

Diagnosis In The Ordinal Logistic Regression Model

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Abstract: The relationship between one or more of the explanatory variables and the ordinal categorical response variable is described as "Ordinal logistic regression (ORS) models." The quality of the fit for the model must be checked when the regression model has been adjusted. Chi-square and probability ratio statistics of Pearson are insufficient to evaluate the suitability of the standard logistic regression model when the model has continuous variables. In the current issue, the diagnostic techniques like Lipsitz tests and other variables were used to determine the relationship between the No Income, minimum wage, and emotional issues in elderly people were applied to Cumulative logit models, Logit adjacent classes, and Logit continuous reason patterns.

Keywords: logistic regression model, Time Up and Go test, Akaike Information Criterion, Lipsitz tests, Logit model, likelihood ratio of Pulkstenis-Robinson

1.0 Introduction

Over the years, methods for analysing categorized data have received considerable attention, due to the increasing use of this type of data in several applications¹. In the field of Psychology or Social Sciences, the use of categorized data is very common to measure attitudes, opinions or preferences. Categorized data can also be found in the areas related to Health, for example, when the interest is to evaluate the improvement or not of a patient. Categorical or qualitative data are divided into three types: nominal, ordinal and interval. The first type occurs when the categorical scale is not ordered, that is, the permutation of the categories does not affect the statistical analysis, for example: marital status (single, married, divorced, widow). The second type, on the other hand, occurs when its categories follow a clear order and their exchange influences statistical analysis. As an illustration we have the variable level of education (elementary school, high school, and undergraduate, graduate). The interval type occurs when a continuous variable is summarized, grouping the values into categories, for example, age (0-20, 20-40, 40-60, 60-80, above 80). Methods developed for the analysis of nominal variables can be used for nominal and ordinal variables, since they require only a categorical scale, regardless of the ordering of the categories. However, different results can be obtained when methods developed for the analysis of ordinal variables are used. Thus, these methods can only be used for ordinal variables, as they take into account the ordering of categories. When the classification of ordinal data is not fully explored, that is, when treating ordinal or interval variables as nominal, the permutations of the categories are irrelevant and, consequently, much information is lost. What distinguishes the use of ordinal models from others is that they produce more powerful inferences about the characteristics of the population.

The cumulative logit model for analysis of ordinal response variables was originally proposed by Walker and Duncan (1967)² and later called the Proportional Chances Model by Cullagh (1980)³. This model is an extension of the Logistic Regression Model, which in turn is a special case of the Generalized Linear Model (MLG), which was proposed by Nelder and Wedderburn (1972)⁴, which allows the use of distributions belonging to the exponential family for the response variable, in addition to the normal distribution⁵.

In addition to the cumulative logit model for data analysis with ordinal response variables, Agresti (2010)⁶ suggested the logit model for adjacent categories and Feinberg (1980)⁷ proposed the logit continuous ratio model. According to Agresti (2010), the choice of the model is associated with the type of comparison that makes the most sense for the study.

The aforementioned models are useful to describe the relationship between the variable ordinal response and one or more explanatory variables. These models differ in how the categories of the response variable are compared. After adjusting the model, it is necessary to check the quality of that adjustment, that is, to check how close the values predicted by this model are to their corresponding observed values. For the models mentioned above, this quality can be ascertained by Pearson's statistics, X^2 ; and by the likelihood ratio statistics, G^2 , as long as the explanatory variables are categorical.

For the case where the model contains at least one continuous explanatory variable, Lipsitz et AL. (1996)⁸ proposed a fit quality test, based partially on the Hosmer-Lemeshow test for logistic regression models. Pulkstenis and Robinson (2004)⁹ modified Pearson's statistics and likelihood ratio for when the model contains continuous and categorical explanatory variables. Fagerland and Hosmer (2013) developed the ordinal version of the Hosmer-Lemeshow test. The aforementioned tests were developed based on the cumulative logit model with proportional

chances, which is the most used model in the analysis of ordinal categorical data. However, Fagerland and Hosmer (2016)¹⁰ extended the tests for Models logito adjacent categories with proportional odds and Model logito continuous ratio with proportional odds. The objective of this work is to illustrate the methods studied to real data sets.

2.0 Data used

The data that illustrate this work were collected in a Specialized Reference Unit of the public service, in the city of Dubai (UAE) from June 2018 to February 2019. The objectives of this research is to study the relation between No Income, Minimum Wage and emotional issues in elderly people.

