Research Article

Effect Of Cerebrospinal Fluid Dynamics With Hydrocephalus In Porous Medium

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Abstract: Hydrocephalus is an excess of fluid in the cavities deep within the brain.It can lead to disturbances of the cerebrospinal fluid (CSF) flow in the hollow places of the brain. This paper presents a Mathematical model based on the principles of biofluid dynamics also this model predict the velocity of fluid flow along with its pressure and the amount of fluid tunneledtoanother body part using lumbar puncture throughout the brain ventricular pathways consistent with intracranial pressure measurements. We analyse the flow of hydrocephalus through porous medium of cerebrospinal fluid. Analytical results with respect to various parameters are presented graphically using MATLAB.

Keywords: Cerebrospinal fluid,Darcy number, Hydrocephalus, Peclet number,Reynolds number.

Introduction

Cerebrospinal fluid is one of the common biofluid that has been handled by most of the mathematicians to predict the pathophysiological disorder. Recently, there has been a numerous interest in the biofluid dynamical studies of various characteristics flow under many different conditions. In this paper we developed a mathematical model for CSF flow for a hydrocephalus patient with respect to velocity, pressure along with its diffusivity.

Bering et. al[1]has been examined the basics of hydrocephalus and the change that occurs in the origination and absorption of cerebrospinal fluid within the cerebral ventricles.

Lininger et al[2]shown the pressure differences between Subarachnoid space and lateral ventricles, differences between the observed and predicted CSF flow velocities in the anterior area point towards brain-CSF interactions in CSF pulsatile flow.

S. Kalyanasundaram et al [3] have been picturised the CSF study when a drug is delivered and to predict the outcome due to the transport of interleukin-2. Edgar A. Bering et al., [4] study designed to measure accurately the changes that occur in the absorption and formation of cerebrospinal fluid within the cerebral ventricles during the development of hydrocephalic dogs.

Mauro Ursino [5] has given a model that explains the intracranial pressure pulse wave as the result of the pulsating changes in cerebral blood volume and the relationship of Cerebral pressure-volume. Nigel Peter Syms [6] simulated the minor pathway hydrocephalus based on the evolution theory of CSF dynamics.

Marmarou et al[7]clearly illustratedmathematical formulation of cerebrospinal fluid (CSF) system was developed to help clarify the kinetics of the intracranial pressure.

Whereas Kauffman et al [8]generalised the Marmarou et al model by using modified Adomian decomposition method to model the nonlinear differential equation regarding CSF dynamics.

Tentiet al [10]reviewed the mathematical models of hydrocephalus using numerical results. Gholampour et.al [11] studied the mean pressure and pressure amplitude for CSF flow using Computational models.

Keith Sharp et al [12], described the mathematical modelto evaluate the significant of Taylor dispersion in the SSS and “glymphatic system” spaces might be clinically controlled to optimize transport.

A. L. Sánchez et al [13] we have analyzed the motion of the CSF in the subarachnoid space to study the characteristics of the flow generated in a simplified model of the spinal canal also the radiological measurements of human adults.

Yamada et.al[14]was analysed Concepts of CSF flow dynamics and the pathophysiology of hydrocephalus on time spatial spin labelling inversion pulse imaging of CSF.

