Mathematical modelling of Transient Free Convection MHD Flow Past a Vertical Plate with Periodic Temperature Using Homotopy Perturbation Method

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ABSTRACT

A mathematical model is used to study the effect of the magnetic field on the temporary free convection flow of the electrically conductive fluid past the vertical plate under periodic temperature. The governing momentum and energy equations are solved analytically using the homotopy perturbation method. An approximate analytical solution using Homotopy perturbation method and Laplace transform is carried out for temperature profile in case of constant plate temperature. The results are obtained the analytical and limiting case results ones, and a good agreement is achieved. The effect of different physical parameters on transient velocity and temperature, such as Grashof number, magnetic parameter, Prandtl number and temperature frequency, is examined. Also, the local Skin-friction and local Nusselt number coefficients are obtained and analyzed.

Keywords: Mathematical modelling; Boundary layer; Laplace transform; Homotopy perturbation method; MHD flows; Vertical plate.

1. Introduction

Convective flows and MHD (Magneto Hydro Dynamics) flows in porous media have been widely studied over the past numerous decades, with many distinct physical effects. Plasmas, liquid metals, saltwater, and electrolytes are examples of magneto fluids. These flows in many natural contexts, such as groundwater flows, geothermal extraction, industrial and agricultural distribution of water, oil recovery systems, cooling of electronic components, food processing, dispersion of chemical contaminants in various functions in the chemical industry and in the environment, soil degradation, etc.

It is noted that the surface temperature depends on many fluid flow problems. A similar study too has become presented to discuss the MHD flow with variable temperature. Proposed to examine the effects of the transverse magnetic field on transient-free Convective nanofluid flow past and impulsively started infinite vertical porous plate with viscous dissipation [1]. Analytical solution of unsteady flow past an accelerated vertical plate at constant temperature has been carried out [2]. Examined the effect of the unsteady MHD mass transfer flow through a vertical porous fixed plate in the presence of viscous dissipation and heat source [3]. Homotopy Analysis Method is used to obtain the analytical expression of the stream function, temperature distribution and volume fraction of nanofluid [4]. To study the transient free convection magnetohydrodynamic flow past an accelerated vertical plate with periodic plate temperature. A numerical analysis of the dimensionless equations is performed and verified by an analytical solution for constant plate temperature [5].

Investigate the effects of MHD as well as Soret on the unsteady free convective mass transfer flow past an infinite vertical plate of variable suction, where the temperatures of a plates oscillate at the same frequency as the one of variable suction velocity [6]. The examined a mathematical model to study the characteristics of heat and mass transfer in mixed convection flow. The Linearly stretching vertical surface in a porous medium filled with visco-elastic fluid, taking into account heat diffusion (Dufour) and temperature diffusion (Soret) effects [7]. To study the free convective flow through a porous medium past a vertical plate with a ramped wall temperature in the presence of a magnetic field is discussed [8].

Samad and Rahman [9] investigated the relationship of thermal radiation with an unstable MHD flow past a vertical porous plate embedded in a porous medium.

Ibrahim et al.[10] investigated the effect of thermal radiation on porous media through optically thick approximat ion using the Newton Scheme process of the Taylor series. Biihler and Zierep[11] studied the flow of an incompressible viscous fluid near a porous oscillating infinite plate with suction or blowing condition. Das et al.[12] performed an analytical investigation to study the radiation effect on natural convective flow past a vertical plate in the presence of a porous medium. Cogley et al. [13] are often used to describe the radiative heat transfer model. Khan et al. [14] solved the long porous slider problem by a homotopy perturbation method, which is coupled with differential equations resulting from the momentum equation. Esmaeilpour and D.D. Ganji [15] have used the homotopy perturbation method to solve boundary layer flow and convection heat transfer over a flat plate.

Recently, Visuvasam et al. [16] derived an analytical expression of the current generated by an electrochemical reaction in the porous rotating disc electrode (PRDE) using the homotopy perturbation method. Kumar [17] investigated a new approximate method, namely homotopy perturbation transform method (HPTM) which is a combination of homotopy perturbation method (HPM) and Laplace transform method (LTM) to provide an approximate analytical solution to time-fractional Cauchy-reaction diffusion equation.

In this study, an analytical approximation to the solution of the problem of forced convection over a horizontal flat plate using a combination of Homotopy perturbation method and Laplace transform is presented. The effect of different physical parameters on transient velocity and temperature, such as Grashof number, magnetic parameter, Prandtl number and temperature frequency is analyzed. Also, the local Skin-friction and local Nusselt number coefficients are obtained.

