Research Article

Total Eccentricity Indices Of A Graph

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Article History: Received: 11 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 20 April 2021

Abstract: We have defined and evaluated total eccentricity indices of polyhex nanotubes $TUAC_6(p, q)$ and $TUZC_6(p, q)$. In thispaper, we compute First Total Eccentricity Index, Second Total Eccentricity Index, First Multiplicative Total Eccentricity Index, Second Multiplicative Total Eccentricity Index, First Hyper Total Eccentricity Index, Second Hyper Total Eccentricity Index, First Multiplicative Hyper Total Eccentricity Index and Second Multiplicative Hyper Total Eccentricity Index of a graph using Total graph of a graph. We evaluate the value of these indices for some standard graphs.

Keyword: Eccentricity Index, Total graph of a graph.

1.Introduction:

Let G be a simple, finite graph with n vertices and m edges with vertex set V(G) and edge set E(G). The edge connecting the vertices u and v$\in$ V(G) is denoted by e = uv. The vertices and edges of a graph are called elements of G.The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. If e = uv is an edge of G, then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) – 2$ with e = uv.

Let G be a connected graph and v be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v); u \in V\}$. The radius r(G) is the minimum eccentricity of the vertices, whereas the diameter diam(G) is the maximum eccentricity.

The Total graph T(G) of G is the graph whose vertex set is $V = V(G) \cup E(G)$ where two elements are adjacent if and only ifthey are adjacent vertices of G or they are adjacent edges of G or one is a vertex of G and another is an edge of G incident with it. Elements of V which are in V(G) are known as point vertices and are in E(G) are known as line vertices. Let $e_{T(G)}(u)$ and $e_{T(G)}(e)$ denote the eccentricity of vertex u and edge e in T(G) respectively.

The topological indices are one of the mathematical models that can be defined by assigning a real number to the chemical molecule. The physical-chemical characteristics of the molecules can be analyzed by taking benefit from the topological indices. Properties such as boiling point, entropy, enthalpy of vaporization, standard enthalpy of vaporization, enthalpy of formation. Acetic factor, etc can be predicted using topological indices..

In 2016, Kulli introduced K Banhatti indices [6].

In 2016, Bhanumathi and Easu Julia Rani introduced K-eccentric indices [2, 4]

In 2020, Bhanumathi and Mariselvi defined and evaluated total eccentricity indices of polyhex nanotubes $TUAC_6(p, q)$ and $TUZC_6(p, q)$ [5].

2.Some Eccentricity based indices of a graph G using Total graph T(G)

Here, we evaluate the First and Second Total Eccentricity Index and First and Second Multiplicative Total Eccentricity Index, First and Second Hyper Total Eccentricity Index and First and Second Multiplicative Hyper Total Eccentricity Index of some particular graphs.

In [5], we define, the First and Second Total Eccentricity Index as

$$BT_1(G) = \sum_{ue}[(e_{T(G)}(u) + e_{T(G)}(e)]$$

$$BT_2(G) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$$

The First and Second Multiplicative Total Eccentricity Index are defined as

$$BT\Pi_1(G) = \prod_{ue}[e_{T(G)}(u) + e_{T(G)}(e)]$$

$$BT\Pi_2(G) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$$

Also, we define the First and Second Hyper Total Eccentricity Indexas

$$HBT_1(G) = \sum_{ue}[(e_{T(G)}(u) + e_{T(G)}(e)]^2$$

$$HBT_2(G) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]^2$$

The First and Second Multiplicative Hyper Total Eccentricity Index are defined as

$$HBT\Pi_1(G) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$HBT\Pi_2(G) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]^2$$

where ue means that the vertex u and edge e are incident in G.

Theorem 2.1: Let K_n be a complete graph with n vertices. Then

(i) $BT_1(K_n) = 4n(n – 1)$.

(ii) $BT_2(K_n) = 4n(n – 1)$.

(iii) $BT\Pi_1(K_n) = (16)^{\frac{n(n-1)}{2}}$.

(iv) $BT\Pi_2(K_n) = (16)^{\frac{n(n-1)}{2}}$.

Proof: Let K_n be a complete graph with n vertices and $m = \frac{n(n-1)}{2}$ edges. Every edge of K_n is incident with exactly two vertices. Every point vertices and line vertices have eccentricity 2 in T(G).

$$BT_1(K_n) = \sum_{ue}[(e_{T(G)}(u) + e_{T(G)}(e)]$$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u) + e_{T(G)}(e) + (e_{T(G)}(v) + e_{T(G)}(e))]$$

$$= \sum[(2 + 2) + (2 + 2)] = \sum 8 = 8\ \frac{n(n-1)}{2} = 4n(n - 1)$$

$$BT_2(K_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e) + (e_{T(G)}(v).e_{T(G)}(e))]$$

$$= \sum[(2 . 2) + (2 . 2)] = \sum 8 = 8\ \frac{n(n-1)}{2} = 4n(n - 1)$$

$$BT\Pi_1(K_n) = \prod_{ue}[e_{T(G)}(u) + e_{T(G)}(e)]$$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u) + e_{T(G)}(e)).(e_{T(G)}(v) + e_{T(G)}(e))]$$

$$= \Pi\ [(2 + 2) . (2 + 2)] = \Pi 16 = (16)^{\frac{n(n-1)}{2}}$$

$$BT\Pi_2(K_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$$

$$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]$$

$= \Pi\ [(2\ .\ 2)\ .\ (2\ .2)] = \Pi 16 = (16)^{\frac{n(n-1)}{2}}$

Theorem 2.2: Let P_n be a path with n vertices. Then

(i) $BT_1(P_n) = 3n^2 - 5n + 2$

(ii) $BT_2(P_n) =$

$$\begin{cases} (n-1)\left(\frac{2n^2+2}{4} + \frac{2n^2+7n+12}{4} + \frac{2n^2+13n+40}{4} + \ldots + \left(2n^2-5n+3\right)\right) if n is odd \\ \frac{n^2}{2} + (n-2)\left[\left(\frac{n^2}{2} + \frac{3n}{2} + 1\right) + \left(\frac{n^2}{2} + \frac{7n}{2} + 6\right) + \ldots + \left(2n^2-5n+3\right)\right] if n is even \end{cases}$$

(iii) $BT\Pi_1(P_n) =$

$$\begin{cases} 2^{\left(\frac{n-1}{2}\right)}\left[(n^2+n)(n^2+5n+6)\ldots(4n^2-10n+6)\right] if n is odd \\ n^2 2^{\left(\frac{n-2}{2}\right)}\left[(n^2+3n)(n^2+7n+16)\ldots(4n^2-10n+6)\right] if n is even \end{cases}$$

(iv) $BT\Pi_2(P_n) =$

$$\begin{cases} 2^{\left(\frac{n-1}{2}\right)}\left(\frac{(n^4+2n^3-2n-1)}{4}\frac{(n^4+7n^3+24n^2+45n+27)}{4}\ldots\left(n^4-5n^3+9n^2-7n+2\right)\right) if n is odd \\ \frac{n}{16}^4 2^{\left(\frac{n-2}{2}\right)}\left[\left(\frac{n^4}{16}+\frac{3n^3}{8}+\frac{3n^2}{4}+\frac{n}{2}\right)\left(\frac{n^4}{16}+\frac{7n^3}{8}+\frac{9n^2}{2}+10n+8\right)\ldots\left(n^4-4n^3+7n^2+2n+2\right) if n is even\right] \end{cases}$$