Mathematical Formation

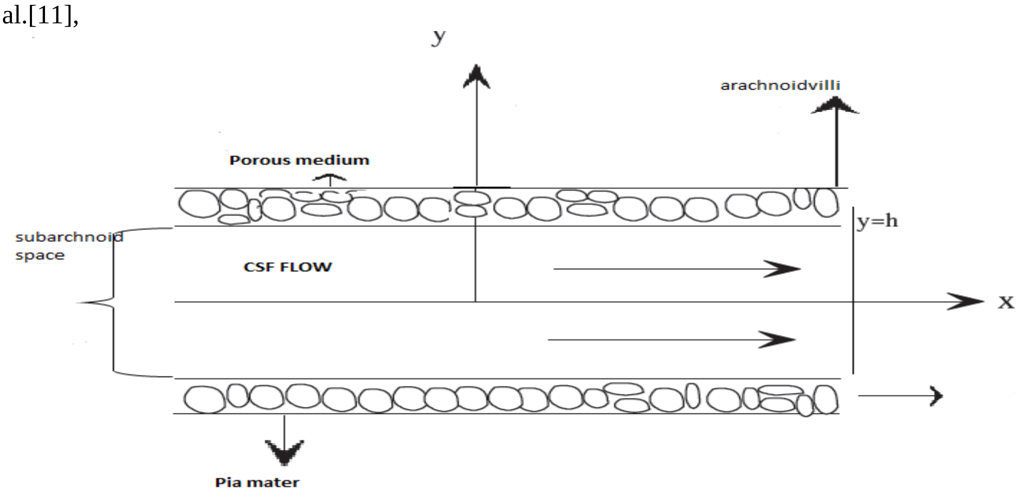
CSF is watery clear fluid [13] and hence it is a Newtonian viscous incompressible fluid [13]. we have considered an unsteady two-dimensional laminar flow bounded by porous layers, saypia matter and Subarachnoid villi. CSF is produced by ventricles of choroid plexus, providing nourishment, waste removal, and it is safeguardof the brain.Thus, secretion range varies between individuals and adult production usually 400 to 600 ml per day.

The spinal SAS, filled with CSF, is a thin annular canal bounded internally by the pia mater, which surrounds the spinal cord, and externally by the deformable dura membrane.The structure of cranial portion of the brain have been observed, then the porous medium SAS and Pia mater is considered non-deformable. Here we discussed about the velocity and porous medium due to hydrocephalus.Let us take the cartesian coordinate as *x* and *y* in which the fluid flow is in normal direction.

The unsteady governing equations are given by,

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

Let us consider the Navier stokes equation of Keith Sharp et. al[13] in which resistance is also addedas in Tenti et al.[11],



$$\frac{\partial \bar{u}}{\partial \bar{t}} + v_0 \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{-1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{RN}{\rho}\bar{u} - \frac{\nu}{k}\bar{u} \quad (2)$$

The followingequationreferred as diffusivity

$$\frac{\partial \bar{c}}{\partial \bar{t}} + v_0 \frac{\partial \bar{c}}{\partial \bar{y}} = D\frac{\partial^2 \bar{c}}{\partial \bar{y}^2} - K\bar{c} \quad (3)$$

Whereas boundary conditions are assumed as,

$$if\ \bar{y} = 0\ \ then\ \ \bar{u} = 0, \bar{c} = c_{ecf}.$$
$$if\ \bar{y} = h\ \ then\ \bar{u} = a, \bar{c} = 0$$

after non dimensionalising the governing equations, we get

$$\frac{\partial v}{\partial y} = 0 \quad (4)$$

$$Re\left[\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + G_{pv}u - S^2u \quad (5)$$

$$\frac{\partial c}{\partial t} + \frac{\partial c}{\partial \bar{y}} = \frac{1}{\beta}\frac{\partial^2 c}{\partial y^2} - kc \quad (6)$$

with the corresponding boundary conditions,

$$if y = 0\ \ then\ \ u = 0, c = c_{ecf}$$
$$if\ y = 1\ then\ \ \ u = a\ , c = 0$$

Introducing the following dimensionless quantities,

$$u = \frac{\bar{u}}{v_0}; P = \frac{\bar{P}}{\rho v_0 \nu};\ t = \frac{v_0 \bar{t}}{h}\ ; y = \frac{\bar{y}}{h}\ ;\ c = \frac{\bar{c}}{c_0}$$

$$G_{pv} = \frac{RNh^2}{\nu\rho}\ ;\ \ Da = \frac{K}{h^2}\ ;\ Re = \frac{v_0 h}{\nu}\ , \beta = \frac{hv_0}{D}, S^2 = \frac{1}{Da}\ , K = \frac{kh}{v_0}$$

<u>**Method of Solution**</u>

Solving the above non-linear partial differential equations by using regular perturbation method. By representing very small perturbation parameterϵ ($\epsilon << 1$) the fluid flow velocity, pressure and concentration as follows

$$u(x,y,t) = u_0 + \epsilon\, e^{\lambda t}u_1 + o(\epsilon^2) \quad (7)$$
$$P(x,y,t) = R + \epsilon\, e^{\lambda t}P_1 + o(\epsilon^2) \quad (8)$$
$$c(x,y,t) = c_0 + \epsilon\, e^{\lambda t}c_1 + o(\epsilon^2) \quad (9)$$