2. Mathematical formulation

Consider a two-dimensional MHD flow of an incompressible electrically conducting viscous fluid past an accelerated infinite vertical plate. A conductive liquid with a density ρ, a dynamic viscosity μ, and an electrical conductivity σ fills the region around the plate. The coordinates x, y is aligned, respectively, along the plate axis, and width. The plate is placed in a uniform transverse magnetic field of flux density Bo in the y-direction. When applying the magnetic field with a presence of periodic plate temperature, the transient governing equations of the MHD flow that present the fluid motion and temperature are as follows [5]:

$$\frac{\partial u}{\partial t} - g \ \beta \,(T - T_\infty) + \nu \ \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \ B_0^2}{\rho} u \tag{1}$$

$$\rho \ C_p \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} \tag{2}$$

with initial and boundary conditions:

$$t \le 0: \ u = 0, \quad T = T_\infty \quad \text{for all} \ \ y$$

$$t > 0: \ u = \left(\frac{u_0^2}{\nu}\right) t, \quad T = T_w + \in (T_w - T_\infty) \ \cos \varpi t \quad \text{at} \ \ y = 0 \tag{3}$$

$$u \to 0, \ T \to T_\infty \quad as \ \ y \to \infty$$

where T, T_w and T_∞ denote the temperature, wall temperature and initial fluid temperature respectively, u is the velocity component in the x-direction, K is the thermal conductivity, C_p is the specific heat, ϖ is the frequency of oscillation and ε is a small reference parameter. The governing equations can be written in dimensionless form using the following non-dimensional quantities:

$$Y = \frac{y \ u_0}{\nu}, \ \tau = \frac{t \ u_0^2}{\nu}, \ \omega = \frac{\nu \ \varpi}{u_0^2}, \ U = \frac{u}{u_0}, \ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \ M = \frac{\sigma \ \nu \ B_0^2}{\rho \ u_0^2},$$
$$Gr = \frac{g \ \beta \nu \ (T_w - T_\infty)}{u_0^2}, \ \Pr = \frac{\mu \ C_p}{k} \tag{4}$$

where τ, ω, U, θ, M, Gr and Pr are dimensionless time, dimensionless frequency, dimensionless velocity, dimensionless temperature, magnetic parameter, Grashof number and Prandtl number, respectively.
The dimensionless of Nonlinear partial differential equations becomes [5]:

$$\frac{\partial U}{\partial \tau} = Gr\,\theta + \frac{\partial^2 U}{\partial Y^2} - M \ \ U \tag{5}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\Pr} \frac{\partial \theta}{\partial Y^2} \tag{6}$$

The corresponding boundary conditions can be specified as follows [5]:
Initially,

$$\tau \le 0: \ U = 0, \ \ \theta = 0 \quad \text{for all} \ \ Y$$

$$\tau > 0: \ \ U = \tau, \ \ \theta = 1 + \in \cos \omega t \quad \text{at} \ \ Y = 0 \tag{7}$$

$$U \to 0, \ \theta \to 0 \quad as \ \ Y \to \infty$$

3. Approximate Analytical Expression for the Concentrations of Velocity and Temperature

Approximate analytical expressions for concentrations of Velocity and Temperature are given by (see information in Appendix A) as follows:

$$U(Y,t) = t\ \exp(-\sqrt{M}\,Y) + \frac{Gr}{M}\ erfc\left(\frac{\sqrt{\Pr}\ Y}{2\ \sqrt{t}}\right) - \frac{Gr}{M}\ \exp(-\sqrt{M}\,Y) \tag{8}$$

$$\theta(Y,t) = \exp(-\sqrt{\Pr\ s}\,Y) + \in \cos(\omega t)\,\frac{\sqrt{\Pr}\ Y\ \exp\left(\frac{\Pr\ Y^2}{4\ t}\right)}{2\ \sqrt{\pi\ t^3}} \tag{9}$$

The dimensionless local Skin friction at the plate is given by [8]

$$C_f = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0} = \frac{t^{3/2}\ M^2\ \sqrt{\pi} + Gr\ \sqrt{\Pr}\,\sqrt{M} - Gr\sqrt{\pi}\,\sqrt{t}}{M^{3/2}\ \sqrt{\pi}\,\sqrt{t}} \tag{10}$$

