Optimization of VAM as Pattern – Connected Edge Weighed Graph

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Abstract: The main objective of this approach is to find the optimal cost of the transportation grid in Balanced and unbalanced ways of the TPM derived from the edge weighted connected non-complete graph. The proposed approach of this article is in a particular path representation of the given transportation problem to ensure the Optimal Basic Feasible Solution by Vogel's Approximation Method for the derived TPM from the edge weighted connected non-complete graph. Optimal Basic Feasible Solution is also provided with the existed methods in operation research field with particular pattern at some extent

Keywords: Degree, Demand, Edge weighted graph, OBFS, Pendent, Supply, Vertex.

1. Introduction

The prime focus of this study is to employ the tenets of Operation Research to settle down the popping up logistics issues and inconveniences [1]-[4]. A consistent study on into various journals and publications has resulted in this review.

This study is broadly divided into

(A) Scheming and Analysis

(B) Nature of local and the disbursement of goods

(C) Snapping and allocating.

The discrepancies in managing logistics can be resolved by a unique kind of linear programming problem [5]. The solution for the discrepancies can be aimed at bringing down/diminishing the exceeding revenue expenses in logistics that are related to transportation of goods. Targeted distribution of every source has a particular demand to be satisfied [6]-[8]. The cost involved in the distribution of products from a manufacturing unit is directly proportional to the units count distributed.

The manifestation of the process in graph theory modeling approach in operation research is one of the common themes that comes as rescue and profitably used. Graph Theoretic models are of great use and profit for analysis. This research article validates some of the initial concepts of graph theory and elaborates certain approaches of operations research problems from graph theory interpretations. Indian railways, Airlines, Defense forces, Textile industries employ operations research techniques to enhance their activities.

In this article the theoretical models of some of the transportation research problems are expanded with the use of graphs. In order to shrink or to enhance the liquidity of the cost these graphs in operation research proves to be a loon in movement of goods from scratch to the estimated destination

At the end of this research, a research agenda that envisions the future is traced in this prime domain [9] & [10].

2. Nomenclature:

VAM –Vogel's Approximation Method

LCM –Least Cost Method

NWC – North West Method.

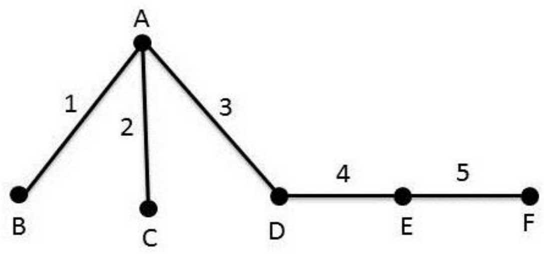
TPM – Transportation Problem Model.

OBFS – Optimal Basic Feasible Solution.

S(a) – Supply of a.

D(a) – Demand of a.

Example: 1.1



Graph $G_{1.1}$

Corresponding TPM :

	D	**F**	**Supply**
E	4	5	**9**
B	4	13	**1**
C	5	14	**2**
A	3	12	**6**
Demand	**7**	**5**	**12** / **18**

Explanation of entries of payoff:

Entry in [1 , 1] : $E \mapsto D$ having weight 4

Entry in [1 , 2] : $E \mapsto F$ having weight 5

Entry in [2 , 1] : $B \mapsto D$ having weight 4 $(B \mapsto A \mapsto D)$

Entry in [2 , 2] : $B \mapsto F$ having weight 13 $(B \mapsto A \mapsto D \mapsto E \mapsto F)$

Entry in [3 , 1] : $C \mapsto D$ having weight 5 $(C \mapsto A \mapsto D)$

Entry in [3 , 2] : $C \mapsto F$ having weight 14 $(C \mapsto A \mapsto D \mapsto E \mapsto F)$

Entry in [4 , 1] : $A \mapsto D$ having weight 3

Entry in [4 , 2] : $A \mapsto F$ having weight 12 $(A \mapsto D \mapsto E \mapsto F)$

