaxed skolam mean label for a graph was defined and coined by V.Balaji et. al.[5]. In the paper [5] he defined the relaxed skolam mean labeling for the first time. In the same paper we can find the basic properties for a graph to be relaxed skolam mean.

2. Preliminaries

Definition 2.1 [4]: A graph G = (V, E) with p vertices and q edges is said to be a **skolam mean graph** if there exists a function $f : V \to \{1,2,3,...,p = |V|\}$ such that the induced map $f^* : E \to \{2,3,...,p = |V|\}$ given by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } \left(f(u)+f(v)\right) \text{is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } \left(f(u)+f(v)+1\right) \text{is even} \end{cases}$$

then, the resulting distinct edge labels are from the set $\{2,3,...,p = |V|\}$.

Definition 2.2 [5]: A graph $G=(V, E)$ with p vertices and q edges is said to be a relaxed skolam mean graph if there exists a function $f : V \to \{1,2,3,...,p+1 = |V|+1\}$ such that the induced edge map $f^* : E \to \{2,3,...,p = |V|+1\}$ given by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } \left(f(u)+f(v)\right) \text{is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } \left(f(u)+f(v)+1\right) \text{is even} \end{cases}$$

. The resulting distinct edge labels are from the set $\{2,3,...,p+1 = |V|+1\}$

Note 2.3: There are p vertices and available vertex labels are p + 1 and hence one number from the set $\{1,2,3,...,p+1=|V|+1\}$ is not used and we call that number as the relaxed label. When the relaxed label is p + 1, the relaxed mean labeling becomes a skolam mean labeling.

Result 2.4: In the relaxed skolam mean labeling p $\geq$ q.

Result 2.5: The three star graph $K_{1,a} \cup K_{1,b} \cup K_{1,c}$ satisfies relaxed skolam mean labeling if $a + b \le c \le a + b + c$.

3. Main Result

Theorem 3.1: The 5 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2}$ where $\alpha_1 \le \alpha_2 \le \alpha_3$ and $\beta_1 \le \beta_2$ is a relaxed skolam mean graph if $|\beta_1 + \beta_2 - \alpha_1 - \alpha_2 - \alpha_3| = 6$.

Proof: Let $\sigma_1 = \alpha_1; \sigma_2 = \alpha_1 + \alpha_2; \sigma_3 = \alpha_1 + \alpha_2 + \alpha_3$ and $\delta_1 = \beta_1; \delta_2 = \beta_1 + \beta_2$.

Consider the 5 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2}$.

The condition $|\beta_1 + \beta_2 - \alpha_1 - \alpha_2 - \alpha_3| = 6$ gives rise to the case $\delta_2 = \sigma_3 + 6$. In this case we will establish that the graph G is relaxed skolam mean.

Let the set of vertices of G be $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$ where $V_k = \{v_{k,i} : 0 \le i \le \alpha_k\}; 1 \le k \le 3$ and $V_4 = \{v_{4,i} : 0 \le i \le \beta_1\}; V_5 = \{v_{5,i} : 0 \le i \le \beta_2\}$. Let the edge set of G be $E = \bigcup_{k=1}^{3} \{v_{k,0}v_{k,i} : 1 \le i \le \alpha_k\} \cup \bigcup_{k=4}^{5} \{v_{k,0}v_{k,i} : 1 \le i \le \beta_{k-3}\}$.

Case 1: Let $\delta_2 = \sigma_3 + 6$.

G has $\sigma_3 + \delta_2 + 5 = 2\sigma_3 + 11$ vertices and $\sigma_3 + \delta_2 = 2\sigma_3 + 6$ edges.

We define the rsv function

$f : V \rightarrow \{1, 2, ..., p+1 = \sigma_3 + \delta_2 + 5 + 1 = 2\sigma_3 + 12\}$ as follows:

$f(v_{1,0}) = 1; \quad f(v_{2,0}) = 3; \; f(v_{3,0}) = 5;$

$f(v_{4,0}) = \sigma_3 + \delta_2 + 5 = 2\sigma_3 + 9;$

$f(v_{5,0}) = \sigma_3 + \delta_2 + 6 = 2\sigma_3 + 11$

$f(v_{1,\kappa}) = 2\kappa + 5 \qquad 1 \le \kappa \le \alpha_1$

$f(v_{2,\kappa}) = 2\sigma_1 + 2\kappa + 5 \qquad 1 \le \kappa \le \alpha_2$

$f(v_{3,\kappa}) = 2\sigma_2 + 2\kappa + 5 \qquad 1 \le \kappa \le \alpha_3$

$f(v_{4,\kappa}) = 2\kappa \qquad 1 \le \kappa \le \beta_1$

$f(v_{5,\kappa}) = 2\delta_1 + 2\kappa \qquad 1 \le \kappa \le \beta_2$

Here $2\sigma_2 + 7$ is the relaxed label.