Proof: Let P_n be a path with n vertices. Then P_n has n – 1 edges. Every edge of $P_n$is incident with exactly two vertices. $T(P_n)$ has n point vertices and n – 1 line vertices.

n is odd

Number of edges e = uv in G	eccentricity of e in T(G) ($e_T(G)$)	eccentricity of end vertices ($e_T(u)$, $e_T(v)$)
2	$\left(\frac{n+1}{2}\right)$	$\left(\left(\frac{n-1}{2}\right), \left(\frac{n-1}{2}\right)+1\right)$
2	$\left(\frac{n+3}{2}\right)$	$\left(\left(\frac{n+1}{2}\right), \left(\frac{n+3}{2}\right)\right)$
2	$\left(\frac{n+5}{2}\right)$	$\left(\left(\frac{n+3}{2}\right), \left(\frac{n+5}{2}\right)\right)$
……..	……..	……………..
2	(n – 1)	((n – 2), (n – 1))

n is even

Number of edges e = uv in G	eccentricity of e in T(G) ($e_T(G)$)	eccentricity of end vertices ($e_T(u)$, $e_T(v)$)
1	$\left(\frac{n}{2}\right)$	$\left(\left(\frac{n}{2}\right), \left(\frac{n}{2}\right)\right)$

2	$\left(\frac{n}{2}\right)+1$	$\left(\left(\frac{n}{2}\right),\left(\frac{n}{2}\right)+1\right)$
2	$\left(\frac{n}{2}\right)+2$	$\left(\left(\frac{n}{2}\right)+1,\left(\frac{n}{2}\right)+2\right)$
……..	……..	……………..
2	(n – 1)	((n – 2), (n – 1))

Case(i): n is odd

Eccentricity of central vertex is $\left(\frac{n-1}{2}\right)$, eccentricity of pendant vertices (n – 1), eccentricity of line vertices incident with central vertex is $\left(\frac{n+1}{2}\right)$, eccentricity of line vertices incident with end vertices is (n – 1) in T(G).

$BT_1(P_n) = \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]$

$= 2\left[\left(\left(\frac{n-1}{2}\right)+\left(\frac{n-1}{2}\right)+1+2\left(\frac{n+1}{2}\right)\right)\right]$

$+2\left(\left(\frac{n+1}{2}\right)+\left(\frac{n+3}{2}\right)+1+2\left(\frac{n+3}{2}\right)\right)+\ldots$

+2[(n–2)+(n–1)+2(n – 1)]

= 2(2n + 1) + 2(2n + 5) + …+2(4n – 5)

= 2(2n + 2n + ….+(n -1) / 2terms) + 2(1+5+ …+(2n – 5))

$= 2\,(2n)\left(\frac{n-1}{2}\right) + 2\left(\frac{n-1}{4}\right)[1 + 2n - 5] = 3n^2 - 5n + 2.$

$BT_2(P_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]$

$=2\left[\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)+\left(\frac{n-1}{2}+1\right)\left(\frac{n+1}{2}\right)\right]$

$+2\left[\left(\frac{n+1}{2}\right)\left(\frac{n+3}{2}\right)+\left(\frac{n+3}{2}\right)\left(\frac{n+3}{2}\right)\right]+\ldots$

+ 2[(n – 2)(n – 1) + (n – 1)(n – 1)]

$= 2\left[\left(\frac{n^2-1}{4}\right)+\left(\frac{n+1}{2}\right)^2\right]+2\left[\left(\frac{n^2+4n+3}{4}\right)+\left(\frac{n+3}{2}\right)^2\right]+\ldots$

$+ 2[(n^2 - 3n + 2) + (n - 1)^2]$

$=(n-1)\left(\frac{2n^2+2}{4}+\frac{2n^2+7n+12}{4}+\frac{2n^2+13n+40}{4}\ldots\left(2n^2-5n+3\right)\right)$

$BT\Pi_1(P_n) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]$

$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]$

$= 2\left[\left(\frac{n-1}{2}+\frac{n+1}{2}\right)\left(\frac{n-1}{2}+1+\frac{n+1}{2}\right)\right]$

$2\left[\left(\frac{n+1}{2}+\frac{n+3}{2}\right)\left(\frac{n+3}{2}+\frac{n+3}{2}\right)\right]$ …2[(n–2)+(n–1)(n–1+n– 1)]

$= 2(n^2 + n)\ 2(n^2 + 5n + 6) \ldots. (4n^2–10n + 6)$

$= 2^{\left(\frac{n-1}{2}\right)}\left[(n^2+n)(n^2+5n+6)....(4n^2-10n+6)\right]$

$BT\Pi_2(P_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$

$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]$

$=2\left[\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)\left(\frac{n-1}{2}+1\right)\left(\frac{n+1}{2}\right)\right]2\left[\left(\frac{n+1}{2}\right)\left(\frac{n+3}{2}\right)\left(\frac{n+3}{2}\right)\left(\frac{n+3}{2}\right)\right]$ …×2[(n–2)(n–1)(n–1)(n–1)]

$= 2\left[\left(\frac{n^2-1}{4}\right)\left(\frac{n+1}{2}\right)^2\right]+2\left[\left(\frac{n^2+4n+3}{4}\right)\left(\frac{n+3}{2}\right)^2\right]$ …2[(n²– 3n + 2)(n – 1)²]

$=2^{\left(\frac{n-1}{2}\right)}\left(\frac{(n^4+2n^3-2n-1)}{4}\frac{(n^4+7n^3+24n^2+45n+27)}{4}....(n^4-5n^3+9n^2-7n+2)\right)$

Case(ii): n is even.

Eccentricity of central vertices is $\left(\frac{n}{2}\right)$ and $\left(\frac{n}{2}\right)$, eccentricity of pendant vertices is (n – 1), eccentricity of line vertices incident with central vertices is $\left(\frac{n}{2}\right)$, eccentricity of line vertices incident with end vertices is (n – 1).

$BT_1(P_n) = \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]$

= (n / 2 + n / 2 +2(n /2)) + 2(n / 2 + n / 2 + 1 + 2(n /2 + 1)) + …+ 2[(n – 2) + (n – 1) + 2(n – 1)] = 2n + 2(2n + 2n +…(n–2)/2 terms] + 2[3 + 7 + …+(2n – 5)]

$= 2n^2– 2n + (n – 1)(n – 2) = 3n^2 – 5n + 2.$

$BT_2(P_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]$

= (n / 2) (n / 2) +(n /2)(n / 2) + 2[(n / 2) (n / 2 + 1) + (n /2 + 1) (n /2 + 1)] + …+ 2[(n – 2)(n – 1) + (n – 1) (n – 1)]

$$= \frac{n^2}{2} + 2\left(\frac{n-2}{2}\right)\left[\left(\frac{n^2}{2}+\frac{3n}{2}+1\right)+\left(\frac{n^2}{2}+\frac{7n}{2}+6\right)\right] \ldots +(2n^2-5n+3)$$

$$= \frac{n^2}{2} + (n-2)\left[\left(\frac{n^2}{2}+\frac{3n}{2}+1\right)+\left(\frac{n^2}{2}+\frac{7n}{2}+6\right)\right] \ldots +(2n^2-5n+3)$$