S(A) : $W(AB) + W(AC) + W(AD) = 1 + 2 + 3 = 6$

S(B) : $W(B \mapsto A) = 1$

S(C) : $W(CA) = 2$

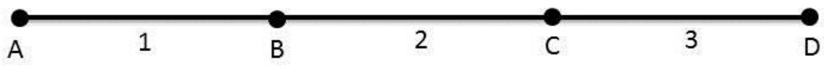
S(E) : $W(EF) + W(ED) = 5 + 4 = 9$

D(D) : $W(DA) + W(DE) = 7$

D(F) : $W(FE) = 5$

The **Graph $G_{1.1}$** provides an unbalanced TPM.

Example: 1.2



Graph G1.2

Corresponding TPM :

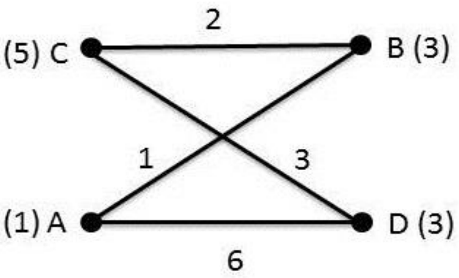
	B	**D**	**Supply**
C	2	3	**5**
A	1	6	**1**
Demand	**3**	**3**	**6**

OBFS by VAM: 14 (ie. $1^2+2^2+3^2$)

OBFS by LCM: 14

OBFS by NWC: 14

Graphical Representation of the given TPM:



Graph G1.3

Allotment table for the path $ABCD$ (Start from A):

Step 1:

	B	**D**	**Supply**
C	2 $(B \to C)$	3 $(C \to D)$	**5**
A	1 $(A \to B)$	6	**1**
Demand	**3**	**3**	**6**

Step 2:

	B	**D**	**Supply**
C	2 $(B \to C)$	3 $(C \to D)$	**5**
A	1_1 (A → B)	6	**0**
Demand	**2**	**3**	**6**

Step 3:

	B	**D**	**Supply**
C	2_2 $(B \to C)$	3 $(C \to D)$	**3**
A	1_1 $(A \to B)$	6	**0**
Deman d	**0**	**3**	**6**

Step 4:

	B	**D**	**Supply**
C	2_2 $(B \to C)$	3_3 $(C \to D)$	**0**
A	1_1 $(A \to B)$	6	**0**
Deman d	**0**	**0**	**6**

Explanation of cost:

Using Step 4,

[(1, 1), 2] = 2 X 2 = 4, [(1, 2), 3] = 3 X 3 = 9, [(2, 1), 1] = 1 X 1 = 1

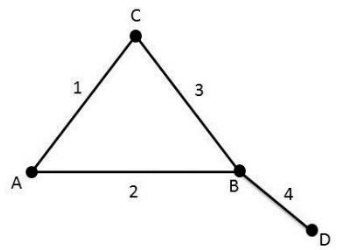
Optimum Cost =4+9+1 = $1^2 + 2^2 + 3^3 = 14$ Units.

This graph provides balanced TPM.

Proposition 1:

Every weighted connected non complete graph not having unique equivalent corresponding TPM (with respect to its given edge labeling).

Example: 1.3



Graph G1.4

Corresponding TPM 1 :

	B	**C**	**Supply**
A	2	1	**3**
D	4	7	**4**
Deman d	**9**	**4**	**13** **7**

Corresponding TPM 2:

	A	**B**	**Supply**
C	1	3	**4**
D	6	4	**4**
Demand	**3**	**9**	**12** **8**

This graph provides an unbalanced two different TPMs.

Theorem 1:

There is no equivalent TPM for any complete connected graph.

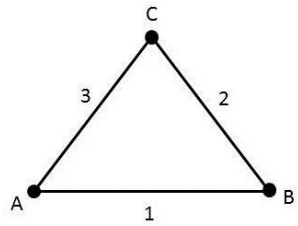
In other words, any (n-1) regular graph with n vertices will not have corresponding TPM.