The data refer to 138 individuals (samples) over the age of 60 and with No Income. Demographic variables include information regarding gender (1 = male, 2 = female), education (0 = illiterate, 1 = elementary school, 2 = high school, 3 = graduation), monthly income (1 = no income, 2 = 1 minimum wage, 3 = greater than 1 to 2 minimum wages, 4 = greater than 2 minimum wages) and age (in years).

To evaluate depression, the Geriatric Depression Scale was formed with 15 affirmative and negative questions in which the result of five or more points characterizes the diagnosis of depression (0 = no, 1 = yes).

Self-reference questions about income were asked, such as when the family started to perceive the No Income, containing the answers 6 months (code 1), 1 year (code 2), 2 years (code 3) and more than two years (code 4).

Static, dynamic balance and mobility tests were performed, including the Unterberg test with eyes open. This test consists of the patient performing a walking movement without leaving his seat and arms extended in front of him. It was considered positive (code 1), if the elderly person could perform the test and negative (code 0) if he could not do it.

Another test of static, dynamic balance and mobility, performed by the elderly, was the Time Up and Go (TUG) test, which consists of timing the time it takes the patient to get up from a chair, make a three-meter walk and return to sit in the chair. The result with less than 11 seconds (code 1) is considered normal, they are independent elderly and without risk of falls; between 11 and 20 seconds (code 2), they are elderly people who have partial independence and with a low risk of falls and time over 20 seconds (code 3), they are elderly people who have significant deficits in physical mobility and risk of falls.

Finally, a self-assessment on income was carried out, through the question in general you would say that your income is bad (code 1), regular (code 2), good (code 3), very good (code 4) and excellent (code 5). The very good and excellent categories were disregarded due to the lack of these results in the sample.

For this study, the variables sex, education, monthly income, age, depression were considered as explanatory variables, when the family started to notice and Unterberg test with eyes open. The response variables considered were the Time Up and Go test and in general you would say your income is. For the model that considered the Time Up and Go test as a response variable, the aforementioned explanatory variables were used, with the exception of the Unterberg test with open eyes variable, which was not of interest in the study.

3.0 Inferential analysis

To illustrate some applications with this database, we chose to adjust the Cumulative logit model with proportional chances for the Time Up and Go test variable and the adjacent logit model with proportional chances for the variable in general you would say your income is. SOFTWARE R was used to perform the analyses. In this SOFTWARE, the test Lipsitz, the ordinal versions of the Hosmer-Lemeshow test and the chi-square Pulkstenis-Robinson tests and the probability relationships are implemented through the "generalhoslem" package with proportional chances for the cumulative logit model. These tests have not been carried out for the Model logito adjacent categories with proportional chances. In this analyses, the number of groups equal to six ($g = 6$) was adopted for the Lipsitz test and for the ordinal version of the Hosmer-Lemeshow test. The significance level of 10% was used for the selection of explanatory variables.

3.2.1 Time Up and Go Test

For the Time Up and Go test variable, the Cumulative logit model with proportional chances was adjusted considering two situations: in the first, the explanatory variables with more than two categories were treated as quantitative and, in the second, as qualitative. For the first case, the values of statistics, freedom levels and p-values of the Lipsitz test are shown in Table 4.1. The standard Hosmer Lemeshow test version and the Pulkstenis-Robinson probability ratio are presented in order to check the quality of the entire model containing all the explanatory variables. The results of this table show how well the entire model fits.

Table 3.1: Adjustment Quality Tests - Time Up and Go Proof - Situation 1

test	Statistical value	Degrees of freedom	p-value
Lipsitz test	6.89	6	0.23
The Hosmer-Lemeshow test	10.89	10	0.29

X_{PR}^2	10.79	10	0.50
G_{PR}^2	14.01	10	0.24

For the selection of explanatory variables, the backward method was used. The variables that had a significant effect were age (Z_1) and monthly income (Z_2).

To assess the validity of the assumption of proportional odds for the two explanatory variables that obtained a significant effect, the proportionality test was applied. The test showed that the cumulative logit model with proportional chances seems adequate (p -value = 0.328).