$BT\Pi_1(P_n) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]$$

= ((n / 2 + n / 2) (n /2 + n /2)) 2((n / 2 + n / 2 + 1) (n /2 + 1 + n /2 + 1)) … ×2[(n – 2 + n – 1) (n – 1 + n – 1)]

$$= n^2\, 2\left(\frac{4n^2+12n}{4}\right) 2\left(\frac{4n^2+28n+48}{4}\right)\ldots\ldots\left(4n^2-10n+6\right)$$

$$= n^2\, 2^{\left(\frac{n-2}{2}\right)}\left[(n^2+3n)(n^2+7n+16)\ldots.(4n^2-10n+6)\right]$$

$BT\Pi_2(P_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$

$$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]$$

= [(n / 2) (n / 2) (n /2)(n / 2)] 2[(n / 2) (n / 2 + 1) (n /2 + 1)
(n /2 + 1)] … ×2[(n – 2)(n – 1) (n – 1) (n – 1)]

$$= \frac{n^4}{16} 2\left[\begin{array}{l}\left(\frac{n^4}{16}+\frac{n^3}{8}+\frac{n^3}{4}+\frac{n^2}{2}+\frac{n^2}{4}+\frac{n}{2}\right)\left(\frac{n^4}{16}+\frac{7n^3}{8}+\frac{n^2}{2}+4n^2+10n+8\right)+\ldots\\ +(n^2-2n+2)(n^2-2n+1)\end{array}\right]$$

$$= \frac{n}{16}^4\, 2^{\left(\frac{n-2}{2}\right)}\left[\begin{array}{l}\left(\frac{n^4}{16}+\frac{3n^3}{8}+\frac{3n^2}{4}+\frac{n}{2}\right)\left(\frac{n^4}{16}+\frac{7n^3}{8}+\frac{9n^2}{2}+10n+8\right)\ldots.\\ \left(n^4-4n^3+7n^2+2n+2\right)\big)\end{array}\right]$$

Theorem 2.3: Let C_n be a cycle with $n \geq 4$ vertices. Then

(i) $BT_1(C_n) = \begin{cases} 2n(n-1) & \text{if n is odd} \\ 2n^2 & \text{if n is even}\end{cases}$

(ii) $BT_2(C_n) = \begin{cases} \frac{n(n+1)^2}{2} & \text{if n is odd} \\ n^3/2. & \text{if n is even}\end{cases}$

(iii) $BT\Pi_1(C_n) = \begin{cases} (n+1)^{2n} & \text{if n is even} \\ n^{2n} & \text{if n is even}\end{cases}$

(iv) $BT\Pi_2(C_n) = \begin{cases} \left(\frac{n+1}{2}\right)^{4n} & \text{if n is odd} \\ (n/2)^{4n} & \text{if n is even}\end{cases}$

Proof : Let C_n be a cycle with n vertices. Then C_n has n edges. Every edge of C_n is incident with exactly two vertices. T(G) has n point vertices and n line vertices.

Case(i): n is odd

Let n = 2k + 1. When n is odd, each point vertex of T(G) has eccentricity (n + 1) / 2 = k + 1, and each line vertex has eccentricity (n + 1) / 2 = k + 1.

$BT_1(C_n) = \sum_{ue}[(e_{T(G)}(u) + e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u) + e_{T(G)}(e) + (e_{T(G)}(v) + e_{T(G)}(e))]$

$= \sum [(k + 1 + k + 1) + (k + 1 + k + 1)] = \sum [4k + 4]$

$= n(4k + 4)$

$= 4n(k + 1) = \frac{4n(n-1)}{2} = 2n(n - 1).$

$BT_2(C_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e) + (e_{T(G)}(v).e_{T(G)}(e))]$

$= \sum [(k + 1) (k + 1) + (k + 1) (k + 1)]$

$= \sum [(k + 1)^2 + (k + 1)^2] = \sum 2(k + 1)^2 = 2n(k + 1)^2$

$= \frac{n(n+1)^2}{2}$

$BT\Pi_1(C_n) = \prod_{ue}[e_{T(G)}(u) + e_{T(G)}(e)]$

$= \prod_{uv\in E(G)}[(e_{T(G)}(u) + e_{T(G)}(e)).(e_{T(G)}(v) + e_{T(G)}(e))]$

$= \Pi [(k + 1 + k + 1) . (k + 1 + k + 1)]$

$= \Pi [(2k + 2) (2k + 2)] = (2k + 2)^{2n} = (n + 1)^{2n}$

$BT\Pi_2(C_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$

$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]$

$= \Pi[(k + 1) (k + 1) . (k + 1) .(k + 1)] = \Pi(k + 1)^4 = (k + 1)^{4n} = \left(\frac{n+1}{2}\right)^{4n}$

Case(ii): n is even.

Let n = 2k. In T(G), each point vertex has eccentricity n / 2and line has eccentricity n / 2.

$BT_1(C_n) = \sum [(k + k) + (k + k)= \sum 4k = n4k = 4n (n / 2) = 2n^2$

$BT_2(C_n) = \sum [(k . k) + (k . k)] =\sum 2k^2 = n2k^2 = n^3 / 2.$

$BT\Pi_1(C_n) = \Pi [(k + k) . (k + k)]= \Pi(2k)^2 = (2k)^{2n} = n^{2n}$

$BT\Pi_2(C_n) = \Pi [(k . k) . (k . k) = \Pi (k)^4 = (k)^{4n} = (n /2)^{4n}$

Theorem 2.4: Let W_n be a wheel with $n \geq 5$ vertices. Then

(i)$BT_1(W_n) = 21n$.

(ii) $BT_1(W_n) = 28n$.

(iii) $BT\Pi1(Wn) = (720)^n$.

(iv) $BT\Pi_2(Wn) = (1944)^n$.

Proof: Let W_n be a wheel with n + 1 vertices. Then W_n has 2n edges. Every edge of W_n is incident with exactly two vertices.

Let $W_n = K_1 + C_n$. Let v be the central vertex of W_n, and $v_1, v_2, \ldots, v_n$ be the vertices of C_n. We have n edges of G which are incident with the central vertex and n edges on the cycle.Let E_1 = {set of all edges incident with central vertex}. E_2 = {set of all edges on with cycle}. For $e_i = vv_i \in E_1(G)$, $e_T(e_i) = 2$ and $e_T(v) = 2$, $e_T(v_i) = 3$. If $e_{ii+1} = v_iv_{i+1} \in E_2(G)$. Then $e_T(e_{ii+1}) = 3$, $e_T(v_i) = 3$. Also, $|E_1| = |E_2| = n$.

$BT_1(W_n) = \sum_{ue}[(e_{T(G)}(u) + e_{T(G)}(e)]$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]$$

$$=\sum_{e\in E_1}((2+2)+(2+3)) + \sum_{e\in E_2}((3+3)+(3+3)) = \sum_{e\in E_1}9 + \sum_{e\in E_2}21 = 21n.$$

$BT_2(W_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]$$

$$= \sum_{e\in E_1}((2.2)+(2.3)) + \sum_{e\in E_2}((3.3)+(3.3)) = \sum_{e\in E_1}10 + \sum_{e\in E_2}18 = 28n.$$

$B T\Pi_1(W_n) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]$$

$$=\prod_{e\in E_1}[(2+2).(2+3)] \prod_{e\in E_2}[(3+3).(3+3)] = \prod_{e\in E_1}20 \prod_{e\in E_2}36$$

$= (20)^n.(36)^n = (720)^n.$

$BT\Pi_2(W_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$

$$= \prod_{uv}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]$$

$$= \prod_{e\in E_1}[(2.2).(2.3)] \prod_{e\in E_2}[(3.3).(3.3)] = \prod_{e\in E_2}24 \prod_{e\in E_2}81 = (24)^n.(81)^n$$

$= (1944)^n.$

Remark: (i)$BT_1(W_4) = 16$. (ii) $BT_2(W_4) = 16$. (iii) $BT\Pi_1(W_4) = 16$. (iv) $BT\Pi_2(W_4) = 16$.