The dimensionless Nusselt number is given by [8]

$$\text{Reduced Nusselt number} = Nu\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} = \frac{\sqrt{\Pr}\ \left(\in \cos(\omega\ t)\ \sqrt{\pi}\,\sqrt{t} - 2\ \sqrt{\pi\, t^3}\right)}{2\sqrt{\pi\, t^3}\ \sqrt{\pi}\ \sqrt{t}} \tag{11}$$

4. Steady-State Solution for the Concentrations of Velocity and Temperature

For $t = 0$, the exact solution of Eqs. (5) -(7) is immediately obtained,

$$U(Y) = \frac{Gr\ (1+\in)\left(\exp\left(-M^{\left(\frac{1}{2}\right)}Y\right) - \exp\left(-\Pr^{\left(\frac{1}{2}\right)}\ Y\right)\right)}{M - \Pr} \tag{12}$$

$$\theta(Y) = (1+\in)\ \exp\left(-\sqrt{\Pr}\ Y\right) + \frac{\sqrt{\Pr}\ \ Y\ (1+\in)\ \exp\left(-\Pr^{\left(\frac{1}{2}\right)}\ Y\right)}{2} \tag{13}$$

The dimensionless local Skin friction at the plate is given by

$$C_f = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0} = \frac{Gr\ (1+\in)\ (-\sqrt{M} + \sqrt{\Pr})}{M - \Pr} \tag{14}$$

The dimensionless rate of heat transfer coefficient in terms of the Nusselt number is given by

$$\text{Reduced Nusselt number} = Nu\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} = \frac{1}{2}\ (1+\in)\ \sqrt{\Pr} \tag{15}$$

5. Results and Discussions

In this study, Transient Free Convection MHD Flow Past a Vertical Plate with Periodic Temperature has been investigated. The governing equation which is a pair of partial differential equations, were solved analytically by using the homotopy perturbation method (HPM) and Laplace transform.

Figs.1(a) ad 3(a), demonstrates the dimensionless velocity profile for different values of magnetic field parameter M for a physical situation with uniform chemical reaction and thermophoretic effects. It shows that the velocity is considerably reduced with the increase of in the value of M because the application of transverse magnetic field opposes the transport phenomena due to the fact that the presence of a magnetic field produces a drag-like force called the Lorentz force in other words, any decrease in the fluid angle on the inclined plate leads to an increase in the flow of the velocity profile.

The result illustrates that the increase in Gr increases the flow velocity. On the Grashof numbers, the increase in the concentration leads to a rise in the concentration gradient existing between the fluids. The increase in the gradient, the fluid particles gain enough energy to become buoyant. The energy gained tends to lose the fluid particles from the grip of viscosity. The buoyancy force acting as lifting force for the fluid particles tends to increase the flow velocity; see Figs. 1(b) and 3(b). This agrees with [18, 19].

Prandtl number is a dimensionless quantity that puts the viscosity of a fluid in correlation with the thermal conductivity. Figs. 1(c) and 3(c) represented an increase in Pr results in a decrease in the velocity profile.

Figs. 2 and 4 indicates that a rise in Pr substantially reduces the temperature in the viscous fluid. It can be found from Figs. 2 and 4 that the solute boundary layer thickness of the fluid enhances with the increase of Pr.