Table 3.2 shows the parameter estimates, respective standard errors and p -values for the final adjusted model.
 Table 3.2: ESTIMATES OF PARAMETERS of the FINAL model - TIME UP AND GO PROOF - SITUATION 1

Variable	Parameters	Estimate	Standard Error	p-value
Intercept 1	α_1	3.8843	2.0056	0.053
Intercept 2	α_2	8.2238	2.1551	< 0.001
Age	γ_1	-0.1034	0.0276	< 0.001
Monthly income	γ_2	0.9283	0.2697	0.001

The final adjusted model can be expressed in terms of the logits, by:

$$\log \left[\frac{\hat{P}(Y_i \leq j | z_1)}{1 - \hat{P}(Y_i \leq j | z_1)} \right] = \hat{\alpha}_j - 0.1034z_1 + 0.9283z_2, j = 1, 2$$

where z_1 represents age in years; $z_2 = 1$, if the elderly person has no income; $z_2 = 2$ if the elderly person has a monthly income of 1 minimum wage; $z_2 = 3$, if the elderly person has an income greater than 1 salary at 2 minimum wages; and $z_2 = 4$, if the elderly has a salary greater than 2 minimum wages.

As for the interpretation, we have that:

- fixed monthly income, the chance of an elderly person with z_1 years taking less than 11 seconds to complete the Time Up and Go test is $\exp(\hat{\gamma}_1) = 0,90$ times the chance of an elderly person ($z_1 - 1$) taking less than 11 seconds to perform the test;
- fixed the age, the chance of an elderly person with monthly income equal to z_2 minimum wages to take less than 11 seconds to perform the Time Up and Go test is $\exp(\hat{\gamma}_2) = 2.53$ times the chance of an elderly person with equal monthly income a ($z_2 - 1$) minimum wages take less than 11 seconds to complete the test.

Due to the assumption of proportional chances assumed for the model adjusted to the data, the same conclusions are reached regarding the chance of an elderly person taking up to 20 seconds to perform the Time Up and Go test.

In the second situation, explanatory variables were treated as qualitative for more than two categories. The Lipsitz test values, the degree of freedom and the p -value of the Hosmer-Lemeshow test and the Pulkstenis-Robinson chi-square test and probability ratio are presented in Table 3.3 for verifying the quality adjustment of the whole model containing all explaining variables. The results of this table show how well the entire model fits.

Table 3.3: Adjustment Quality Tests - Time Up and Go Proof - Situation 2

Test	Statistical value	Degrees of freedom	p-value
Lipsitz test	3.50	6	0.701
The Hosmer-Lemeshow Test	4.80	8	0.752

For the selection of explanatory variables, the BACKWARD method was used. The variables that had a significant effect were age (z_1 and monthly income (z_2)). To assess the validity of the assumption of proportional chances for the two explanatory variables that obtained a significant effect, the proportionality test was applied. The test showed that the cumulative logit model with proportional chances seems adequate (p -value = 0.147). Table 3.4 shows the estimates of the parameters, respective standard errors and p -values for the final adjusted model.

Table 3.4: Estimates of Parameters of the final model - proof Timed Up and Go - situation 2

Variable	Parameters	Estimate	Standard Error	p-value
Intercept 1	α_1	4.8272	1.90	0.015
Intercept 2	α_2	9.1651	2.21	<0.001
Age	γ_1	-0.1053	0.09	<0.001
Monthly income of 1 minimum wage	γ_2	1.1229	0.84	0.179
Monthly income > 1 to 2 minimum wages	γ_3	1.6628	0.92	0.075

The final adjusted model can be expressed in terms of the logits, by:

$$\log \left[\frac{\hat{P}(Y_i \leq j|z_1)}{1 - \hat{P}(Y_i \leq j|z_1)} \right] = \hat{\alpha}_j - 0.1053z_1 + 1.1229z_2 + 1.6628z_3 + 3.1578 z_4, j = 1,2$$

where z_1 represents age in years; $z_2=1$, if the elderly person has a monthly income of 1 minimum wage; $z_2=0$, otherwise; $z_3=1$, if the elderly person has a monthly income greater than 1 to 2 minimum wages; $z_3 = 0$, otherwise; and $z_4 = 1$, if the elderly has a salary greater than 2 minimum wages; $z_4 = 0$, otherwise.