We get the edge labels as follows:

The edge labels of $v_{1,0}v_{1,\kappa}$ is $\kappa + 3$ for $1 \le \kappa \le \alpha_1$ ($4, 5, ..., \alpha_1 + 2 = \sigma_1 + 3$), $v_{2,0}v_{2,j}$ is $\sigma_1 + \kappa + 4$ for $1 \le \kappa \le \alpha_2$ ($\sigma_1 + 5, \sigma_1 + 6, ..., \sigma_1 + \alpha_2 + 4 = \sigma_2 + 4$), $v_{3,0}v_{3,j}$ is $\sigma_2 + \kappa + 5$ for $1 \le \kappa \le \alpha_3$ ($\sigma_2 + 6, \sigma_2 + 7, ..., \sigma_2 + \alpha_2 + 5 = \sigma_3 + 5$), $v_{4,0}v_{4,\kappa}$ is $\sigma_3 + \kappa + 5$ for $1 \le \kappa \le \beta_1$ ($\sigma_3 + 6, \sigma_3 + 7, ..., \sigma_3 + \beta_1 + 5 = \sigma_3 + \delta_1 + 5$), $v_{5,0}v_{5,\kappa}$ is $\sigma_3 + \delta_1 + \kappa + 6$ for $1 \le \kappa \le \beta_2$ ($\sigma_3 + \delta_1 + 7, \sigma_3 + \delta_1 + 8, ..., \sigma_3 + \delta_1 + (\beta_2) + 6 = \sigma_3 + \delta_2 + 6 = 2\sigma_3 + 12$).

The edge labels are therefore $4, 5, ..., \sigma_1 + 3$, $\sigma_1 + 5, \sigma_1 + 6, ..., \sigma_2 + 4$, $\sigma_2 + 6, \sigma_2 + 7, ..., \sigma_3 + 5$, $\sigma_3 + 6, \sigma_3 + 5, ..., \sigma_3 + \delta_1 + 5$, $\sigma_3 + \delta_1 + 6, \sigma_3 + \delta_1 + 7, ..., 2\sigma_3 + 12$.

These edge labels, the images of the rse function of the graph G are therefore distinct. Hence G is a relaxed skolam mean graph.

Example:

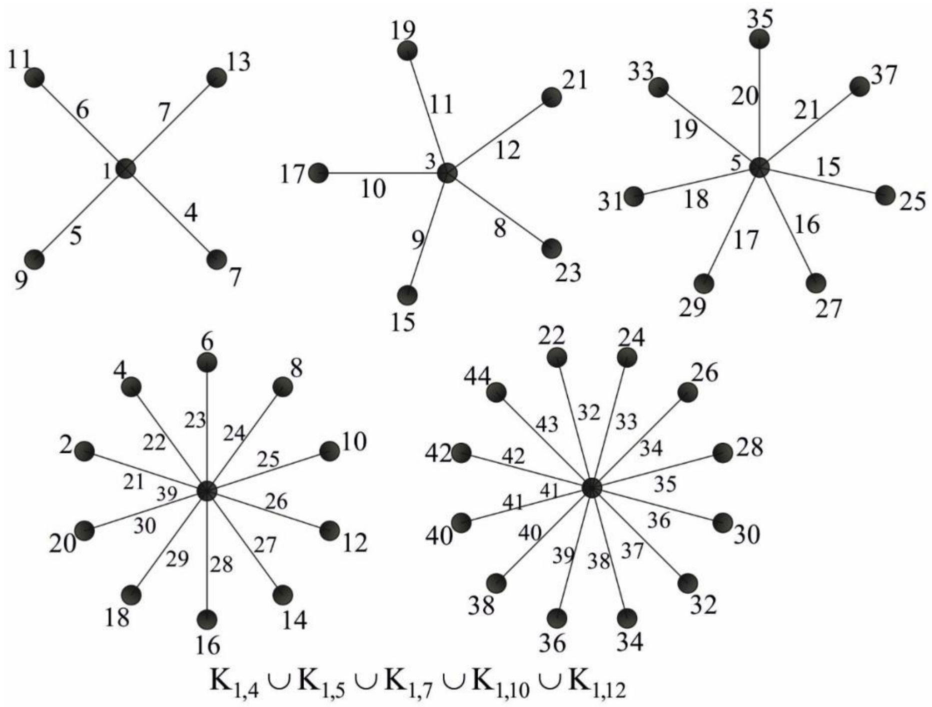

Figure 3.2

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