On Solving Transportation Problem – Linear Path Approach

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Abstract: Aim of this study is to determine the optimal cost for the given rectangular (or) square grid corresponding transportation problem along with balanced and unbalanced manner. We proposed the algorithmic way to provide cost from supply to the corresponding demand of the transportation problem in graph theoretical way to obtain minimum cost than the existed method in Operation Research. The proposed research brings out anoptimal basic feasible solution derived through graph theoretical method. It provides more than hundred percentage matching with so called existed method's optimal basic feasible solution.

Keywords: Maximum, Minimum, Optimization, Path, Pivot, Vertex

1. Introduction

Organizations need to fix lot of problems and make fruitful decisions in order to manage proper administration. Operation Research is one of the domains that help as a rescue to solve complications by the application of a set of analytical methods [1]-[4]. These analytical methods involve an advanced range of mathematical models to get an optimal solution of the given task.

Visual representations like graph help a better understanding of data. Graphs are normally presented by points structured both directed and undirected ways so as to capture the image for analysis in different scientific and real time problems [5].

Expansion of any business that relies on transportation can be successful or failure [8]-[10] based on economic management of cost. Mobility of products from one end to another end may lead to some issues. Issues that are related to monetary reduction and economizing can be sort out by unique kind of Linear Programming problem [6]& [7].

Nomenclature:

VAM - Vogel's Approximation Method

LCM - Least Cost Method

NWC - North West Corner Method

TPM - Transportation ProblemModel

OBFS - Optimal Basic Feasible Solution

BFS - Basic Feasible Solution

Cor - Corollary

Theorem:

There exists a path of length $(p-1)$ as an equivalent graph of the corresponding transportation pay off matrix where p is a number of vertices of G is connected.

Proof:

We know that for the TPM having solution satisfied degeneracy condition

(i.e) $m+n-1$

(i.e) $m+n-1=(p-1)$

Where m is number of rows in TPM, n is number of columns in TPM, p is number of vertices of G (as a solution of connected graph). Hence the proof.

Cor 1:

Every TPM will have an unique weighted graphical representation.

Cor 2:

There exist several OBFS for a TPM in its corresponding graphical representation with different starting point and also consists of the same point.

Algorithm:

Step 1: Drawan equivalent edge weighted connected graph $G(V, E)$ corresponding to given TPM.

Step 2: List out all possible paths from certain starting point which covers maximum number of other points once.

Step 3: Shade the cost which is weight of the corresponding two vertices of the paths.

Step 4: Choose the least element which occur in supplyor demand and allotthat cost to the minimum value corresponding to the row and column of the shaded cell if possible.

Step 5: Repeat Step 4 until degeneracy condition is satisfied.

Step 6: Compute the cost value based on Step 5.

Example :

TPM :

	J	**K**	**L**	**Supply**
A	2	3	1	**10**
B	5	4	8	**35**
C	5	6	8	**25**
Demand	**20**	**25**	**25**	**70**

Table: 1

Graphical Representation of the given TPM:

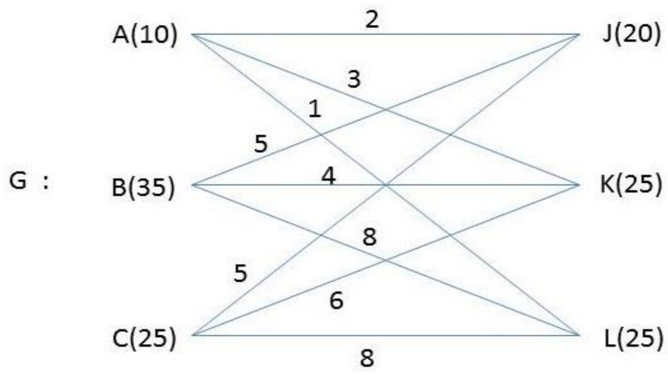

Figure: 1

All Corresponding paths (possible) of G:

Paths from A:

S.NO	PATHS	WEIGHT	COST	ALOTTED CELLS
1	P_1 : AJBKCL	25	370	{[(1,1),10] , [(2,1),10] , [(2,2),25] , [(3,3),25]}
2	P_2 : AJCKBL	25	400	{[(1,1),10] , [(2,2),10] , [(2,3),25] , [(3,1),10] , [(3,2),15]}
3	P_3 : AJBLCK	29	420	{[(1,1),10] , [(2,1),10] , [(2,3),25] , [(3,2),25] }
4	P_4 : AJCLBK	27	370	{[(1,1),10] , [(2,2),25] , [(2,3),10] , [(3,1),10] , [(3,3),15]}
5	P_5 : AKBJCL	25	390	{[(1,2),10] , [(2,1),20] , [(2,2),15] , [(3,3),25]}
6	P_6 : AKCJBL	27	420	{[(1,2),10] , [(2,1),10] , [(2,3),25] , [(3,1),10] , [(3,2),15]}
7	P_7 : AKBLCJ	28	390	{[(1,2),10] , [(2,2),15] , [(2,3),20] , [(3,1),20] , [(3,3),5]}
8	P_8 : AKCLBJ	30	420	{[(1,2),10] , [(2,1),20] , [(2,3),15] , [(3,2),15] , [(3,3),10]}
9	P_9 : ALBJCK	25	380	{[(1,3),10] , [(2,1),20] , [(2,3),15] , [(3,2),25]}
10	P_{10} : ALCJBK	23	330	{[(1,3),10] , [(2,1),10] , [(2,2), 25] , [(3,1),10] , [(3,3),15]}
11	P_{11} : ALBKCJ	24	340	{[(1,3),10] , [(2,2),20] , [(2,3),15] , [(3,1),20] , [(3,2),5]}
12	P_{12} : ALCKBJ	24	350	{[(1,3),10] , [(2,1),20] , [(2,2),15] , [(3,2),10] , [(3,3),15]}

Table: 2

Allotment table for the path P1:

Step 1: Shade the cells with the edge weight of the corresponding path.

	J	K	L	Supply
A	2 $(A \to J)$	3	1	**10**
B	5 $(J \to B)$	4 $(B \to K)$	8	**35**
C	5	6 $(K \to C)$	8 $(C \to L)$	**25**
Demand	**20**	**25**	**25**	**70**

Table: 3

Step 2: Choose the least cost from supply& demand and allot the possible cost to the minimum element corresponding to the row and column of the shaded cell if possible.

	J	K	L	Supply
A	2_{10}	3	1	**0**
B	5	4	8	**35**
C	5	6	8	**25**
Demand	**10**	**25**	**25**	**70**

Table: 4

Step 3: Repeat Step 2 until the degeneracy condition satisfied for all shaded cells in Step 1 if possible.(which is explained in step 3.1 to step 3.3)

Step 3.1:

	J	K	L	Supply
A	2_{10}	3	1	**0**
B	5_{10}	4	8	**25**
C	5	6	8	**25**
Demand	**0**	**25**	**25**	**70**

Table: 5

Step 3.2:

	J	**K**	**L**	**Supply**
A	2_{10}	3	1	**0**
B	5_{10}	4_{25}	8	**0**
C	5	6	8	**25**
Demand	**0**	**0**	**25**	**70**

Table: 6

Step 3.3:

	J	**K**	**L**	**Supply**
A	2_{10}	3	1	**0**
B	5_{10}	4_{25}	8	**0**
C	5	6	8_{25}	**0**
Demand	**0**	**0**	**0**	**70**

Table: 7

Step 4:

Explanation of cost:

Cost for P_1: Using Step 3.3,[(1, 1), 10] = 2X10 = 20, [(2, 1), 10] = 5X10 = 50,

[(2, 2), 25] = 4X25 = 100, [(3, 3), 25] = 8X25 = 200

Cost = 20 + 50 + 100 + 200 = 370

Paths from B:

S.NO	**PATHS**	**WEIGHT**	**COST**	**ALOTTED CELLS**
1	P_1 : BJAKCL	24	420	{[(1,1),10] , [(2,1),10] , [(2,3),25] , [(3,2),25]}
2	P_2 : BJALCK	22	350	{[(1,3),10] , [(2,1),20] , [(2,2),15] , [(3,2),10] , [(3,3),15]}
3	P_3 : BJCKAL	20	340	{[(1,3),10] , [(2,2),20] , [(2,3),15] , [(3,1),20] , [(3,2),5]}
4	P_4 : BJCLAK	22	330	{[(1,3),10] , [(2,1),10] , [(2,2),25] , [(3,1),10] , [(3,3),15]}
5	P_5 : BKALCJ	21	330	{[(1,3),10] , [(2,1),10] , [(2,2),25] , [(3,1),10] , [(3,3),15]}
6	P_6 : BKCLAJ	21	350	{[(1,3),10] , [(2,1),20] , [(2,2),15] , [(3,2),10] , [(3,3),15]}
7	P_7 : BKAJCL	22	370	{[(1,1),10] , [(2,2),25] , [(2,3),10] , [(3,1),10] , [(3,3),15]}
8	P_8 : BKCJAL	18	340	{[(1,3),10] , [(2,2),20] , [(2,3),15] , [(3,1),20] , [(3,2),5]}
9	P_9 : BLCKAJ	27	420	{[(1,1),10] , [(2,1),10] , [(2,3),25] , [(3,2),25]}
10	P_{10} : BLAJCK	22	340	{[(1,3),10] , [(2,2),20] , [(2,3),15] , [(3,1),20] , [(3,2),5]}
11	P_{11} : BLCJAK	26	370	{[(1,1),10] , [(2,2),25] , [(2,3),10] , [(3,1),10] , [(3,3),15]}
12	P_{12} : BLAKCJ	23	340	{[(1,3),10] , [(2,2),20] , [(2,3),15] , [(3,1),20] , [(3,2),5]}