As for the interpretation, we have that:

- fixed monthly income, the chance of an elderly person with z_1 years taking less than 11 seconds to complete the Time Up and Go test is $\exp(\hat{\gamma}_1) = 0.90$ times the chance of an elderly person ($z_1 - 1$) taking less than 11 seconds to perform the test;
- fixed age, the chance for an elderly person with a monthly income of 1 minimum wage to take less than 11 seconds to complete the Time Up and Go test is $\exp(\hat{\gamma}_2) = 3,07$ times the chance of an elderly person with no income take less than 11 seconds to complete the Time Up and Go test;
- fixed age, the chance of an elderly person with a monthly income greater than 1 salary at 2 minimum wages taking less than 11 seconds to perform the test Time Up and Go is $\exp(\hat{\gamma}_3 - \hat{\gamma}_2) = 1,72$ times the chance an elderly person who has a monthly income of 1 minimum wage takes less than 11 seconds to complete the Time Up and Go test;
- fixed age, the chance of an elderly person with a monthly income greater than 2 minimum wages taking less than 11 seconds to perform the Time Up and Go test is $\exp(\hat{\gamma}_4 - \hat{\gamma}_3) = 4.46$ times the chance of an elderly person with income monthly over 1 salary at 2 minimum wages take less than 11 seconds to complete the Time Up and Go test.

Due to the assumption of proportional chances assumed for the model adjusted to the data, the same conclusions are reached regarding the chance of an elderly person taking up to 20 seconds to perform the Time Up and Go test.

Finally, the Akaike Information Criterion (AIC) was used to compare the model considering explanatory variables with more than two categories as quantitative with the model that considers explanatory variables as qualitative. From Table 3.5, it can be seen that the smallest AIC went to the simplest model, which considers the explanatory variables with more than two categories as quantitative. However, this method should be used with caution, since this criterion favors the simplest model as it penalizes the model with more parameters. To finish, the Logito Model was adjusted to adjacent categories with proportional chances for the same explanatory variables and response and the same significant explanatory variables were obtained, both for the model that considers the explanatory variables with more than two categories as quantitative and for the model it considers qualitative.

Table 3.5: AIC of the ADJUSTED models - PROOF Timed Up AND Go

Model	AIC
Model (qualitative)	204.03
Model (quantitative)	199.44

3.2.2 In general you would say that your income is

For the variable in general you would say that your income is, the Logito Model was adjusted with adjacent categories with proportional chances, and two situations were considered: in the first, the explanatory variables with more than two categories were treated as quantitative and, in the second, as qualitative.

Table 3.6 presents for the first situation the values of the statistics, the degree of freedom and the p-values for the Lipsitz test, the standard version of the Hosmer-Lemeshow test, the chi-square test of Pulkstenis-Robinson, and the probability proportion, to check the fitness of the complete model containing all explanatory variables. The results in this table show that the complete model fits well, with the exception of the X^2 test.

Table 3.6: Quality Testing - Overall You Would Say Your Income Is - Situation 1

Test	Statistical value	Degrees of freedom	p-value
Lipsitz test	4.501	6	0.49
The Hosmer-Lemeshow test	9.049	8	0.44
X^2_{PR}	33.30	20	0.06
G^2_{PR}	12.49	20	0.30

To assess the validity of the assumption of proportional odds in the complete model, the proportionality test was applied. The test showed that the Model logito adjacent categories with proportional chances seems adequate (p-value = 0.179).

Excluding the Unterberg test variable with eyes open, which is 93.5% negative responses, all tests of quality of the adjustment showed p-values greater than 10%. To select the variables, the backward method was used. The explanatory variable that had a significant effect was the depression variable (X_1).

Table 3.7 shows the parameter estimates, respective standard errors and p-values for the final adjusted model.

The final adjusted model can be expressed in terms of the logits, by:

Table 3.7: ESTIMATES OF PARAMETERS of the FINAL model ADOPTED

Variable	Parameters	Estimate	Standard Error	p value
Intercept 1	α_1	-0.6427	0.2785	0.021
Intercept 2	α_2	1.899	0.3942	<0.001
Depression	β_1	0.5927	0.3096	0.056

$$\log \left[\frac{\hat{P}(Y_i = j|x_1)}{\hat{P}(Y_i = j + 1|x_1)} \right] = \hat{\alpha}_j - 0.5927x_1, j = 1,2,$$

where $X_1 = 0$, if the elderly person does not have depression; and $X_1 = 1$, if the elderly person has depression.