Theorem 2.5: Let $K_{1,n}$ be a star graph. Then

(i)$BT_1(K_{1,n}) = 7n$.

(ii) $BT_2(K_{1,n}) = 6n$.

(iii) $BT\Pi_1(K_{1,n}) = (12)^n$.

(iv) $BT\Pi_2(K_{1,n}) = (8)^n$.

Proof : Let $K_{1,n}$ be a star graph with n + 1 vertices and n edges. Every edge of $K_{1,n}$ is incident with exactly two vertices. Let u be a central vertex. Eccentricity of u is one and all other point and line vertices are of eccentricity two in T(G).

$BT_1(K_{1,n}) = \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]$$

$= \sum[(1 + 2) + (2 + 2)] = \sum 7 = 7n.$

$BT_2(K_{1,n}) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]$$

$= \sum[(1 . 2) + (2 . 2)] = \sum 6 = 6n.$

$BT\Pi_1(K_{1,n}) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]$$

$= \Pi [(1 + 2) . (2 + 2)] = \Pi\ 12 = (12)^n.$

BTΠ$_2$(K$_{1,n}$) = $\prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$

$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]$

$= \Pi[(1 . 2) . (2 . 2)] = \Pi 8 = (8)^n$.

Theorem 2.6: Let $K_{m,n}$ be a complete graph with $2 \le m \le n$. Then

(i)BT$_1$(K$_{m,n}$) = 8mn.

(ii) BT$_2$(K$_{m,n}$) = 8mn.

(iii) BTΠ$_1$(K$_{m,n}$) = $(16)^{mn}$.

(iv) BTΠ$_2$(K$_{m,n}$) = $(16)^{mn}$.

Proof : Let $K_{m,n}$ be a complete bipartite graph with m + n vertices, mn edges and $|V_1| = m$, $|V_1| = n$, V(K$_{m,n}$) = V1∪V2. Every edge of $K_{m,n}$ is incident with exactly two vertices. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Every point vertices and line vertices have eccentricity 2 in T(G).

BT$_1$(K$_{m,n}$) = $\sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]$

$= \Sigma[(2 + 2) + (2 + 2)] = \Sigma\, 8 = 8mn$.

BT$_2$(K$_{m,n}$) = $\sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]$ = $\Sigma[(2 . 2) + (2 . 2)]$

$= \Sigma\, 8 = 8mn$.

BTΠ$_1$(K$_{m,n}$) = $\prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]$

$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]$ = $\Pi\, [(2 + 2).(2 + 2)]$

$= \Pi\, 16 = (16)^{mn}$.

BTΠ$_2$(K$_{m,n}$) = $\prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$

$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]$

$= \Pi\, [(2 . 2) . (2 . 2)] = \Pi\, 16 = (16)^{mn}$.

Corollary 2.1: Let $K_{n,n}$ be a complete bipartite graph. Then

(i)BT$_1$(K$_{n,n}$) = $8n^2$.

(ii) BT$_2$(K$_{n,n}$) = $8n^2$.

(iii) BTΠ$_1$(K$_{n,n}$) = $(16)^{n^2}$

(iv) BTΠ$_2$(K$_{n,n}$) = $(16)^{n^2}$.

Proof: Put m = n in the previous theorem.

Theorem 2.7: Let F_n be a fan graph. Then

(i)BT$_1$(F$_n$) = 21n – 12.

(ii) BT$_2$(F$_n$) = 28n – 18.

(iii) BTΠ$_1$(F$_n$) = $(20)^n\, (36)^{(n-1)}$.

(iv) BTΠ$_2$(F$_n$) = $(24)^n(81)^{(n-1)}$.

Proof : Let F_n be a fan graph with n + 1 vertices and 2n – 1 edges.

Let $F_n = K_1 + P_n$. Let v be a central vertex of F_n, and $v_1, v_2, v_3, \ldots, v_n$ be the vertices of P_n. We have n edges of G which are incident with the central vertex and n – 1 edges on the path. Let E$_1$(G) = {set of all edges incident with central vertex v}. E$_2$(G) = {set of all edges on the path P_n}. If $e_i = vv_i \in E_1(G)$, then $e_T(e_i) = 2$ and $e_T(v) = 2$, $e_T(v_i) = 3$. If $e_{ii+1} = v_iv_{i+1} \in E_2(G)$. Then $e_T(e_{ii+1}) = 3$, $e_T(v_i) = 3$.

$BT_1(F_n) = \sum_{ue}[(e_{T(G)}(u) + e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u) + e_{T(G)}(e) + (e_{T(G)}(v) + e_{T(G)}(e))]$

$= \sum_{e\in E_1}((2+2)+(3+2)) + \sum_{e\in E_2}((3+3)+(3+3)) = \sum_{e\in E_1} 9 + \sum_{e\in E_2} 12$

$= 9n + 12(n – 1) = 9n + 12n – 12 = 21n – 12.$

$BT_2(F_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e) + (e_{T(G)}(v).e_{T(G)}(e))]$

$= \sum_{e\in E_1}((2.2)+(3.2)) + \sum_{e\in E_2}((3.3)+(3.3)) = \sum 10 + \sum 18$

$= 10n + 18(n – 1) = 10n + 18n – 18 = 28n – 18.$

$BT\Pi_1(F_n) = \prod_{ue}[e_{T(G)}(u) + e_{T(G)}(e)]$

$= \prod_{uv\in E(G)}[(e_{T(G)}(u) + e_{T(G)}(e)).(e_{T(G)}(v) + e_{T(G)}(e))]$

$= \prod_{e\in E_1}[(2+2).(2+3)] \prod_{e\in E_2}[(3+3).(3+3)] = \Pi\, 20\, \Pi 36$

$= (20)^n(36)^{(n – 1)}.$

$BT\Pi_2(F_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$

$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]$

$= \prod_{e\in E_1}[(2.2).(2.3)] \prod_{e\in E_2}[(3.3).(3.3)] = \Pi 24\, \Pi\, 81 = (24)^n(81)^{(n – 1)}.$

Theorem 2.8:(i) $BT_1(K_{2n} – F) = 16n(n – 1)$.

(ii) $BT_2(K_{2n} – F) = 16n(n – 1)$.

(iii) $BT\Pi_1(K_{2n} – F) = (16)^{2n^2-2n}$.

(iv) $BT\Pi_2(K_{2n} – F) = (16)^{2n^2-2n}$.

Proof : Let K_{2n} be a complete graph with 2n vertices and $\frac{2n(2n-1)}{2} = 2n^2 – n$ edges. F is a 1-factor of K_{2n}. K_{2n} – F has $2n^2 – n – n$ edges = $2n^2 – 2n$ edges. Every point vertices and line vertices have eccentricity two in $T(K_{2n} – F)$.

