Some Labelings On Cycle With Parallel P_4 Chord

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Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 28 April 2021

Abstract: In this paper we focused, to obtain some results on labeling of cycle graph, Cycle C_{2m} ($m \geq 3$) and C_{2m+1} ($m \geq 3$) with parallel (path) P_4 chords. We have proved, every cycle C_{2m} ($m \geq 3$) and C_{2m+1} ($m \geq 3$) with parallel (path) P_4 chords is a vertex odd mean graph and vertex even mean graph, though it satisfied their labeling. And also graph is proved for Square Sum labeling and Square Difference Labeling on cycle C_{2m} ($m \geq 3$) and C_{2m+1} ($m \geq 3$) with parallel (path) P_4 chords.

Keywords: Cycle with parallel (path) P_4 chords, Vertex odd mean labeling, Vertex even mean labeling, Square sum labeling, Square Difference Labeling

Subject Classification: 05C78

Introduction

Rosa introduced the labeling of graph in the year 1967[1]. A.Gallian has given survey for graph labeling in detail [3]. S.Somasundaram and R. Ponraj, found mean labeling and published results for some graphs[7]. Revathi [6] has established and proved the graphs for vertex even mean and vertex odd mean labeling. In [8], Ajitha, Arumugam & Germina, established results for some graphs which admits square sum labeling. Square difference labeling is introduced and proved by J.Shiama [9]. We can able to study mean labeling for cycle graphs[4]. In [5] Graceful labeling is proposed for C_n with parallel (path) P_k chords.

Labelings on C_n where ($n \geq 6$) attains parallel Chords with path P_3 proved by A.Uma Maheswari & V.Srividya [2]. In [10], [11] further results are proposed for vertex even & odd mean labeling, for new families of cycle with chord (parallel). In [12] certain labeling are proved for C_n ($n \geq 6$) with parallel (path) P_3 as a Chord.

Throughout this paper, consider the cycle C_{2m} ($m \geq 3$) and C_{2m+1} ($m \geq 3$) with parallel P_4 chords. We have proved that the Cycle C_{2m} **(**$m \geq 3$) and C_{2m+1} ($m \geq 3$) with parallel P_4 chords admits labeling for Vertex Odd mean and Even mean. In addition, we also proved that these Graphs satisfies Square Sum and square difference labeling.

Definition 1.1: [6]

A graph G, with vertices (p) and edges (q), if there exist function (injective) $f: V(G) \rightarrow \{1, 3, 5, \ldots, 2q-1\}$ such that the induced function $f^*: E(G) \rightarrow N$ is given by $f^*(uv) = \frac{f(u)+f(v)}{2}$ where each edge uv are distinct by the vertex odd mean labeling.

Definition 1.2: [8]

A graph G is called square sum graph, if it admits an 1-1 and onto labeling mapping $f: V(G) \rightarrow \{0, 1, 2 \ldots, p-1\}$ given by the induced function $f^* = E(G) \rightarrow N$, defined by $f^* = [f(u)]^2 + [f(v)]^2$ is injective for every edge uv are distinct.

Definition 1.3: [6]

A graph G, with vertices (p) and edges (q), if there exist an injective function f : $V(G) \rightarrow \{2, 4, 6, \ldots 2q\}$ such that the induced mapping f^*: $E(G) \rightarrow N$ is given by $f^*(uv) = \frac{f(u)+f(v)}{2}$ are where all edges uv are distinct is said to be vertex even mean graph by its labeling.

Definition 1.4: [9]

A graph G, is called square difference graph, if it admits labeling with an 1-1 and onto mapping f : $V(G) \rightarrow \{0, 1, 2, \ldots, p-1\}$ given by the induced function f^*: E(G) → N, defined by $f^*(uv) = |[f^*(u)]^2 - [f^*(v)]^2|$ is injective for every edge uv are distinct.