Because the final model does not have continuous variables, Pearson's statistics (X^2) and the likelihood ratio (G^2) were used to verify the quality of the fit of this model. The results of the fit quality tests can be seen in Table 3.8, indicating evidence in favor of the model.

Table 3.8: ADJUSTMENT QUALITY Tests - OVERALL YOU WOULD SAY YOUR INCOME IS - SITUATION 1 - FINAL model

test	Statistical value	Degrees of freedom	p value
Pearson (X^2)	1.56	1	0.212
Likelihood Ratio (G^2)	1.56	1	0.212

Figure 3.1 shows the graph of Pearson's residuals as a function of the combinations of the categories of the depression variable with the categories of the response variable. As the values are around zero, there is evidence in favour of the model.

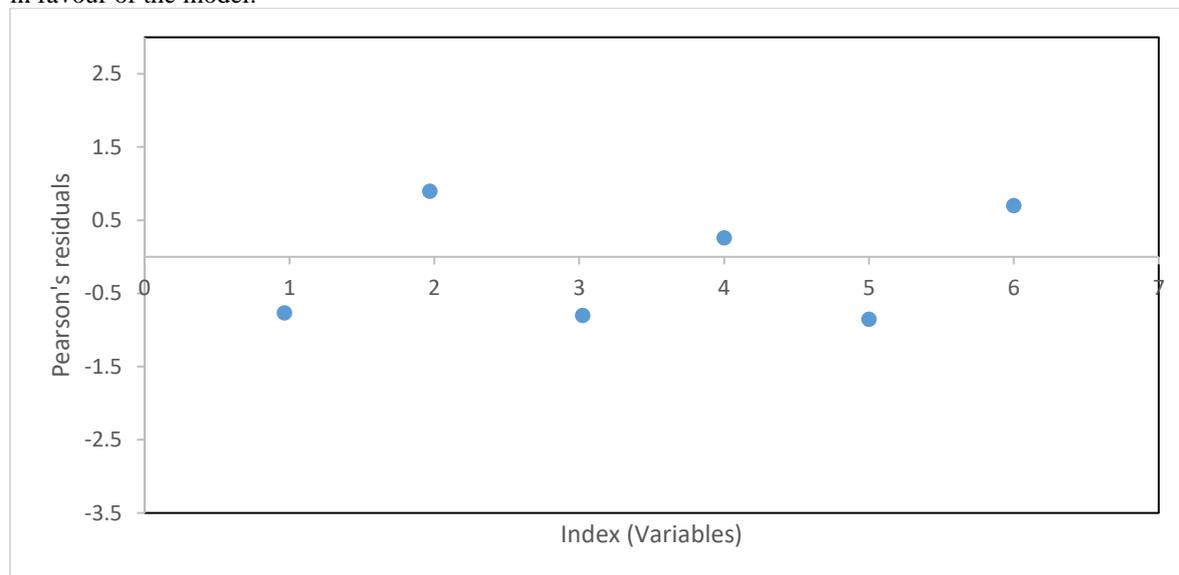


Figure 3.1: Residues of the Logito Model ADJACENT CATEGORIES - OVERALL YOU WOULD SAY YOUR INCOME IS - SITUATION 1

As for the interpretation, we have that:

- the chance of an elderly person with depression to say that their income is bad in relation to regular is $\exp(\hat{\beta}_1) = 1.8$ times the chance of an elderly person who does not have depression to say that their income is bad in relation to regular.

Due to the assumption of proportional chances assumed for the model adjusted to the data, the same conclusion is reached regarding the chance of an elderly person to say that their income is regular in relation to good. It is also possible to verify the elderly person's chance to say that their income is bad in relation to good. In this case the model can be expressed by:

$$\log \left[\frac{\hat{P}(Y_i = j|x_1)}{\hat{P}(Y_i = j + 1|x_1)} \right] = \sum_{j=1}^2 \hat{\alpha}_j + 0.5927(3 - j)x_1, j = 1,2$$

Where $X_1 = 0$, if the individual does not have depression; and $X_1 = 1$, if the individual has depression.

As for the interpretation, we have that:

- The chance of an elderly person with depression saying that their income is bad in relation to good is $\exp(2 \cdot \hat{\beta}_1) = 3,3$ times the chance of an elderly person who does not have depression to say that their income is bad in relation to good.