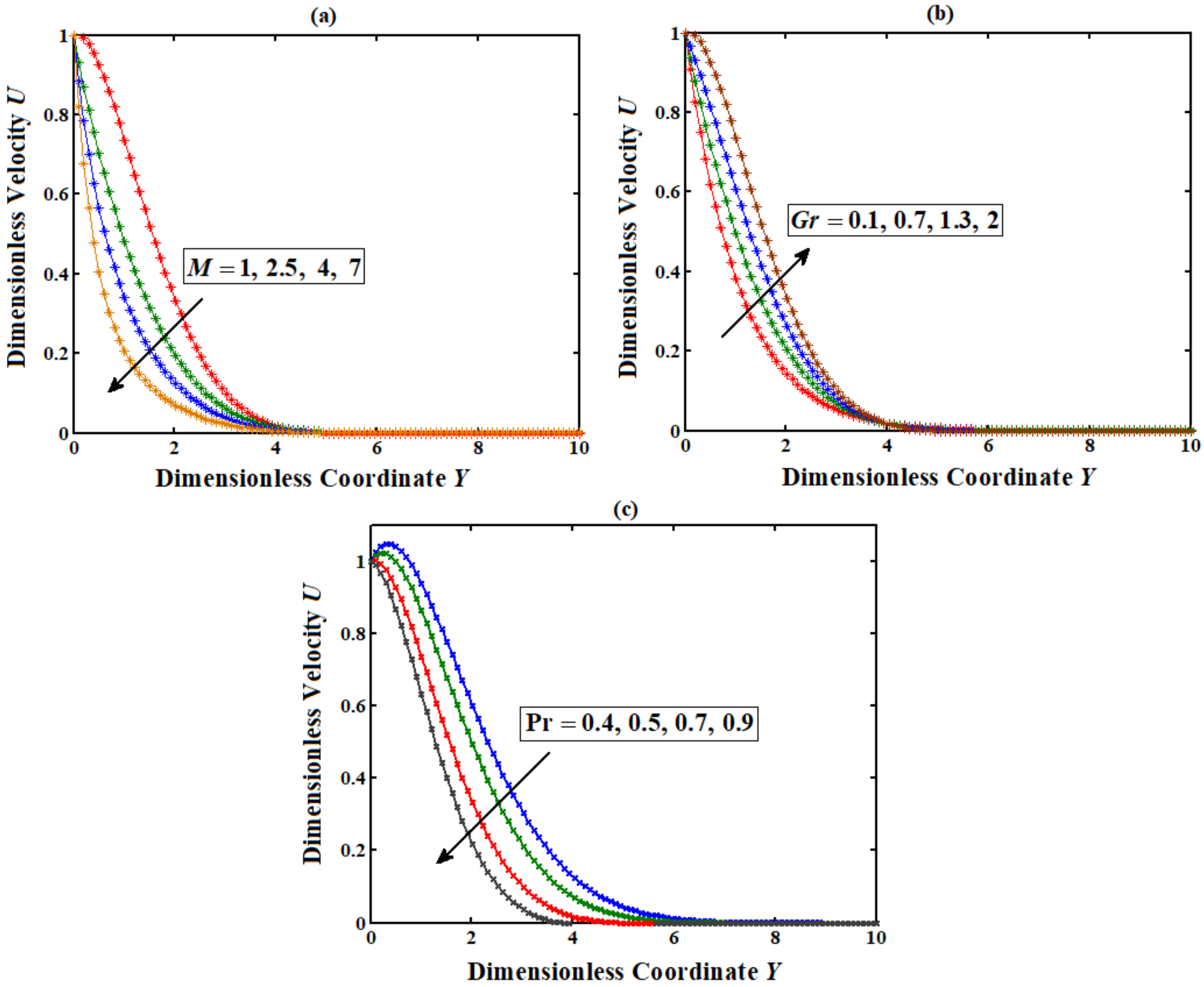

Fig.1. Dimensionless concentration of velocity profile (U) for against Y for $\Pr = 0.7,\ Gr = 2,\ M = 1$ and $t = 1$

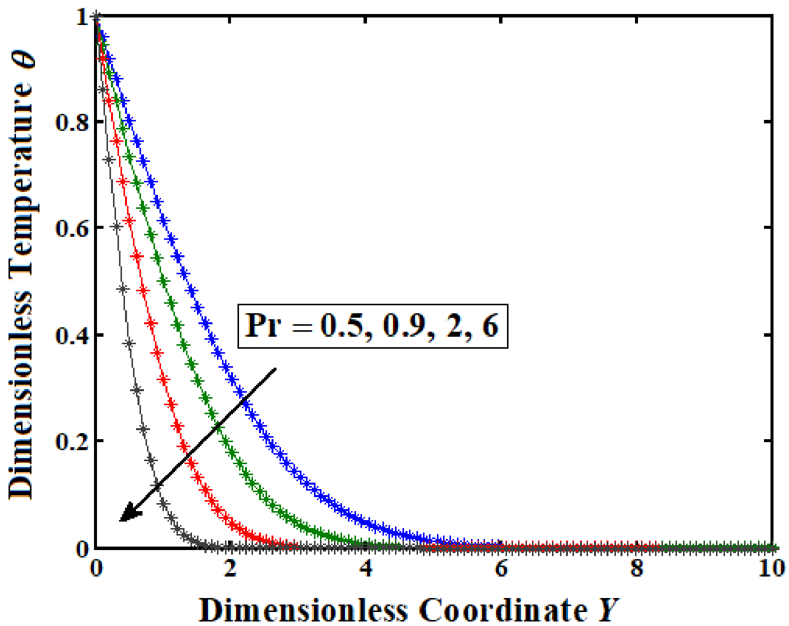

Fig.2. Dimensionless concentration of temperature profile (θ) for against Y for $\in = 0.1,\ \omega = 30$ and $t = 1$