$BT_1(K_{2n} – F) = \sum_{ue}[(e_{T(G)}(u) + e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u) + e_{T(G)}(e) + (e_{T(G)}(v) + e_{T(G)}(e))]$

$= \sum[(2 + 2) + (2 + 2)] = \sum 8 = 8(2n^2 – 2n) = 16n(n – 1).$

$BT_2(K_{2n} – F) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e) + (e_{T(G)}(v).e_{T(G)}(e))] = \sum[(2 . 2) + (2 . 2)]$

$= \sum 8 = 8(2n^2 – 2n) = 16n(n – 1).$

$BT\Pi_1(K_{2n} – F) = \prod_{ue}[e_{T(G)}(u) + e_{T(G)}(e)]$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]$$

$= \Pi$ [(2 + 2). (2 + 2)] = $\Pi 16 = (16)^{2n^2-2n}$.

$$BT\Pi_2(K_{2n} - F) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]$$

$$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]$$

$= \Pi$ [(2 . 2). (2 .2) = $\Pi 16 = (16)^{2n^2-2n}$.

Theorem 2.9: Let K_n be a complete graph with n vertices. Then

(i)$HBT_1(K_n) = 16n(n-1)$.

(ii) $HBT_2(K_n) = 16n(n-1)$.

(iii) $HBT\Pi_1(K_n) = (16)^{n(n-1)}$.

(iv) $HBT\Pi_2(K_n) = (16)^{n(n-1)}$.

Proof : Let K_n be a complete graph with n vertices and m = $\frac{n(n-1)}{2}$ edges. Every edge of K_n is incident with exactly two vertices. Every point vertices and line vertices have eccentricity 2 in T(G).

$$HBT_1(K_n) = \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2$$

$$= \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2 + \sum_{ve}[(e_{T(G)}(v)+e_{T(G)}(e)]^2$$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]^2$$

$= \sum[(2+2)^2 + (2+2)^2] = \sum(16+16) = 16n(n-1)$

$$HBT_2(K_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$= \sum[(2\,.\,2)^2 + (2\,.\,2)^2] = \sum(16+16) = 32\frac{n(n-1)}{2} = 16n(n-1)$.

$$HBT\Pi_1(K_n) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]^2$$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]^2$$

$= \Pi [(2+2)^2 . (2+2)^2] = \Pi (16)^2 = (16)^{n(n-1)}$.

$$HBT\Pi_2(K_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$= \Pi [(2\,.\,2)^2.(2\,.2)^2] = \Pi (16)^2 = (16)^{n(n-1)}$.

Theorem 2.10: Let P_n be a path with n vertices. Then

(i) $HBT_1(P_n)$ =

$$\begin{cases} (n-1)\left(\left(2n^2+2n+1\right)+\left(2n^2+10n+13\right)+\ldots+\left(4n^2-10n+6\right)\right) & if\, n\, is\, odd \\ 2n^2+(n-2)\left(\left(2n^2+6n+5\right)+\ldots+\left(8n^2-20n+13\right)^2\right) & if\, n\, is\, even \end{cases}$$

(ii) $HBT_2(P_n)$ =

$$\begin{cases} (n-1)\left(\left(\frac{4n^4+2n^3+2n^2+2n+1}{8}\right)^2+....+\left(n^2-3n+2\right)\left(n^2-2n+1\right)^2\right) ifnisodd \\ \frac{n^4}{8}+(n-2)\left[\left(\frac{10n^4}{16}+\frac{2n^3}{4}+\frac{3n^2}{2}+2n+1\right)+...+\left(2n^4-2n^3+19n^2+16n+5\right)\right] ifniseven \end{cases}$$ (iii)

HBTΠ$_1$(P$_n$) = $\begin{cases} 2^{\left(\frac{n-1}{2}\right)}\left[(n^2+n)^2(n^2+5n+6)^2....(4n^2-10n+6)^2\right] ifnisodd \\ n^4 2^{\left(\frac{n-2}{2}\right)}\left[(n^2+3n+2)^2....(n^2-5n+6)^2\right] ifniseven \end{cases}$

(iv) HBTΠ$_2$(P$_n$) =

$$\begin{cases} 2^{\left(\frac{n-1}{2}\right)}\left(\left(\frac{(n^2-1)}{4}\frac{(n^2+1+2n)}{4}\right)^2\left(\frac{(n^2+4n+3)}{4}\frac{(n^2+9+3n)}{4}\right)^2 ... \left(n^2-3n+2)(n^2-2n+1\right)^2\right) ifnisodd \\ \left(\frac{n^4}{16}\right)^2 2^{\left(\frac{n-2}{2}\right)}\left[\left(\left(\frac{n^2}{4}+\frac{n}{2}\right)\left(\frac{n^2}{4}+n+1\right)\right)^2....\left(\left(n^2-3n+2\right)\left(n^2-2n+1\right)\right)^2\right) ifniseven\right] \end{cases}$$

Proof: Let P$_n$ be a path with n vertices. Then P$_n$ has n – 1 edges. Every edge of P$_n$is incident with exactly two vertices. T(P$_n$) has n point vertices and n – 1 line vertices.

Case(i): n is odd

Eccentricity of central vertex is $\left(\frac{n-1}{2}\right)$, eccentricity of pendant vertices (n – 1), eccentricity of line vertices incident with central vertex is $\left(\frac{n+1}{2}\right)$, eccentricity of line vertices incident with end vertices is (n – 1).

HBT$_1$(P$_n$) = $\sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2$

$$=\sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2+\sum_{ve}[(e_{T(G)}(v)+e_{T(G)}(e)]^2$$

$$=\sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]^2$$

$$=\left[\left(2\left(\left(\frac{n-1}{2}\right)+\left(\frac{n+1}{2}\right)\right)^2+\left(\left(\frac{n+1}{2}\right)+\left(\frac{n+1}{2}\right)\right)^2+\left(2\left(\left(\frac{n+1}{2}\right)+\left(\frac{n+3}{2}\right)\right)^2+\left(\left(\frac{n+3}{2}\right)+\left(\frac{n+3}{2}\right)\right)^2\right]\right.\right.$$

+ ….+[2[((n – 2) + (n –1))^2 + ((n – 1)(n – 1))^2]

$$=(n-1)\left(\left(2n^2+2n+1\right)+\left(2n^2+10n+13\right)+...+\left(4n^2+6-10n\right)\right)$$

HBT$_2$(P$_n$) = $\sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]^2$

$$=\sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$$=2\left[\left(\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)\right)^2+\left(\left(\frac{n-1}{2}+1\right)\left(\frac{n+1}{2}\right)\right)^2\right]$$

$$+2\left[\left(\left(\frac{n+1}{2}\right)\left(\frac{n+3}{2}\right)\right)^2+\left(\left(\frac{n+3}{2}\right)\left(\frac{n+3}{2}\right)\right)^2\right]+\ldots$$

$+ 2[((n – 2)(n – 1))^2 + ((n – 1)(n – 1))^2]$

$$= 2\left[\left(\frac{n^2-1}{4}\right)^2+\left(\frac{n+1}{2}\right)^4\right]+2\left[\left(\frac{n^2+4n+3}{4}\right)^2+\left(\frac{n+3}{2}\right)^4\right]+\ldots$$

$+ 2[(n^2 – 3n + 2)^2 + (n – 1)^4]$

$$= (n-1)\left(\left(\frac{4n^4+2n^3+2n^2+2n+1}{8}\right)^2+\ldots.+\left(n^2-3n+2\right)^2\left(n^2-2n+1\right)^2\right)$$

$$HBT\Pi_1(P_n) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]^2$$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]^2$$

$$=\left[2\left(\left(\frac{n-1}{2}+\frac{n+1}{2}\right)^2\left(\frac{n-1}{2}+1+\frac{n+1}{2}\right)^2\right)\right]\quad\left[2\left(\left(\frac{n+1}{2}+\frac{n+3}{2}\right)^2\left(\frac{n+3}{2}+\frac{n+3}{2}\right)^2\right)\right]$$