Definition 1.5: [5]

Cycle with parallel P_4 chords is obtained from the graph, C_n: $u_0u_1u_2\ldots\ldots u_{n-1}u_0$ by attaching disjoint paths P_4's between two vertices u_1u_{n-1}, u_2u_{n-2}, ...$u_\alpha u_\beta$ of C_n where $\alpha = \left\lfloor \frac{n}{2} \right\rfloor -1$, $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 2$ (or) $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 1$, when n is odd & even as shown in Fig. 1a and 1b.

In this paper, C_{2m} ($m \geq 3$) has $4m$-2 vertices and $5m - 3$ edges and for C_{2m+1} ($m \geq 3$), has $4m-1$ vertices and $5m - 2$ edges.

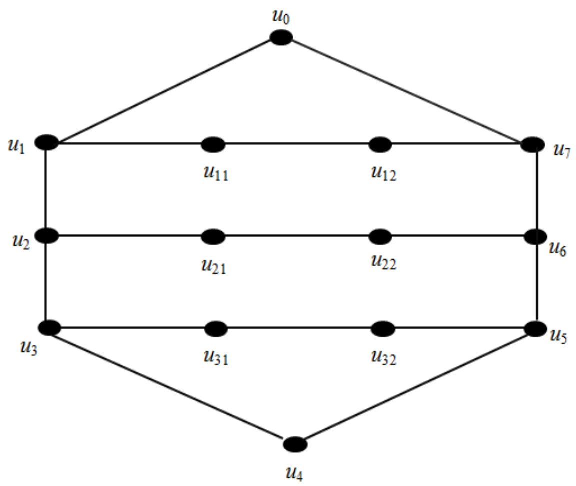

Fig.1a Cycle C_8 with parallel P_4 chord

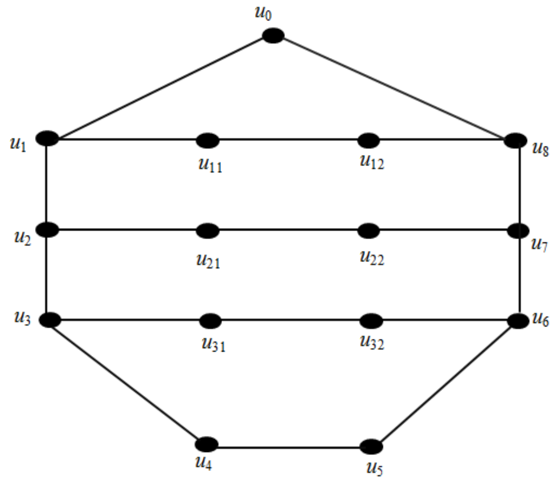

Fig.1b Cycle C_9 with parallel P_4 chord

II. Main Results

Theorem 1: For $m \geq 3$ every cycle C_{2m} with (path) P_4 which are parallel chords admits vertex even mean labeling.

Proof: Consider the graph G, $C_{2m}(m \geq 3)$ with parallel P_4 chords. Let $v_0, v_1, v_2, \ldots, v_{4m-3}$ are the vertices of Graph. Labeling of vertex are $f : V(G) \rightarrow \{2,4,6,\ldots 2(5m-3)\}$,

$$f\ (v_{4j-4}) = 2(4j - 3)\ \ ;\ 1 \leq j \leq m$$
$$f\ (v_{4j-3}) = 4(2j - 1)\ \ ;\ 1 \leq j \leq m$$
$$f\ (v_{4j-2}) = 8j - 2\ \ ;\ 1 \leq j \leq m-1$$
$$f(v_{4j-1}) = 8j\ \ ;\ 1 \leq j \leq m-1$$

The above labeling function of vertices ensures the labeling are unique.