In the second situation, explanatory variables were treated as qualitative for more than two categories. Table 3.9 presents the statistics values, freedom degrees and p-value of Lipsitz test, the ordinary Hosmer-Lemeshow test version and Pulkstenis-Robinson's chi-square tests and likelihood ratio to verify quality adjustment for the complete model with all explanatory variables. The results of this table show that the complete model is in good shape, except the chi-square test Pulkstenis-Robinson.

Table 3.9: Quality Testing - Overall You Would Say Your Income Is - Situation 2

test	Statistical value	Degrees of freedom	p-value
The Lipsitz test	5.01	6	0.43
The Hosmer-Lemeshow test	5.02	10	0.84

To assess the validity of the assumption of proportional chances in the complete model, the proportionality test was applied. The test showed that the Model logito adjacent categories with proportional odds seems adequate (p-value = 0.197).

Once again, when excluding the Unterberg test variable with open eyes, the model with adjacent categories with proportional chances was well adjusted in the four tests.

For the selection of explanatory variables, the BACKWARD method was used. The variables that had a significant effect were depression (X_1) and when the family started to perceive the low income (X^2).

Table 3.10 shows the parameter estimates, respective standard errors and p-values for the final adjusted model.

Table 3.10: Estimates of Parameters of The Selected Model

Variable	Parameters	Estimate	Standard Error	p-value
Intercept 1	α_1	-2.392	0.9566	0.012
Intercept 2	α_2	0.329	0.8874	0.711
Depression	β_1	0.596	0.3179	0.068
Family (1 year)	β_2	2.2474	1.0166	0.027
Family (2 years)	β_3	1.3361	0.9515	0.16
Family (> 2 years)	β_4	1.8146	0.9201	0.049

The final adjusted model can be expressed in terms of the logits, by:

$$\log \left[\frac{\hat{\pi}(x)}{\hat{\pi}_{j+1}(x)} \right] = \hat{\alpha}_j + 0.5927x_1 + 2.2472x_2 + 1.3361x_3 + 1.8146x_4, j = 1,2$$

where $X_1 = 0$, if the individual does not have depression and $X_1 = 1$, if the individual has depression; $X_2 = 1$, if the family started to notice the No Income in 1 year and $X_2 = 0$, otherwise; $X_3 = 1$, if the family started to notice the No Income in 2 years and $X_3 = 0$, otherwise; and $X_4 = 1$, if the family started to perceive the No Income after 2 years and $X_4 = 0$, otherwise. As in the previous model, Pearson's statistics (X^2) and likelihood ratio (G^2) were used to verify the quality of the final model's fit. The result of the adjustment can be seen in Table 3.11, indicating evidence in favour of the model.

Table 3.11: Adjustment Quality Tests - Overall You Would Say Your Income Is - Situation 2 - Final Model

test	Statistical value	Degrees of freedom	p value
Pearson (X^2)	9.1	10	0.523
Likelihood ratio (G^2)	8.59	10	0.571

Figure 3.2 shows the graph of Pearson's residuals as a function of the combinations of the categories of the depression variable and the variable when the family began to perceive the lack of income with the categories of the response variable. As the values are around zero, there is evidence in favour of the model.