$\ldots 2[((n–2)+(n–1))^2(n–1+n– 1)^2]$

$= 2(n^2 + n)^2\ 2(n^2 + 5n + 6)^2 \ldots.\ 2(4n^2 -10n + 6)^2$

$$= 2^{\left(\frac{n-1}{2}\right)}\left[(n^2+n)^2(n^2+5n+6)^2\ldots.(4n^2-10n+6)^2\right]$$

$$HBT\Pi_2(P_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$$=\left[2\left(\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)\right)^2\left(\left(\frac{n-1}{2}+1\right)\left(\frac{n+1}{2}\right)\right)^2\right]$$

$$\left[2\left(\left(\frac{n+1}{2}\right)\left(\frac{n+3}{2}\right)\right)^2\left(\left(\frac{n+3}{2}\right)\left(\frac{n+3}{2}\right)\right)^2\right]\ldots 2[((n–2)(n–1))^2((n – 1)(n – 1))^2]$$

$$= 2\left[\left(\frac{n^2-1}{4}\right)^2\left(\frac{n+1}{2}\right)^4\right]2\left[\left(\frac{n^2+4n+3}{4}\right)^2\left(\frac{n+3}{2}\right)^4\right]\ldots$$

$\times 2[(n^2 – 3n + 2)^2(n – 1)^4]$

$$= 2^{\left(\frac{n-1}{2}\right)}\left[\left(\frac{n^2-1}{4}\right)\left(\frac{n^2+1+2n}{4}\right)\right]^2\left[\left(\frac{n^2+4n+3}{4}\right)\left(\frac{n^2+9+3n}{4}\right)\right]^2 \ldots [(n^2 – 3n + 2)(n^2 – 2n + 1)]^2$$

Case(ii): n is even.

Eccentricity of central vertices $\left(\frac{n}{2}\right)$ and $\left(\frac{n}{2}\right)$, eccentricity of pendant vertices (n – 1), eccentricity of line vertices incident with central vertices $\left(\frac{n}{2}\right)$, eccentricity of line vertices incident with end vertices is (n – 1).

$HBT_1(P_n) = \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]^2$

$= (n/2 + n/2)^2 + (n/2 + n/2)^2 + 2\{(n/2 + n/2 + 1)^2 + (n/2 + 1 + n/2 + 1)^2\} + \ldots + 2[((n-2) + (n-1))^2 + ((n-1)+(n-1))^2)$

$= 2n^2 + 2\left((n+1)^2 + (n+2)^2\right) + \ldots + 2\left((2n-3)^2 + (2n-2)^2\right)$

$= 2n^2 + (n-2)\left(\left(2n^2+6n+5\right) + \ldots + \left(8n^2-20n+13\right)\right)$

$HBT_2(P_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]^2$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]^2$

$= ((n/2)\ (n/2))^2 + ((n/2)(n/2))^2 + 2[((n/2)\ (n/2+1))^2 + ((n/2+1)(n/2+1))^2] + \ldots + 2[((n-2)(n-1))^2 + ((n-1)\ (n-1))^2]$

$= \frac{n^4}{8} + 2\left(\frac{n-2}{2}\right)\left[\left(\frac{10n^4}{16}+\frac{2n^3}{4}+\frac{3n^2}{2}+2n+1\right)+\ldots+\left(2n^4-2n^3+19n^2+16n+5\right)\right]$

$= \frac{n^4}{8} + (n-2)\left[\left(\frac{10n^4}{16}+\frac{2n^3}{4}+\frac{3n^2}{2}+2n+1\right)+\ldots+\left(2n^4-2n^3+19n^2+16n+5\right)\right]$

$HBT\Pi_1(P_n) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]^2$

$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]^2$

$= ((n/2 + n/2)^2\ (n/2 + n/2)^2)$

$[2((n/2 + n/2+1)^2(n/2 + 1 + n/2 + 1)^2]\ldots$

$[2[(n-2 + n-1)^2(n-1 + n-1)^2]$

$= n^4\, 2\left(\left((n+1)(n+2)\right)^2 \ldots.. \left(n^2-5n+6\right)^2\right)$

$= n^4\, 2^{\left(\frac{n-2}{2}\right)}\left[(n^2+3n+2)^2 \ldots. (n^2-5n+6)^2\right]$

$HBT\Pi_2(P_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]^2$

$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$

$= [((n/2)(n/2))^2((n/2)(n/2))^2][2[((n/2)(n/2+1))^2((n/2+1)(n/2+1))^2] \ldots [2((n-2)(n-1))^2((n-1)(n-1))^2]$

$$= \left(\frac{n^4}{16}\right)^2 2\left[\left(\left(\frac{n^2}{4}+\frac{n}{2}\right)\left(\frac{n^2}{4}+\frac{n}{2}+\frac{n}{2}+1\right)\right)^2 + \ldots + ((n^2-2n+2)(n^2-2n+1))^2\right] =$$

$$\left(\frac{n^4}{16}\right)^2 2^{\left(\frac{n-2}{2}\right)}\left[\left(\left(\frac{n^2}{4}+\frac{n}{2}\right)\left(\frac{n^2}{4}+n+1\right)\right)^2 \ldots.\left(\left(n^2-3n+2\right)\left(n^2-2n+1\right)\right)^2\right)\right]$$

Theorem 2.11: Let C_n be a cycle with n ≥ 4 vertices. Then

(i) $HBT_1(C_n) = \begin{cases} 8n\left(\frac{n+1}{2}\right)^2 & \text{if n is even} \\ 2n^3 & \text{if n is even} \end{cases}$

(ii) $HBT_2(C_n) = \begin{cases} 2n\left(\frac{n+1}{2}\right)^4 & \text{if n is odd} \\ \frac{n^5}{8} & \text{if n is even} \end{cases}$

(iii) $HBT\Pi_1(C_n) = \begin{cases} n^{4n} & \text{if n is even} \\ (n+1)^{4n} & \text{if n is odd} \end{cases}$

(iv) $HBT\Pi_2(C_n) = \begin{cases} \left(\frac{n}{2}\right)^{8n} & \text{if n is even} \\ \left(\frac{n+1}{2}\right)^{8n} & \text{if n is odd} \end{cases}$

Proof: Let C_n be a cycle with n vertices. Then C_n has n edges. Every edge of C_n is incident with exactly two vertices.

Case(i): n is odd

Let n = 2k + 1. When n is odd, each point vertex of T(G) has eccentricity (n + 1) / 2 and each line vertex has eccentricity (n + 1) / 2.

$$HBT_1(C_n) = \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2$$

$$= \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2 + \sum_{ve}[(e_{T(G)}(v)+e_{T(G)}(e)]^2$$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]^2$$

$$= \sum [(k + 1 + k + 1)^2 + (k + 1 + k + 1)^2] = \sum [2(2k + 2)^2]$$

$$= 2n(2k + 2)^2 = 8n(k + 1)^2 = 8n\left(\frac{n+1}{2}\right)^2$$

$$HBT_2(C_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$$= \sum [((k + 1) (k + 1))^2 + ((k + 1) (k + 1))^2]$$

$$= \sum [(k + 1)^4 + (k + 1)^4] = \sum [2(k + 1)^4]$$

$$= 2n(k + 1)^4 = 2n\left(\frac{n+1}{2}\right)^4$$

$$HBT\Pi_1(C_n) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]^2$$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]^2$$

$$= \Pi [((k + 1 + k + 1))^2 . ((k + 1 + k + 1))^2]$$

$$= \Pi [(2k + 2)^2 (2k + 2)^2] = (2k + 2)^{4n} = (n + 1)^{4n}$$

$HBT\Pi_2(C_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]^2$

$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$

$= \Pi\ [((k + 1)(k + 1))^2((k + 1)(k + 1))^2] = \Pi\ (k + 1)^8$

$= (k + 1)^{8n} = \left(\frac{n+1}{2}\right)^{8n}$

Case(i): n is even.