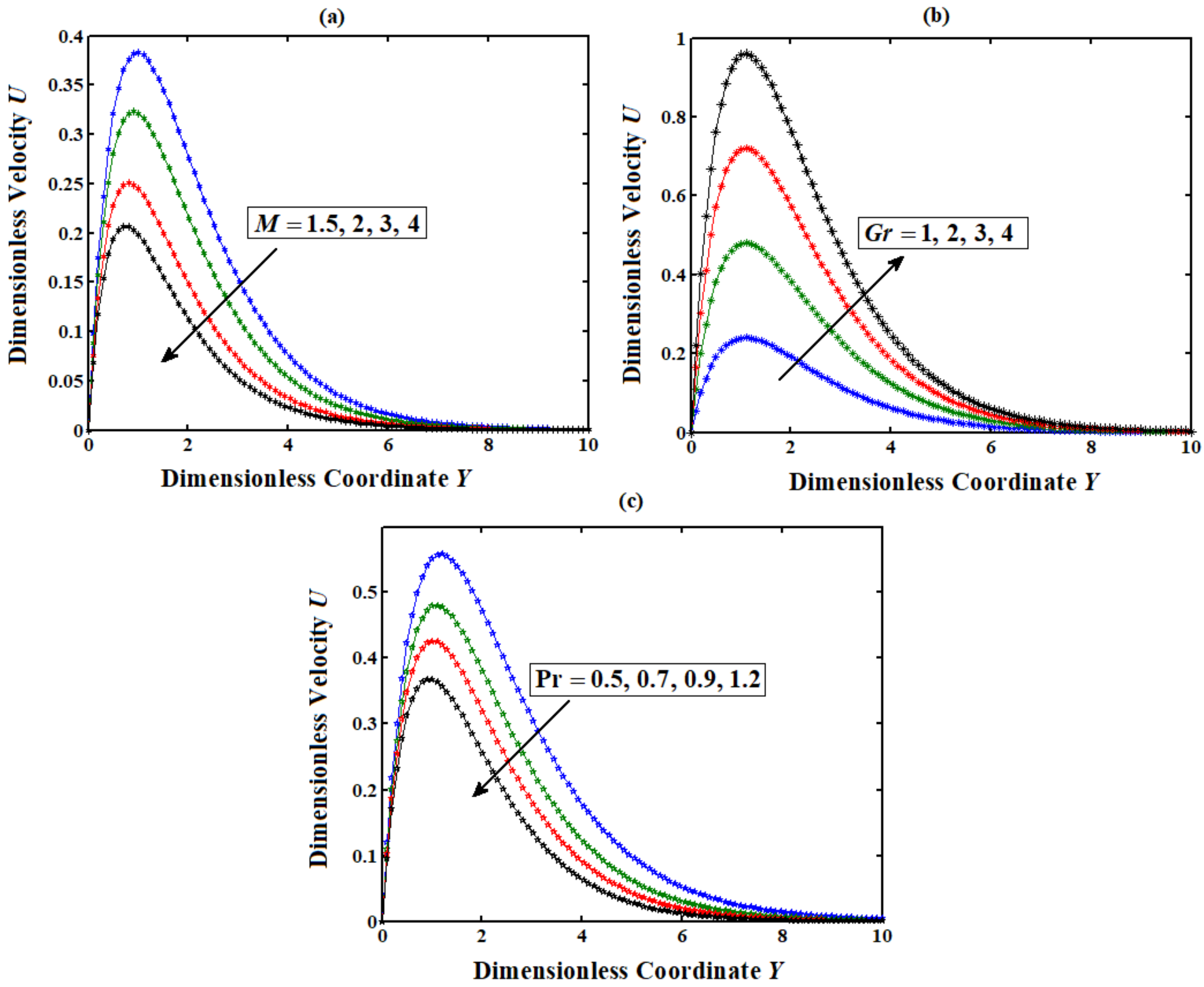

Fig.3. Dimensionless concentration of velocity profile (U) for against Y for $\Pr = 0.7,\ Gr = 2,\ M = 1$ and $t = 0$