Let $E(G)$, given by $E(G) = \cup_{i=1}^{7} E_i$ where,

$$E_1 = \{(v_{4j-4}v_{4j-3})\ ;\ j = 1\}$$
$$E_2 = \{(v_{4j-4}v_{4j})\ ;\ 1 \leq j \leq m-1\}$$
$$E_3 = \{(v_{4j-3}v_{4j+1})\ ;\ 1 \leq j \leq m-1\}$$
$$E_4 = \{(v_{4j-3}v_{4j-2})\ ;\ 1 \leq j \leq m-1\}$$
$$E_5 = \{(v_{4j-1}v_{4j})\ ;\ 1 \leq j \leq m-1\}$$
$$E_6 = \{(v_{4j-2}v_{4j-1})\ ;1 \leq j \leq m-1\}$$
$$E_7 = \{(v_{4m-4}v_{4m-3})\}$$

Induced function $f^* : E(G) \rightarrow N$, is defined as,

$$f^*(v_{4j-4}v_{4j-3}) = 8j - 5\ ;\ j = 1$$
$$f^*(v_{4j-4}v_{4j}) = 2(4j - 1)\ ;\ 1 \leq j \leq m-1$$
$$f^*(v_{4j-3}v_{4j+1}) = 8j\ ;\ 1 \leq j \leq m-1$$
$$f^*(v_{4j-3}v_{4j-2}) = 8j - 3\ ;\ 1 \leq j \leq m-1$$
$$f^*(v_{4j-1}v_{4j}) = 8j + 1\ ;1 \leq j \leq m-1$$
$$f^*(v_{4j-2}v_{4j-1}) = 8j - 1\ ;\ 1 \leq j \leq m-1$$
$$f^*(v_{4m-4}v_{4m-3}) = 8m - 5$$

It is clear that, labeling of the edges are distinct by the induced function. Hence, C_{2m} ($m \geq 3$) with path P_4 chords which are parallel is a vertex even mean graph.

□

Example 1: Vertex even mean labeling for C_8 with parallel P_4 chords, illustrated in Fig 2.

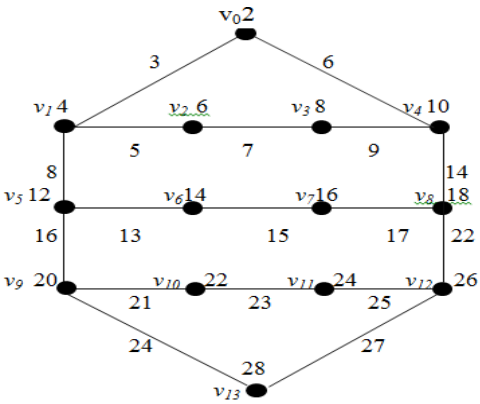

Fig. 2 Cycle C_8 with parallel P_4 chord

Theorem 2: For $m \geq 3$ every cycle C_{2m+1} with (path) P_4 which are parallel chords admits vertex even mean labeling.

Proof: Let the graph G, cycle C_{2m+1} ($m \geq 3$) with parallel P_4 chords. Let $v_0, v_1, v_2, \ldots, v_{4m-2}$ are the vertices of G. Define the labeling for vertex $f : V(G) \to \{2,4,6,\ldots 2(5m-2)\}$ as follows:

$f(v_{4j-4}) = 2(4j-3)$; $1 \leq j \leq m$

$f(v_{4j-3}) = 4(2j-1)$; $1 \leq j \leq m$

$f(v_{4j-2}) = 2(4j-1)$; $1 \leq j \leq m-1$

$f(v_{4j-1}) = 8j$; $1 \leq j \leq m$

$f(v_{4m-2}) = 8m-2$

The above labeling function of vertices ensures the labeling are unique.

Let $E(G) = \cup_{i=1}^{7} E_i$ where,

$E_1 = \{(v_{4j-4}\, v_{4j-3})$; $j = 1\}$

$E_2 = \{(v_{4j-4}\, v_{4j})$; $1 \leq j \leq m-1\}$

$E_3 = \{(v_{4j-3}\, v_{4j+1})$; $1 \leq j \leq m-1\}$

$E_4 = \{(v_{4j-3}\, v_{4j-2})$; $1 \leq j \leq m-1\}$

$E_5 = \{(v_{4j-1}\, v_{4j})$; $1 \leq j \leq m-1\}$

$E_6 = \{(v_{4j-2}\, v_{4j-1})$; $1 \leq j \leq m-1\}$

$E_7 = \{(v_{4m-4}\, v_{4m-2})\}$ and $E_8 = \{(v_{4m-3}\, v_{4m-2})\}$

Let us define the values of induced function $f^* : E(G) \to N$, as follows to label the edges