Omitting the higher order of ϵ,Using equations (7) to (9) in (5) & (6) we get,

$$Re\frac{\partial u_0}{\partial y} = -R + \frac{\partial^2 u_0}{\partial y^2} + \left(G_{pv} - S^2\right)u_0 \quad (10)$$

$$Re\left[\lambda u_1 + \frac{\partial u_1}{\partial y}\right] = \frac{\partial^2 u_1}{\partial y^2} + \left(G_{pv} + S^2\right)u_1 \quad (11)$$

$\frac{\partial c_0}{\partial y} = \frac{1}{\beta}\frac{\partial^2 c_0}{\partial y^2} - kc_0$ (12)

$\lambda c_1 + \frac{\partial c_1}{\partial y} = \frac{1}{\beta}\frac{\partial^2 c_1}{\partial y^2} - kc_1$ (13)

$u_0 = A_1 e^{m_1 y} + A_2 e^{m_2 y} + \frac{R}{b_1}$ (14)

Were $b_1 = S^2 - G_{pv}$

$c_0 = A_3 e^{m_3 y} + A_4 e^{m_4 y}$ (15)

$u_1 = A_5 e^{m_5 y} + A_6 e^{m_6 y}$ (16)

$c_1 = A_7 e^{m_7 y} + A_8 e^{m_8 y}$ (17)

$u'(x,y,t) = A_1 e^{m_1 y} + A_2 e^{m_2 y} + \frac{R}{b_1} + \epsilon\, e^{\lambda t}(A_5 e^{m_5 y} + A_6 e^{m_6 y})$ (18)

$c'(x,y,t) = A_3 e^{m_3 y} + A_4 e^{m_4 y} + \epsilon\, e^{\lambda t}(A_7 e^{m_7 y} + A_8 e^{m_8 y})$ (19)

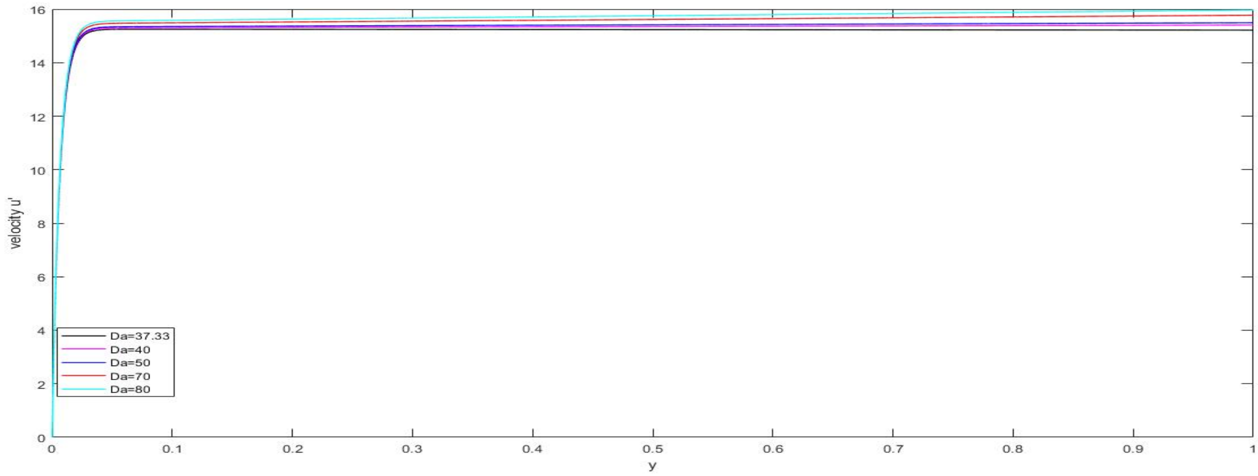
Result and discussion

Analytical solutions of this problem are performed and the results are portrayed graphically in Figs. (2) to (19) to shows the interesting features of significant physical parameters on the velocity, concentration distributions. Throughout the computations we employ $(t = 0.01, R = 20[3], \rho = 998.2$ [2]&[3]$, \lambda = 0.3, \epsilon = 0.01, \beta = 0.33[12]$