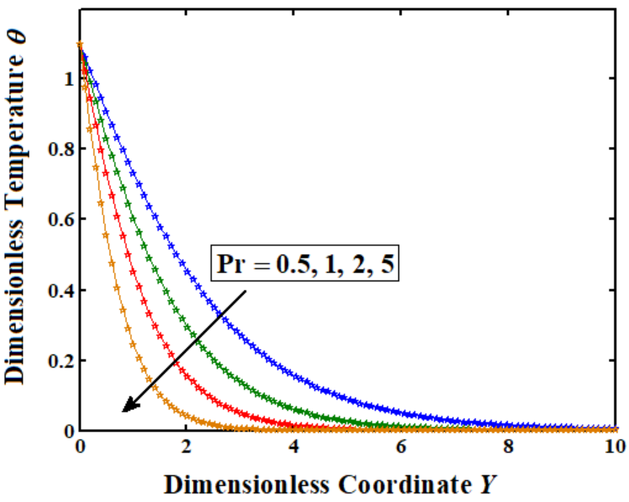

Fig.4. Dimensionless concentration of temperature profile (θ) for against Y for $\in = 0.1,\ \omega = 30$ and $t = 0$

6. Conclusion

An analytical method is performed to predict the velocity and temperature distribution in a free convection flow past a vertical plate with periodic temperature. The effect of Grashof number, magnetic parameter, Prandtl number, and temperature frequency on the transient velocity and temperature profiles is studied. Also, the local Skin-friction and local Nusselt number coefficients are obtained. The velocity is shown to decrease as the magnetic parameter (M) increases, which means that a somewhat conductive fluid is not disturbed by the influence of a magnetic field. The increase in the Grashof number is shown to produce an increase in the velocity across the plate.

It is seen that the velocity and temperature will be decreased by the (Pr) Prandtl number. In the transient temperature profile, the periodic behaviour of the plate temperature is expressed, while in the velocity profile, this behaviour is not measurable.

Appendix - A

Analytical solution of equations (5) - (7) using the Homotopy perturbation method as follows:

Equations (5) – (7) can be written as follows:

$$\frac{\partial U}{\partial \tau} = Gr\,\theta + \frac{\partial^2 U}{\partial Y^2} - M\ U \tag{A1}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1+}{\Pr}\frac{\partial \theta}{\partial Y^2} \tag{A2}$$

The corresponding boundary conditions can be specified as follows:

Initially,

$$\tau \le 0:\ U = 0,\quad \theta = 0 \quad \text{for all } Y$$

$$\tau > 0:\ U = \tau,\ \theta = 1 + \in \cos\omega t \quad \text{at } Y = 0 \tag{A4}$$

$$U \to 0,\ \theta \to 0 \quad as\ \ Y \to \infty$$

To find the solution of Equation (A1) and (A2) take Laplace transformation.

$$\frac{\partial^2 \overline{U}}{\partial Y^2} - M\,\overline{U} + Gr\,\overline{\theta} - s\overline{U}(x,s) + \overline{U}(x,0) = 0 \tag{A5}$$

$$\frac{\partial \overline{\theta}}{\partial Y^2} - \Pr\left(s(x,s) + \overline{\theta}(x,0)\right) = 0 \tag{A6}$$

under the boundary conditions are:

$$\tau \le 0:\ \overline{U} = 0,\quad \overline{\theta} = 0 \quad \text{for all } Y \tag{A7}$$

$$\tau > 0:\ \overline{U} = \frac{1}{s^2},\ \overline{\theta} = \frac{1}{s} + \in \frac{s}{s^2+\omega^2} \quad \text{at } Y = 0 \tag{A8}$$

$$\overline{U} \to 0,\ \overline{\theta} \to 0 \quad as\ \ Y \to \infty \tag{A9}$$

Using the boundary condition (A7) in equation (A5) and (A6) we write

$$\frac{\partial^2 \overline{U}}{\partial Y^2} - M\,\overline{U} + Gr\,\overline{\theta} - s\overline{U} = 0 \tag{A10}$$

$$\frac{\partial \overline{\theta}}{\partial Y^2} - \Pr s\overline{\theta} = 0 \tag{A11}$$

Homotopy for the above equations (A10) and (A11) can be constructed as follows:

$$\left(1-p\right)\left[\frac{\partial^2 \overline{U}}{\partial Y^2} - M\,\overline{U}\right] + p\left[\frac{\partial^2 \overline{U}}{\partial Y^2} - M\,\overline{U} + Gr\,\overline{\theta} - s\overline{U}\right] = 0 \tag{A12}$$

$$\left(1-p\right)\left[\frac{\partial \overline{\theta}}{\partial Y^2} - \Pr s\overline{\theta}\right] + p\left[\frac{\partial \overline{\theta}}{\partial Y^2} - \Pr s\overline{\theta}\right] = 0 \tag{A13}$$