Let n = 2k. In T(G), each point vertex has eccentricity n / 2and line has eccentricity n / 2.

Let n = 2k.

$HBT_1(C_n) = \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]^2$

$= \sum\ [\ (k + k)^2 + (k + k)^2] = \sum\ 2(2k)^2 = n8k^2 = 8n(n / 2)^2 = 2n^3$.

$HBT_2(C_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]^2$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]^2$

$= \sum\ [(k .k\ )^2 + (k\ .\ k)^2] = \sum(2k^4) = n2k^4 = n^5 / 8$.

$HBT\Pi_1(C_n) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]^2$

$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]^2$

$= \Pi\ [\ (k + k)^2\ .\ (k + k)^2] = \Pi\ (2k)^4 = (2k)^{4n} = n^{4n}$

$HBT\Pi_2(C_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]^2$

$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$

$= \Pi\ [(k \times k\ )^2(k\ \times k)^2] = \Pi\ (k)^8$

$= (k)^{8n} = \left(\frac{n}{2}\right)^{8n}$

Theorem 2.12: Let W_n be a wheelwith $n \geq 5$ vertices. Then

(i)$HBT_1(W_n) = 113n$

(ii) $HBT_2(W_n) = 214n$

(iii) $HBT\Pi1(Wn) = (720)^{2n}$.

(iv) $HBT\Pi_2(Wn) = (1944)^{2n}$.

Proof: Let W_n be a wheel with n + 1 vertices. Then W_n has 2n edges. Every edge of W_n is incident with exactly two vertices.

Let $W_n = K_1 + C_n$. Let v be the central vertex of W_n, and $v_1, v_2, \ldots, v_n$ be the vertices of C_n. We have n edges of G which are incident with the central vertex and n egdes on the cycle.

Let $E_1(G)$ = {set of all edges incident with central vertex}. $E_2(G)$ = {set of all edges on with cycle}. If $e_i = vv_i \in E_1(G)$, then $e_T(e_i) = 2$ and $e_T(v) = 2$, $e_T(v_i) = 3$. If $e_{ii+1} = v_iv_{i+1} \in E_2(G)$. Then $e_T(e_{ii+1}) = 3$, $e_T(v_i) = 3$.

$HBT_1(W_n) = \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2$

$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]^2$

$$= \sum_{e\in E_1}((2+2)^2+(2+3)^2) + \sum_{e\in E_2}((3+3)^2+(3+3)^2) = \Sigma(41) + \Sigma(72) = 113n.$$

$$HBT_2(W_n) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$$= \sum_{e\in E_1}((2.2)^2+(2.3)^2) + \sum_{e\in E_2}((3.3)^2+(3.3)^2) = \Sigma(52) + \Sigma(162) = 214n.$$

$$HBT\Pi_1(W_n) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]^2$$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]^2$$

$$= \prod_{e\in E_1}[(2+2)^2.(2+3)^2]\prod_{e\in E_2}[(3+3)^2.(3+3)^2] = \Pi(20)^2\Pi(36)^2$$

$$= (20)^{2n}.(36)^{2n} = (720)^{2n}.$$

$$HBT\Pi_2(W_n) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \prod_{ue}[(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$$= \prod_{e\in E_1}[(2.2)^2.(2.3)^2]\prod_{e\in E_2}[(3.3)^2.(3.3)^2] = \Pi(24)^2\Pi(81)^2$$

$$= (24)^{2n}.(81)^{2n} = (1944)^{2n}.$$

Remark: (i)$HBT_1(W_4) = 64$ (ii) $HBT_2(W_4) = 64$ (iii) $HBT\Pi_1(W_4) = (256)^2$(iv) $HBT\Pi_1(W_4) = (256)^2$

Theorem 2.13: Let $K_{1,n}$ be a star graph. Then

(i)$HBT_1(K_{1,n}) = 25n$.

(ii) $HBT_2(K_{1,n}) = 20n$.

(iii) $HBT\Pi_1(K_{1,n}) = (12)^{2n}$.

(iv) $HBT\Pi_2(K_{1,n}) = (8)^{2n}$.

Proof : Let $K_{1,n}$ be a star graph with n + 1 vertices and n edges. Every edge of $K_{1,n}$ is incident with exactly two vertices. Let u be a central vertex. Eccentricity of u is one and all other point and line vertices are of eccentricity two in T(G).

$$HBT_1(K_{1,n}) = \sum_{ue}[(e_{T(G)}(u)+e_{T(G)}(e)]^2$$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u)+e_{T(G)}(e)+(e_{T(G)}(v)+e_{T(G)}(e))]^2$$

$$= \Sigma[(1+2)^2 + (2+2)^2] = \Sigma(25) = 25n.$$

$$HBT_2(K_{1,n}) = \sum_{ue}[(e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \sum_{uv\in E(G)}[((e_{T(G)}(u).e_{T(G)}(e)+(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$$= \Sigma[(1.\,2)^2 + (2\,.\,2)^2] = \Sigma(20) = 20n.$$

$$HBT\Pi_1(K_{1,n}) = \prod_{ue}[e_{T(G)}(u)+e_{T(G)}(e)]^2$$

$$= \prod_{uv\in E(G)}[(e_{T(G)}(u)+e_{T(G)}(e)).(e_{T(G)}(v)+e_{T(G)}(e))]^2$$

$$= \Pi\,[(1+2)^2\,(2+2)^2] = \Pi\,(12)^2 = (12)^{2n}.$$

$$HBT\Pi_2(K_{1,n}) = \prod_{ue}[e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \prod_{ue} [(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$= \Pi\ [(1\ .\ 2)^2(2\ .\ 2)^2] = \Pi\ (8)^2 = (8)^{2n}$.

Theorem 2.14: Let $K_{m,n}$ be a complete graph with $2 \le m \le n$. Then

(i)$HBT_1(K_{m,n}) = 32mn$.

(ii) $HBT_1(K_{m,n}) = 32mn$.

(iii) $HBT\Pi_1(K_{m,n}) = (16)^{2mn}$.

(iv) $BT\Pi_2(K_{m,n}) = (16)^{2mn}$.