Table: 8

Allotment table for the path P1:

	J	K	L	Supply
A	2_{10}	3	1	**10**
B	5_{10}	4	8_{25}	**35**
C	5	6_{25}	8	**25**
Demand	**20**	**25**	**25**	**70**

Table: 9

Note: Yellow shaded indicates the allotment from path P_1; Red shaded indicates the new adjusted allotment with respect to supply & demand.

Paths from C:

S.NO	PATHS	WEIGHT	COST	ALOTTED CELLS
1	P_1 : CJALBK	20	330 370	{[(1,3),10] , [(2,2),25] , [(2,3),10] , [(3,1),20] , [(3,3),5]} {[(1,1),10] , [(2,2),25] , [(2,3),10] , [(3,1),10] , [(3,3),15]}
2	P_2 : CJAKBL	22	370	{[(1,1),20] , [(2,2),25] , [(2,3),10] , [(3,1),10] , [(3,3),15]}
3	P_3 : CJBKAL	18	330	{[(1,3),10] , [(2,1),10] , [(2,2),25] , [(3,1),10] , [(3,3),15]}
4	P_4 : CJBLAK	22	380	{[(1,3),10] , [(2,1),20] , [(2,3),15] , [(3,2),25]}
5	P_5 : CKALBJ	23	380	{[(1,3),10] , [(2,1),20] , [(2,3),15] , [(3,2),25]}
6	P_6 : CKAJBL	24	420	{[(1,1),10] , [(2,1),10] , [(2,3),25] , [(3,2),25]}
7	P_7 : CKBJAL	18	330	{[(1,3),10] , [(2,1),10] , [(2,2),25] , [(3,1),10] , [(3,3),15]}
8	P_8 : CKBLAJ	21	330	{[(1,3),10] , [(2,2),25] , [(2,3),10] , [(3,1),20] , [(3,3),5]}
9	P_9 : CLAJBK	20	330	{[(1,3),10] , [(2,1),10] , [(2,2),25] , [(3,1),10] , [(3,3),15]}
10	P_{10} : CLAKBJ	21	330	{[(1,3),10] , [(2,1),10] , [(2,2),25] , [(3,1),10] , [(3,3),15]}
11	P_{11} : CLBKAJ	25	390	{[(1,2),10] , [(2,1),20] , [(2,2),15] , [(3,3),25]}
12	P_{12} : CLBJAK	26	420	{[(1,1),10] , [(2,1),10] , [(2,3),25] , [(3,2),25]}

Table: 10

OBFS allotment table of P3 :

	J	K	L	Supply
A	2	3	1_{10}	**10**
B	5_{10}	4_{25}	8	**35**
C	5_{10}	6	8_{15}	**25**
Demand	**20**	**25**	**25**	**70**

Table: 11

2. Results &Conclusion:

S.NO	STARTUP	PATHS	RESULT
1.	A	P_{10}	**OBFS**is 100% matching with **VAM** and **LCM**
	B	P_4 , P_5	
	C	P_1 , P_3 , P_7 , P_8 , P_9 , P_{10}	
2.	A	P_1 , P_4	**OBFS**solution with **NWC** in **100%** matching
	B	P_7, P_{11}	
	C	P_1, P_2	
3.	A	P_2 , P_3 , P_5 , P_6 , P_7, P_8 and P_9	**Not providing solution**
	B	P_1 , P_9	
	C	P_4 , P_5 , P_6 , P_{11} , P_{12}	
4.	A	Remaining all possible paths and paths other than S.NO 3	**BFS** for **NWC**providing **107.173%** matching .
	B		
	C		

Table: 12

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