Proof:

We cannot find any independent set other than (1xn) or (nx1) order.

This completes the proof.

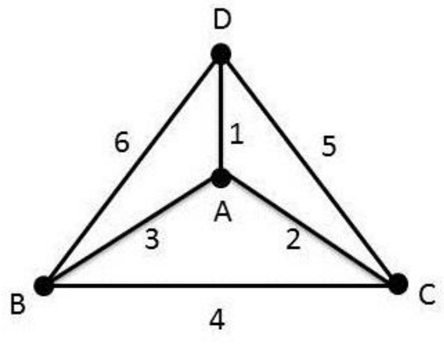
Example: 2.1



Graph G2.1

This is connected complete graph. So, it will not provide Equivalent TPM by theorem 1.

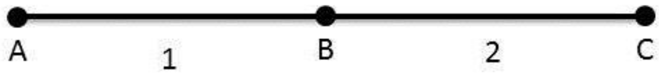
Example: 2.2



Graph G2.2

By theorem 1, there is no equivalent TPM for this graph.

Example: 2.3



Graph G2.3

By theorem 1, this graph will not Providing Equivalent TPM.

Theorem: 2

All star graphs will not provide TPM.

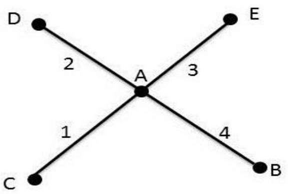
Proof:

We cannot find independent set.

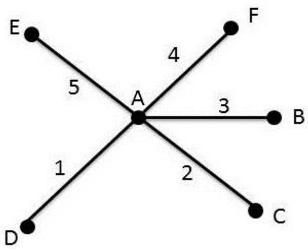
Ie. (1xn) or (nx1) payoff will be existed.

Hence the proof.

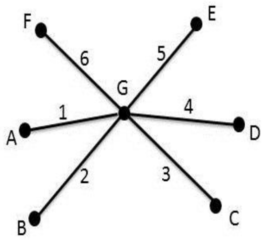
Example: 3.1



Graph G3.1



Graph G3.2



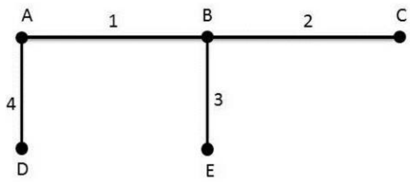
Graph G3.3

The above three graphs **Graph $G_{3.1}$, Graph $G_{3.2}$ & Graph $G_{3.3}$** not providing any TPM other than (1xn) or (nx1).

Theorem: 3

All star graphs with only one extension at its any one of the pendent vertex will provide corresponding balanced TPM.

Example: 4.1



Graph G4.1

Corresponding TPM:

	A	**C**	**E**	**Supply**
B	1	2	3	**6**
D	4	7	8	**4**
Demand	**5**	**2**	**3**	**10**

BFS by NWC: 38

OBFS by LCM: 38

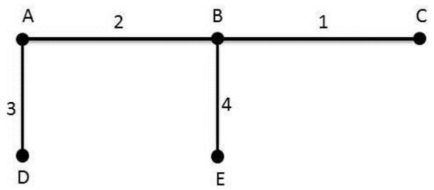
OBFS by VAM: 30 (ie. $1^2+2^2+3^2+4^2$)

OBFS by Path $(E \rightarrow B \rightarrow C)$: 30

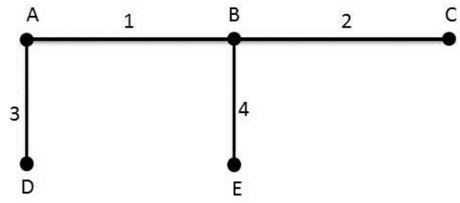
OBFS by Path $(D \rightarrow A \rightarrow B \rightarrow C)$: 30