Proof : Let $K_{m,n}$ be a complete bipartite graph with m + n vertices, mn edges and $|V_1| = m$, $|V_1| = n$, $V(K_{m,n}) = V_1 \cup V_2$. Every edge of $K_{m,n}$ is incident with exactly two vertices. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Every point vertices and line vertices have eccentricity 2 in T(G).

$$HBT_1(K_{m,n}) = \sum_{ue} [(e_{T(G)}(u) + e_{T(G)}(e)]^2$$

$$= \sum_{uv\in E(G)} [((e_{T(G)}(u) + e_{T(G)}(e) + (e_{T(G)}(v) + e_{T(G)}(e))]^2$$

$= \sum[(2 + 2)^2 + (2 + 2)^2] = \sum (32) = 32mn$.

$$HBT_2(K_{m,n}) = \sum_{ue} [(e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \sum_{uv\in E(G)} [((e_{T(G)}(u).e_{T(G)}(e) + (e_{T(G)}(v).e_{T(G)}(e))]^2$$

$= \sum[(2\ .\ 2)^2 + (2\ .\ 2)^2] = \sum (32) = 32mn$.

$$HBT\Pi_1(K_{m,n}) = \prod_{ue} [e_{T(G)}(u) + e_{T(G)}(e)]^2$$

$$= \prod_{uv\in E(G)} [(e_{T(G)}(u) + e_{T(G)}(e)).(e_{T(G)}(v) + e_{T(G)}(e))]^2$$

$= \Pi\ [(2 + 2)^2\ .\ (2 + 2)^2]= \Pi\ (16)^2 = (16)^{2mn}$.

$$HBT\Pi_2(K_{m,n}) = \prod_{ue} [e_{T(G)}(u).e_{T(G)}(e)]^2$$

$$= \prod_{ue} [(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$$

$= \Pi\ [(2\ .2)^2\ .\ (2.2)^2] = \Pi\ (16)^2 = (16)^{2mn}$.

Corollary 2.2: Let $K_{n,n}$ be a complete bipartite graph. Then

(i)$HBT_1(K_{n,n}) = 32n^2$.

(ii) $HBT_2(K_{n,n}) = 32n^2$.

(iii) $HBT\Pi_1(K_{n,n}) = (16)^{2n^2}$.

(iv) $HBT\Pi_2(K_{n,n}) = (16)^{2n^2}$

Proof: Put m = n in the previous theorem.

Theorem 2.15: Let F_n be a fan graph. Then

(i)$HBT_1(F_n) = 41n + 72(n - 1)$

(ii) $HBT_2(F_n) = 52n + 162(n - 1)$.

(iii) $HBT\Pi_1(F_n) = (20)^{2n}\ (36)^{2(n-1)}$.

(iv) $HBT\Pi_2(F_n) = (24)^{2n}(81)^{2(n-1)}$.

Proof : Let F_n be a fan graph with n + 1 vertices and 2n – 1 edges.

Let $F_n = K_1 + P_n$. Let v be a cetral vertex of F_n, and $v_1, v_2, v_3, \ldots, v_n$ be the vertices of P_n. We have n edges of G which are incident with the central vertex and n – 1 edges on the path. Let $E_1(G)$ = {set of all edges incident with central vertex v}. $E_2(G)$ = {set of all edges on the path P_n}. If $e_i = vv_i \in E_1(G)$, then $e_T(e_i) = 2$ and $e_T(v) = 2$, $e_T(v_i) = 3$. If $e_{ii+1} = v_iv_{i+1} \in E_2(G)$. Then $e_T(e_{ii+1}) = 3$, $e_T(v_i) = 3$.

$$HBT_1(F_n) = \sum_{ue} [(e_{T(G)}(u) + e_{T(G)}(e)]^2$$

$= \sum_{uv \in E(G)} [((e_{T(G)}(u) + e_{T(G)}(e) + (e_{T(G)}(v) + e_{T(G)}(e))]^2$

$= \sum_{e \in E_1} ((2+2)^2 + (3+2)^2) + \sum_{e \in E_2} ((3+3)^2 + (3+3)^2) = \sum(41) + \sum(72)$

$= 41n + 72(n - 1)$.

$HBT_2(F_n) = \sum_{ue} [(e_{T(G)}(u).e_{T(G)}(e)]^2$

$= \sum_{uv \in E(G)} [((e_{T(G)}(u).e_{T(G)}(e) + (e_{T(G)}(v).e_{T(G)}(e))]^2$

$= \sum_{e \in E_1} ((2.2)^2 + (3.2)^2) + \sum_{e \in E_2} ((3.3)^2 + (3.3)^2) = \sum(52) + \sum(162)$

$= 52n + 162(n - 1)$.

$HBT\Pi_1(F_n) = \prod_{ue} [e_{T(G)}(u) + e_{T(G)}(e)]^2$

$= \prod_{uv \in E(G)} [(e_{T(G)}(u) + e_{T(G)}(e)).(e_{T(G)}(v) + e_{T(G)}(e))]^2$

$= \prod_{e \in E_1} [(2+2)^2.(2+3)^2] \prod_{e \in E_2} [(3+3)^2.(3+3)^2] = \Pi (20)^2 \Pi (36)^2$

$= (20)^{2n}(36)^{2(n-1)}$.

$HBT\Pi_2(F_n) = \prod_{ue} [e_{T(G)}(u).e_{T(G)}(e)]^2$

$= \prod_{ue} [(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$

$= \prod_{e \in E_1} [(2.2).(2.3)]^2 \prod_{e \in E_2} [(3.3).(3.3)]^2 = \Pi (24)^2 \Pi (81)^2$

$= (24)^{2n}(81)^{2(n-1)}$.

Theorem 2.16: (i) $HBT_1(K_{2n} - F) = 64n(n - 1)$.

(ii) $HBT_2(K_{2n} - F) = 64n(n - 1)$.

(iii) $HBT\Pi_1(K_{2n} - F) = (16)^{2(2n^2 - 2n)}$.

(iv) $HBT\Pi_2(K_{2n} - F) = (16)^{2(2n^2 - 2n)}$.

Proof : Let K_{2n} be a complete graph with 2n vertices and $\frac{2n(2n-1)}{2} = 2n^2 - n$ edges. F is a 1-factor of K_{2n}. $K_{2n} - F$ has $2n^2 - n - n$ edges $= 2n^2 - 2n$ edges. Every point vertices and line vertices have eccentricity two in $T(K_{2n} - F)$.

$HBT_1(K_{2n} - F) = \sum_{ue} [(e_{T(G)}(u) + e_{T(G)}(e)]^2$

$= \sum_{uv \in E(G)} [((e_{T(G)}(u) + e_{T(G)}(e) + (e_{T(G)}(v) + e_{T(G)}(e))]^2$

$= \sum[(2 + 2)^2 + (2 + 2)^2] = \sum 32 = 32(2n^2 - 2n) = 64n(n - 1)$.

$HBT_2(K_{2n} - F) = \sum_{ue} [(e_{T(G)}(u).e_{T(G)}(e)]^2$

$= \sum_{uv \in E(G)} [((e_{T(G)}(u).e_{T(G)}(e) + (e_{T(G)}(v).e_{T(G)}(e))]^2$

$= \sum[(2 . 2)^2 + (2 . 2)^2] = \sum 32 = 32(2n^2 - 2n) = 64n(n - 1)$.

$HBT\Pi_1(K_{2n} - F) = \prod_{ue} [e_{T(G)}(u) + e_{T(G)}(e)]^2$

$= \prod_{uv \in E(G)} [(e_{T(G)}(u) + e_{T(G)}(e)).(e_{T(G)}(v) + e_{T(G)}(e))]^2$

$= \Pi\ [(2 + 2)^2 . (2 + 2)^2] = \Pi\ (16)^2 = (16)^{2(2n^2 - 2n)}$.

$HBT\Pi_2(K_{2n} - F) = \prod_{ue} [e_{T(G)}(u).e_{T(G)}(e)]^2$

$= \prod_{ue} [(e_{T(G)}(u).e_{T(G)}(e)).(e_{T(G)}(v).e_{T(G)}(e))]^2$

$= \Pi\ [(2\ .\ 2)^2 . (2\ .2)^2] = \Pi\ (16)^2 = (16)^{2(2n^2 - 2n)}$.

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