$f^*(v_{4j-4}\, v_{4j-3}) = 8j-5$; $j = 1$

$f^*(v_{4j-4}\, v_{4j}) = 2(4j-1)$; $1 \leq j \leq m-1$

$f^*(v_{4j-3}\, v_{4j+1}) = 8j$; $1 \leq j \leq m-1$

$f^*(v_{4j-3}\, v_{4j-2}) = 8j-3$; $1 \leq j \leq m-1$

$f^*(v_{4j-2}\, v_{4j-1}) = 8j-1$; $1 \leq j \leq m-1$

$f^*(v_{4j-1}\, v_{4j}) = 8j+1$; $1 \leq j \leq m-1$

$f^*(v_{4m-4}\, v_{4m-2}) = 4(2m-1)$ and $f^*(v_{4m-3}\, v_{4m-2}) = 8m-3$

It is clear that, labeling of the edges are distinct by the induced function. □

Example 2: Vertex even mean graph, C_9 with (path) P_4 as parallel chords, illustrated in Fig 3

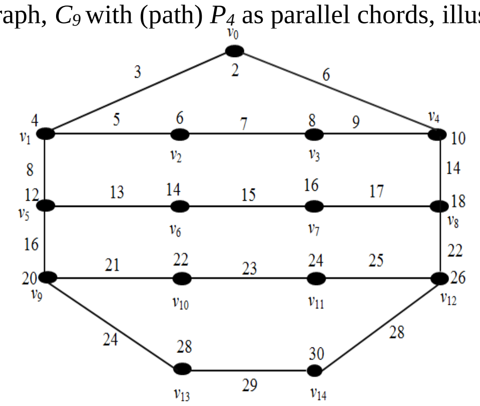

Fig. 3 Cycle C_9 with parallel P_4 chord

Theorem 3: For $m \geq 3$ every cycle C_{2m} with (path) P_4 which are parallel chords admits vertex odd mean labeling.

Proof: Consider G, as C_{2m} ($m \geq 3$) with parallel (path) P_4 chords. Let $v_0, v_1, v_2, \ldots, v_{4m-3}$ are the vertices of G. Labeling for vertices are defined by $f: V(G) \rightarrow \{1, 3, 5, \ldots 2(5m-3)-1\}$,

$f(v_{4j-4}) = 8j - 7 \quad ; 1 \leq j \leq m$
$f(v_{4j-3}) = 8j - 5 \quad ; 1 \leq j \leq m-1$
$f(v_{4j-2}) = 8j - 3 \quad ; 1 \leq j \leq m-1$
$f(v_{4j-1}) = 8j - 1 \quad ; 1 \leq j \leq m-1$

It implies that vertices are labeled and distinct.
Let $E(G) = \cup_{i=1}^{7} E_i$ where,

$E_1 = \{(v_{4j-4}\, v_{4j-3}) ; j = 1\}$
$E_2 = \{(v_{4j-4}\, v_{4j}); 1 \leq j \leq m-1\}$
$E_3 = \{(v_{4j-3}\, v_{4j+1}); 1 \leq j \leq m-1\}$
$E_4 = \{(v_{4j-3}\, v_{4j-2}); 1 \leq j \leq m-1\}$
$E_5 = \{(v_{4j-2}\, v_{4j-1}); 1 \leq j \leq m-1\}$
$E_6 = \{(v_{4j-1}\, v_{4j}) ; 1 \leq j \leq m-1\}$
$E_7 = \{(v_{4m-4}\, v_{4m-3})$