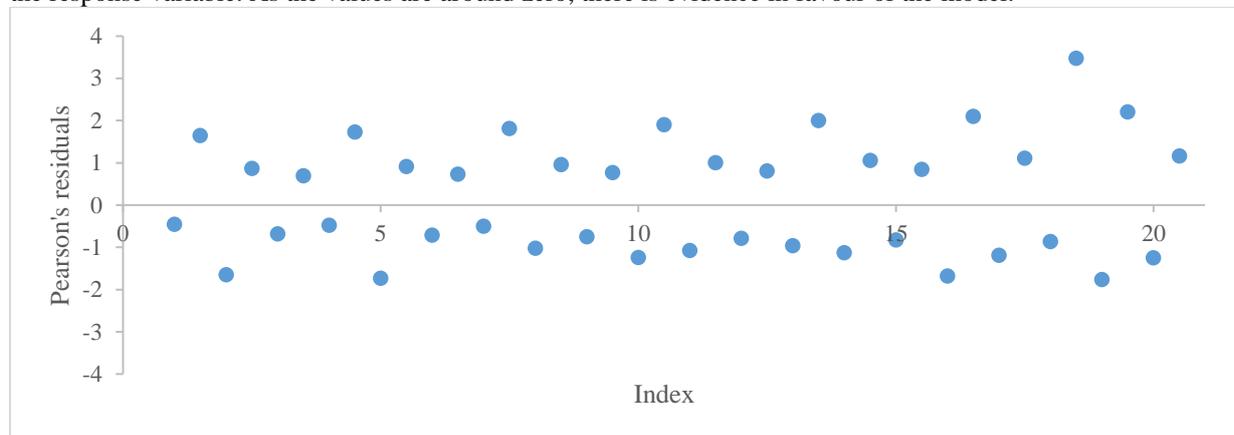


Figure 3.2: Residues of the logit Model Adjacent Categories - Overall You Would Say Your Income is - Situation 2

As for the interpretation, we have that:

- fixed the variable when the family started to perceive the No Income, the chance of an elderly person with depression saying that their income is bad in relation to regular income is $\exp(\hat{\beta}_1) = 1.79$ times the chance of the elderly person without depression saying that your income is poor compared to regular income;
- fixed the depression variable, the chance of an elderly person whose family started to perceive the problem of income in 1 year say that income is bad in relation to regular income is $\exp(\hat{\beta}_2) = 9.46$ times the chance of an elderly person whose family started to perceive the lack of income in 6 months say that income is poor compared to regular income;
- when the depression variable is fixed, there is no evidence that the chance of an elderly person saying that income is bad in relation to regular income is different from when the family started to perceive the No Income in 6 months or 2 years;
- fixed the variable depression, the chance of an elderly person whose family took more than 2 years to perceive the lack of income saying that income is bad in relation to regular income is $\exp(\hat{\beta}_4) = 6.14$ times the chance of an elderly person whose family began to notice the lack of income in 6 months saying that income is bad compared to regular income.

Due to the assumption of proportional chances assumed for the model adjusted to the data, the same conclusions are obtained regarding the chance of an elderly person to say that their income is regular in relation to good.

It is also possible to check the chance of the elderly to say that their income is bad in relation to good.

In this case the model is expressed by:

$$\log \left[\frac{\hat{P}(Y_i = j|x_i)}{\hat{P}(Y_i = 3|x_i)} \right] = \sum_{j=1}^2 \hat{\alpha}_j + 0.5927(3-j)x_1 + 2.2472(3-j)x_2 + 1.3361(3-j)x_3 + 1.8146(3-j)x_4, j = 1,2$$

Where $X_1 = 0$, if the individual does not have depression and $X_1 = 1$, if the individual has depression; $X_2 = 1$, if the family started to notice the No Income in 1 year and $X_2 = 0$, otherwise; $X_3 = 1$, if the family started to notice the No Income in 2 years and $X_3 = 0$, otherwise; and $X_4 = 1$, if the family started to notice the No Income after 2 years; $X_4 = 0$, otherwise.

As for the interpretation, we have that:

- fixed the variable when the family started to perceive the No Income, the chance of an elderly person with depression saying that their income is bad in relation to good is $\exp(2 \cdot \hat{\beta}_1) = 3.19$ times the chance of an elderly person who has no depression to say that their income is bad in relation to good;

- Fixed the variable depression, the chance of an elderly person whose family started to perceive the No Income in 1 year to say that income is bad in relation to good is $\exp(2. \hat{\beta}_2) = 89.55$ times the chance of an elderly person whose family started to perceive the lack of income in 6 months to say that the income is bad in relation to the good;
- when the depression variable is fixed, there is no evidence that the chance of an elderly person saying that income is bad in relation to good is different from when the family started to perceive the No Income in 6 months or 2 years;
- fixed the depression variable, the chance of an elderly person whose family took more than 2 years to perceive the lack of income saying that income is bad in relation to good is $\exp(2. \hat{\beta}_4) = 37.68$ times the chance of an elderly whose family started to perceive the lack of income in 6 months to say that the income is bad in relation to the good.

The Akaike Information Criterion (AIC) was used to compare the model considering the explanatory variables with more than two categories as quantitative with the model that considers the explanatory variables as qualitative. From Table 3.12 and in the same way as in the previous models, it is noted that the smallest AIC was for the simplest model, which considers the explanatory variables with more than two categories as quantitative. However, the model which considers variables with more than two categories to be quantitative presented only one significant variable. The model that considers variables with more than two categories as qualitative presented two significant explanatory variables.