OBFS by Path $(D \rightarrow A \rightarrow B \rightarrow E)$: 30

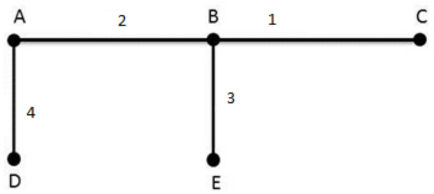
Example: 4.2



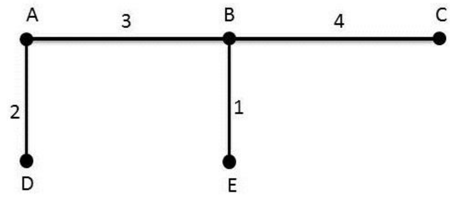
Graph G4.2



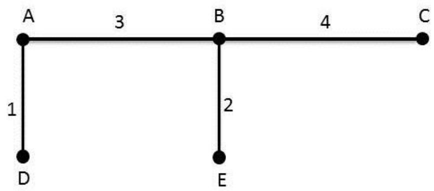
Graph G4.3



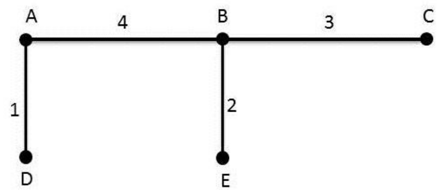
Graph G4.4



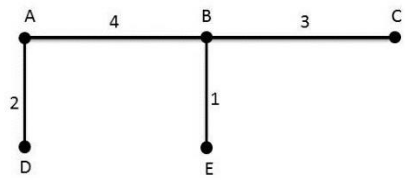
Graph G4.5



Graph G4.6



Graph G4.7



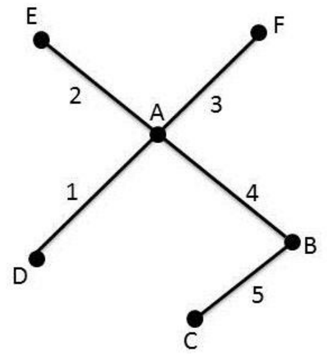
Graph G4.8

GRAPH	**OBFS BY**					
	NWC	**LCM**	**VAM**	**Path (E→B→C)**	**Path (D→A→B→C)**	**Path (D→A→B→E)**
Graph$_{4.1}$	38	38	30	30	30	30
Graph$_{4.2}$	42	42	30	30	30	30
Graph$_{4.3}$	36	36	30	30	30	30
Graph$_{4.4}$	42	46	30	30	30	30
Graph$_{4.5}$	42	30	30	30	30	30
Graph$_{4.6}$	36	30	30	30	30	30
Graph$_{4.7}$	38	30	30	30	30	30
Graph$_{4.8}$	44	30	30	30	30	30

Table: 1

Patent for all the above graphs will be $1^2+2^2+3^2+4^2=30$ (wherever 30 appears in the table 1)

Example: 4.3



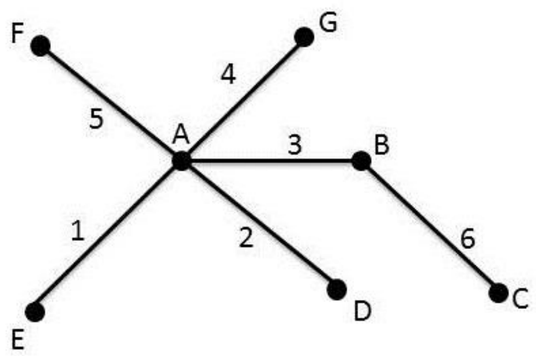
Graph G4.9

Corresponding TPM:

	A	C	Supply
B	4	5	**9**
D	1	10	**1**
F	3	12	**3**
E	2	11	**2**
Demand	**10**	**5**	**15**

OBFS by VAM: 55 (ie. $1^2+2^2+3^2+4^2+5^2$)