$\vartheta = 0.8[3], G_{pv} = 1.3[5], k = 0.67 \times 10^{-16}[2], Da = 37.33[12], Re = 150 - 420[11],$

Fig:2Reynolds number variation with velocity profile

Fig:2 Particle mass parameter with velocity profile



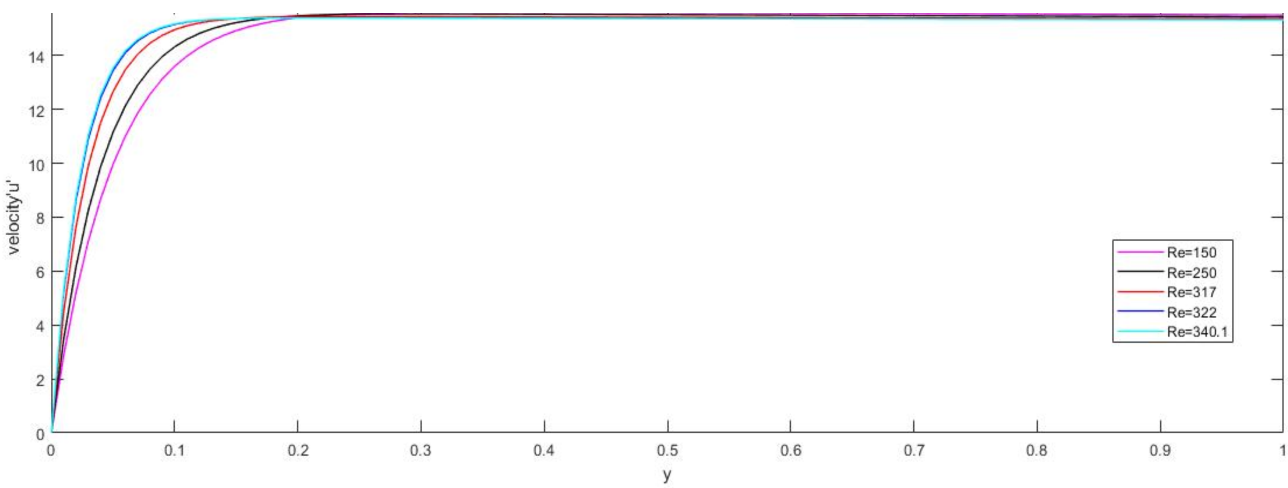
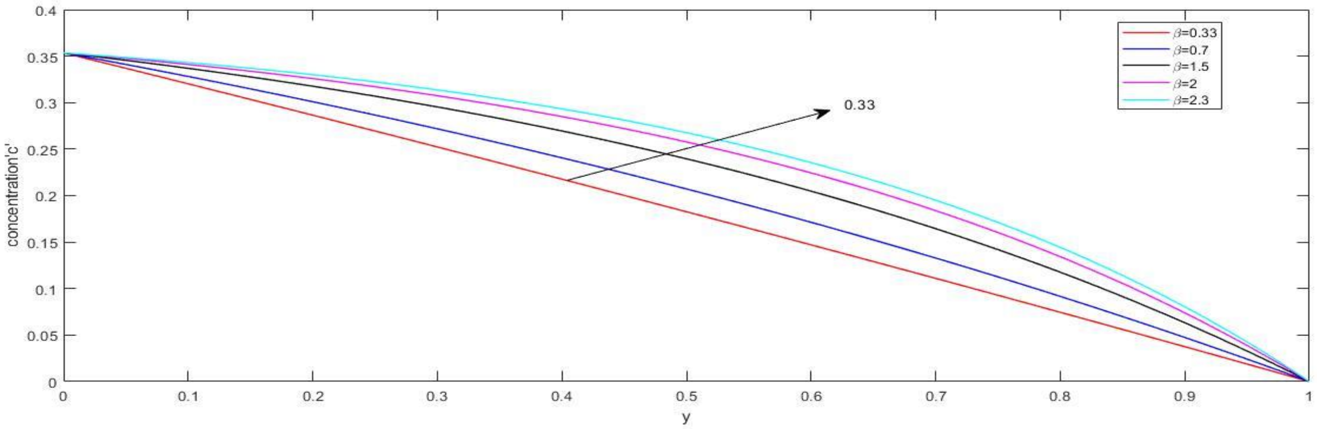


Fig: 4Darcy number variation with respect to velocity profile

Fig: 5Peclet number variation with respect to concentration profile

Result and discussion:



To understand the behaviour of the flow characteristics, velocity *(u')* and Concentration*(c')* are calculated by varying the emerging flow parameters like Reynolds Number, Peclet number, Darcy number, particle mass parameter and so on.