Table 3.12: AIC of the adjusted models - overall you would say your income is

Model	AIC
Model (qualitative)	253.04
Model (quantitative)	250.75

Finally, the cumulative logit model was adjusted with proportional chances for the same explanatory variables and response and the same significant explanatory variables were obtained, both for the model that considers explanatory variables with more than two categories as quantitative as for the model it considers qualitative. In addition, the effects agreed on both models.

4.0 Conclusions and suggestions for future research

There are some options to check the quality of the fit for the selected models when the explanatory variables are categorical and there are no sparse values, such as Pearson's test, likelihood ratio test and Pearson's residuals. When continuous predictor variables are present in the model or when values are sparse, Pearson's X^2 statistic and G^2 of the likelihood ratio is not adequate to check the quality of the fit. In these cases the Lipsitz test, the chi-square test and the Pulkstenis-Robinson probability ratio, and the ordinal Hosmer-Lemeshow test version are used. The current cumulative logit models, adjacent logit categories and continuous logit models can be adjusted in the main computer packages. However, the Lipsitz test, the chi-square tests and Pulkstenis-Robinson likelihood ratios and the ordinal version of the Hosmer-Lemeshow test in SOFTWARE R are only available for the Cumulative logit model with proportional odds ("generalhoslem" package). As these tests are not implemented for the Logito Model adjacent categories with proportional chances, these were developed in this dissertation. Recently, Fagerland and Hosmer (2017)¹¹ presented the ologitgof command implemented in Stata which calculates the tests mentioned to assess the adequacy of the aforementioned models. For the application presented in this work, the In order to examine the relationship between No Income, minimum wages, and emotional issues for elderly, cumulative logit template with proportional chance and the adjacent logit model were chosen. The proportional oddity models were better than the proportional odds models.

When the cumulative logit models and adjacent categories were adjusted, both with proportional odds, and with the same explanatory variables and response, the same explanatory variables were selected in both models.

There is still much to be developed in future research for the diagnostic techniques of ordinal models, both in the theoretical part and in SOFTWARE development. For example, there are still no methods available to access the quality of fit for models without proportional odds or with partial proportional odds and which consider continuous explanatory variables. One option would be to categorize the continuous explanatory variable or use the multinomial logistic regression model, which have diagnostic techniques. However, a lot of information can be lost by categorizing the variable or using the multinomial logistic regression model, since this model ignores the order of the categories of the response variable.

References

1. Bogdan, R. C., & Biklen, S. K. (2003). Qualitative research in education: An introduction to theory and methods (4th ed.). Needham Heights, MA: Allyn & Bacon.
2. Walker, S.H., Duncan D.B. 1967. Estimation of the probability of an event as a function of several independent variables. Biometrika; 54:167-179.

3. McCullagh, P. (1980). Regression models for ordinal data. *Journal of the Royal Statistical Society, Series B*, 42, 109-142.
4. Nelder, J. A., & Wedderburn, R. W. M. (1972). Generalized linear models. *Journal of the Royal Statistical Society, Series A*, 135, 370-384.
5. Bercedis Peterson and Frank E. Harrell, Jr, Partial Proportional Odds Models for Ordinal Response Variables, *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, Vol. 39, No. 2 (1990), pp. 205-217, DOI: 10.2307/2347760
6. Agresti, A. (2010). *Analysis of ordinal categorical data* (2nd ed.), Wiley Series in Probability and Statistics. Hoboken, New Jersey: Wiley
7. Feinberg, S., 1980. *The Analysis of Cross-Classified Categorical Data*, Cambridge, MIT Press.
8. Fagerland, Morten & Hosmer, David. (2013). A goodness-of-fit test for the proportional odds regression model. *Statistics in medicine*. 32. 10.1002/sim.5645.
9. Erik Pulkstenis, Timothy J. Robinson, Goodness-of-fit tests for ordinal response regression models, Volume 23, Issue 6, 2004, Pages 999-1014
10. Fagerland, Morten & Hosmer, David. (2016). Tests for goodness of fit in ordinal logistic regression models. *Journal of Statistical Computation and Simulation*. 86. 1-21. 10.1080/00949655.2016.1156682.
11. Fagerland, Morten & Hosmer, David. (2017). How to Test for Goodness of Fit in Ordinal Logistic Regression Models. *The Stata Journal: Promoting communications on statistics and Stata*. 17. 668-686. 10.1177/1536867X1701700308.