The approximate solution of the equations (A12) and (A13) are

$$\overline{U} = \overline{U}_0 + p\overline{U}_1 + p^2\overline{U}_2 + ... \tag{A14}$$

$$\overline{\theta} = \overline{\theta}_0 + p\overline{\theta}_1 + p^2\overline{\theta}_2 + ... \tag{A15}$$

Substituting Equations (A14) and (A15) into Equations (A12) and (A13) and comparing the coefficients of like powers $'p'$

$$p^0 : \frac{\partial^2 \overline{U_0}}{\partial Y^2} - M\,\overline{U_0} = 0 \tag{A16}$$

$$p^1 : \frac{\partial^2 \overline{U_1}}{\partial Y^2} - M\,\overline{U_1} + Gr\,\overline{\theta_0} - s\overline{U_0} = 0 \tag{A17}$$

$$p^0 : \frac{\partial \overline{\theta_0}}{\partial Y^2} - \Pr s\overline{\theta_0} = 0 \tag{A18}$$

The initial conditions are

$$\tau > 0: \; \overline{U_0} = 0, \overline{U_0} = \frac{1}{s^2}, \; \overline{\theta_0} = \frac{1}{s}, \overline{\theta_1} = \in \; \frac{s}{s^2 + \omega^2} \; \text{ at } \; Y = 0 \tag{A19}$$

$$\overline{U_0} \to 0, \overline{U_1} \to 0, \; \overline{\theta_0} \to 0, \overline{\theta_1} \to 0 \;\; as \;\; Y \to \infty \tag{A20}$$

Solving Equations (A16) - (A18) with initial condition (A19) and (A20), yields

$$\bar{U}_0(Y, \; s) = 0 \tag{A21}$$

$$\bar{U}_1(Y, \; s) = \frac{1}{s^2} \exp(-\sqrt{M} \;\; Y) + \frac{Gr}{s \; M} \left(\exp(-\sqrt{\Pr s} \;\; Y) - \exp(-\sqrt{M} \;\; Y \right) \tag{A22}$$

$$\bar{\theta}_0(Y, s) = \frac{\exp(-\sqrt{\Pr} Y)}{s} \tag{A23}$$

$$\bar{\theta}_1(Y, s) = \frac{\in s}{s^2 + \omega^2} \left(\exp(-\sqrt{\Pr} Y) \right) \tag{A24}$$

Taking Inverse Laplace transform in equation (A21) – (A24)

$$\bar{U}_0(Y, \; t) = 0 \tag{A25}$$

$$\bar{U}_1(Y, \, t) = t \; \exp(-\sqrt{M} \, Y) + \frac{Gr}{M} \; erfc\left(\frac{\sqrt{\Pr} \;\; Y}{2 \; \sqrt{t}} \right) - \frac{Gr}{M} \; \exp(-\sqrt{M} \, Y) \tag{A26}$$

$$\bar{\theta}_0(Y, t) = \exp(-\sqrt{\Pr \; s} \, Y) \tag{A27}$$

$$\bar{\theta}_1(Y, \, t) = \in \cos(\omega t) \frac{\sqrt{\Pr} \;\; Y \;\; \exp\left(\frac{\Pr \; Y^2}{4 \; t} \right)}{2 \; \sqrt{\pi \;\; t^3}} \tag{A28}$$

The solutions are

$$U(Y, t) = t \; \exp(-\sqrt{M} \, Y) + \frac{Gr}{M} \; erfc\left(\frac{\sqrt{\Pr} \;\; Y}{2 \; \sqrt{t}} \right) - \frac{Gr}{M} \; \exp(-\sqrt{M} \, Y) \tag{A29}$$

$$\theta(Y, t) = \exp(-\sqrt{\Pr \; s} \, Y) + \in \cos(\omega t) \frac{\sqrt{\Pr} \;\; Y \;\; \exp\left(\frac{\Pr \; Y^2}{4 \; t} \right)}{2 \; \sqrt{\pi \;\; t^3}} \tag{A30}$$