- Reynolds number increases gradually and then becomes consistent as the flow velocity increased.
- Darcy number increases slowly as velocity increases which signifies that there is an ample flowin porous medium say pia mater, due to pressure gradient.
- Particle mass parameter enhancedthe resistance due to excess fluid when compared to the normal flow.
- Peclet number increases for 0.33,it representsthat there is a bulk motion caused theincrease in size of ventricles.

Conclusions

This paper introduces a fluid dynamics model of the CSF flow of intercranial. The equations of motion for hydrocephalus CSF flow in subarachnoid space inside the porous parenchyma were solved. The boundary conditions for the brain physiology were formulated. The simulations proposed in this paper that the increase in Darcy numberreflects the increase in ventricles that increase in size of head of certain kind of peoples. All this makes the dynamics of CSF extremely difficult to process under realistic conditions. However, we take into consideration that further research including some features of hydrocephalus in this model variations with respect toflow velocity to facilitate the task. Future this can be extended for COVID -19 as it was determined that the pandemic virus affects the nervous system.

APPENDIX

$$m_1 = \frac{-Re + \sqrt{(Re)^2 - 4(G_{pv} - S^2)}}{2} \quad , m_2 = \frac{-Re - \sqrt{(Re)^2 - 4(G_{pv} - S^2)}}{2}$$

$$m_3 = \frac{-\beta + \sqrt{(\beta)^2 - 4(\beta * k)}}{2} \quad , m_4 = \frac{-\beta - \sqrt{(\beta)^2 - 4(\beta * k)}}{2}$$

$$m_5 = \frac{-Re + \sqrt{(Re)^2 - 4\left(G_{pv} + S^2 - \lambda Re\right)}}{2} ,$$

$$m_6 = \frac{-Re - \sqrt{(Re)^2 - 4(G_{pv} + S^2 - \lambda Re)}}{2}$$

$$m_7 = \frac{-\beta + \sqrt{(\beta)^2 - 4\beta^2(\lambda + k)}}{2} \quad , m_8 = \frac{-\beta - \sqrt{(\beta)^2 - 4\beta(\lambda + k)}}{2}$$

$$A_1 = -\left(A_2 + \frac{R}{b_1}\right) , A_2 = \frac{1}{e^{m_2} - e^{m_1}}\left[a_1 - \frac{R}{b_1}(1 - e^{m_1})\right], \; S^2 = \frac{1}{Da}$$

$$A_5 = -A_6 A_6 = \frac{a_1}{e^{m_6} - e^{m_5}}$$

$$A_3 = d - A_4 \qquad A_4 = \frac{de^{m_3}}{e^{m_3} - e^{m_4}}$$

$$A_7 = d - A_8 \qquad A_8 = \frac{de^{m_7}}{e^{m_7} - e^{m_8}}$$

NOMENCLATURE

$CSF \rightarrow$ *Cerebrospinal Fluid*
$SSS \rightarrow$ *Superior Subarachnoid Space*
$ICP \rightarrow$ *Intracranial pressure*
$u, v \rightarrow$ *velocity of the fluid*
$\rho \rightarrow$ *density of the fluid*
$P \rightarrow$ *pressure (Systolic and Diastolic)*
$k \rightarrow$ *permeability of the porous medium*
$\nu \rightarrow$ *kinematic viscosity*
$R \rightarrow$ *pressure gradient*
$C \rightarrow$ *concentration of the fluid*
$D \rightarrow$ *coefficient of mass diffusivity of the fluid*
$c_{ecf} \rightarrow$ *excess of cerebrospinal fluid due to Hydrocephalus*
$\beta \rightarrow$ *Peclet number*
$Re \rightarrow$ *Reynolds Number*
$G_{pv} \rightarrow$ *Particle mass parameter*
$a_1 \rightarrow$ *intial velocity of fluid*

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