Example: 4.4



Graph G4.10

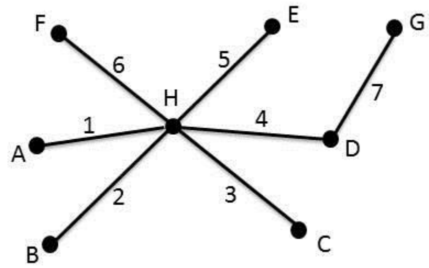
Corresponding TPM:

	A	C	Supply
B	3	6	**9**
D	2	9	**2**

E	1	10	**1**
F	5	14	**5**
G	4	13	**4**
Demand	**15**	**6**	**21**

OBFS by VAM: 91 (ie. $1^2+2^2+3^2+4^2+5^2+6^2$)

Example: 4.5



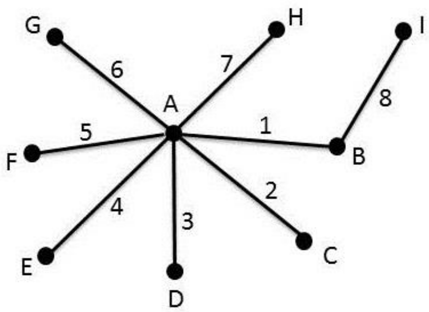
Graph G4.11

Corresponding TPM:

	H	G	Supply
D	4	7	**11**
A	1	12	**1**
B	2	13	**2**
C	3	14	**3**
E	5	16	**5**
F	6	17	**6**
Demand	**21**	**7**	**28**

OBFS by VAM: 140 (ie. $1^2+2^2+3^2+4^2+5^2+6^2+7^2$)

Example: 4.6



Graph G4.12

Corresponding TPM:

	A	**I**	**Supply**
B	1	8	**9**
C	2	11	**2**
D	3	12	**3**
E	4	13	**4**
F	5	14	**5**
G	6	15	**6**
H	7	16	**7**
Demand	**28**	**8**	**36**

OBFS by VAM: 204 (ie. $1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2$)

Theorem: 4

The OBFS by Vogel's Approximation Method of corresponding TPM of all star graphs with n+1 vertices is $1^2+2^2+3^2+\ldots..+n^2$ if and only if

(i) (n-1) vertices are pendent.

(ii) One vertex will have degree two.

(iii) One vertex will have degree (n-1).

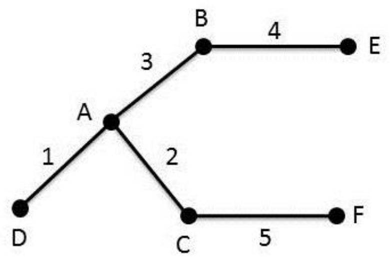
Proof:

By observation of the graphs **Graph $G_{4.1}$** to **Graph $G_{4.12}$.**

Also we can prove the above theorem by induction method.

Corollary: 1

The following **Graph $G_{5.1}$** having OBFS by VAM is 55.



Graph G5.1

Corresponding TPM:

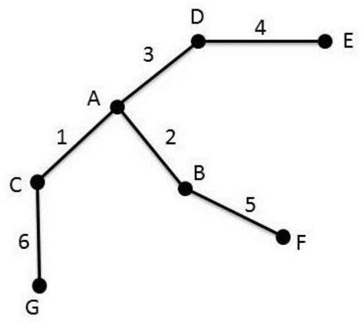
	D	**C**	**B**	**Supply**
A	1	2	3	**6**
E	8	9	4	**4**
F	8	5	10	**5**

Demand	**1**	**7**	**7**	**15**

OBFS by VAM is 55 (ie. $1^2+2^2+3^2+4^2+5^2$)

Corollary: 2

The following **Graph $G_{5.2}$** having OBFS by VAM is 101.



Graph G5.2

Corresponding TPM:

	B	C	D	Supply
A	2	1	3	**6**
G	9	6	10	**6**
F	5	8	10	**5**
E	9	8	4	**4**
Demand	**7**	**7**	**7**	**21**

OBFS by VAM is 101 (Pattern Not appears)

3. Acknowledgement

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