References

1. V. Rajesh, M. P. Mallesh, O. Anwar Bég, Transient MHD free convection flow and heat transfer of nanofluid past an impulsively started vertical porous plate in the presence of viscous dissipation, Procedia Materials Science 10 (2015) 80 – 89.
2. Uwanta I., Yale I., 2014, Analytical Solution of Unsteady Flow Past an Accelerated Vertical Plate with Constant Temperature and Variable Mass Transfer, International Journal of Computational Engineering Research, 4, 50-55.
3. Mahender D, Rao P., 2015, Unsteady MHD Free Convection and Mass Transfer Flow Past a Porous Vertical Plate in Presence of Viscous Dissipation, Journal of physics: Conference Series, 662:1-6.
4. R. Angel Joy, J. Visuvasam, V. Rajendran, L. Rajendran, Theoretical Analysis of Flow of Nanofluid over a Stretching Surface, International Journal of Mathematical Archive-9(1), 2018, 95-112.
5. Abdullah M.R., 2018, Transient free convection mhd flow past an accelerated vertical plate with periodic temperature, Chemical Engineering Transactions, 66, 331-336 DOI:10.3303/CET1866056.
6. G. V. Ramana Reddy, Ch. V. Ramana Murthy, N. Bhaskar Reddy, Unsteady MHD Free Convective Mass Transfer Flow Past an Infinite Vertical Porous Plate with Variable Suction and Soret Effect, Int. J. Of Appl. Math and Mech. 7 (21): 70-84, 2011.
7. Hayat T, Mustafa M and Pop I (2010). Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco-elastic fluid. Communications in Nonlinear Science and Numerical Simulation, 15(5): 1183–1196.
8. A. Sinha, N. Ahmed and S. Agarwalla, MHD Free Convective Flow Through a Porous Medium Past a Vertical Plate with Ramped Wall Temperature, Applied Mathematical Sciences, Vol. 11, 2017, No. 20, 963 – 974.

9. M.D. Abdus Samad and M. Mansur Rahman, Thermal Radiation Interaction with Unsteady MHD Flow past a Vertical Porous Plate Immersed in a Porous Medium, Journal of Naval Architecture and Marine Engineering, **3** (2006), no. 1, 7-14. https://doi.org/10.3329/jname.v3i1.924.
10. M.O. Ibrahim, K.K. Asogwa, I.J. Uwanta, B.G. Dan Shehu, Thermal Radiation Effecton a Porous Media under Optically Thick Approximation, American Journal of Numerical Analysis, **1** (2013), no. 1, 15-21.
11. K. Bühler and J. Zierep, Instationäre Plattenströmung mit Absaugung and Ausblasen, ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik, **70** (1990), 589-590.
12. Sanatan Das, Mrinal Jana, Rabindra Nath Jana, Radiation Effect on Natural Convection near a Vertical Plate Embedded in a Porous Medium with Ramped Wall Temperature, Open Journal of Fluid Dynamics, **1** (2011), 1-11.
13. A.C.L. Cogley, W.G. Vincenti and E.S. Gilles, Differential Approximation for Radiative Transfer in a Nongray Gas near Equillibrium, American Institute of Aeronautics and Astronautics Journal, **6** (1968), no. 3, 551-553.
14. Y. Khan, N. Faraz, A. Yildirim and Q. Wu, A Series Solution of the Long Porous Slider, Tribology Transactions,54, 2, (2011) 187-191.
15. M. Esmaeilpour, D.D. Ganji, Application of He’s homotopy perturbation method to boundar layer flow and convection heat transfer over a flat plate, Physics Letters A 372, (2007) 33–38.
16. J. Visuvasam, A. Molina, E. Laborda, L. Rajendran, Mathematical Models of the Infinite Porous Rotating Disk Electrode, Int. J. Electrochem. Sci.,13(2018) 9999
17. S. Kumar, A new fractional modelling arising in Engineering Sciences and its analytical approximate solution, Alexandria Eng. J. 52 (4) (2013) 813–819, http://dx.doi.org/10.1016/j.aej.2013.09.005.
18. F.G. Awad, P. Sibanda, M. Narayana, S.S. Motsa, Convection from a semi-finite plate in a fluid saturatedporous medium with cross-diffusion and radiative heattransfer, Int. J. Phys. Sci. 6 (21) (2011) 4910–4923, https://doi.org/10.5897/IJPS11.295.
19. S.M.M. Hossain, SdRahman, Md.M. Hafatue, Natural convective mass transfer MHD flow of chemically reacting micropolar fluid past vertical porous plate, Phys. Sci. Int. J. 13 (1) (2017) 1–11.