Defining the induced function $f^*: E(G) \rightarrow N$, as follows

$f^*(v_{4j-4}\, v_{4j-3}) = 2(4j - 3) ; \ j = 1$
$f^*(v_{4j-4}\, v_{4j}) = 8i - 3 ; 1 \leq j \leq m-1$
$f^*(v_{4j-3}\, v_{4j+1}) = 8i - 1 ; 1 \leq j \leq m-1$
$f^*(v_{4j-3}\, v_{4j-2}) = 4(2j - 1) ; 1 \leq j \leq m-1$
$f^*(v_{4j-2}\, v_{4j-1}) = 2(4j - 1) ; 1 \leq j \leq m-1$
$f^*(v_{4j-1}\, v_{4j}) = 8j \ ; 1 \leq j \leq m-1$
$f^*(v_{4m-4}\, v_{4m-3}) = 8m - 6$

It is clear that, labeling of the edges are distinct by the induced function. Hence G, C_{2m} ($m \geq 3$) with parallel (path) P_4 as a chords is said to be vertex odd mean graph.

□

Example 3: Vertex odd mean graph, C_8 with parallel (path) P_4 chords is, illustrated in Fig.4

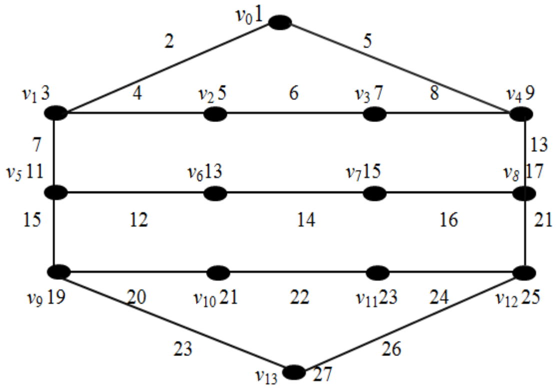

Fig. 4 Cycle C_8 with parallel P_4 chord

Theorem 4: For $m \geq 3$ every cycle C_{2m+1} with (path) P_4 which are parallel chords is admits vertex odd mean labeling.

Proof: Consider G, as C_{2m+1} ($m \geq 3$) with (path) P_4 chords as a parallel. Let $v_0, v_1, v_2, \ldots, v_{4m-2}$ are vertices of G. where the vertex labeling $f: V(G) \rightarrow \{1, 3, 5, \ldots, 2(5m-2)-1\}$ as follows:

$f(v_{4j-4}) = 8j - 7 \quad ; 1 \leq j \leq m$
$f(v_{4j-3}) = 8j - 5 \quad ; 1 \leq j \leq m$
$f(v_{4j-2}) = 8j - 3 \quad ; 1 \leq j \leq m-1$
$f(v_{4j-1}) = 8j - 1 \quad ; 1 \leq j \leq m-1$
$f(v_{4m-2}) = 8m - 3$

The vertices are distinctly labeled.
Let $E(G) = \cup_{i=1}^{8} E_i$ where,

$E_1 = \{(v_{4j-4}\, v_{4j-3}) ; j = 1\}$
$E_2 = \{(v_{4j-4}\, v_{4j}) ; 1 \leq j \leq m-1\}$
$E_3 = \{(v_{4j-3}\, v_{4j+1}) ; 1 \leq j \leq m-1\}$
$E_4 = \{(v_{4j-3}\, v_{4j-2}) ; 1 \leq j \leq m-1\}$
$E_5 = \{(v_{4j-2}\, v_{4j-1}) ; 1 \leq j \leq m-1\}$
$E_6 = \{(v_{4j-1}\, v_{4j}) ; 1 \leq j \leq m-1\}$
$E_7 = \{(v_{4m-4}\, v_{4m-2})\}$ &

$E_8 = \{(v_{4m-3}\ v_{4m-2})\}$

Defining the induced edges by the function $f^* : E(G) \to N$, follows

$f^*(v_{4j-4}\ v_{4j-3}) = 2(4j - 3)$; $j = 1$

$f^*(v_{4j-4}\ v_{4j}) = 8j - 3$; $1 \le j \le m-1$

$f^*(v_{4j-3}\ v_{4j+1}) = 8j - 1$; $1 \le j \le m-1$

$f^*(v_{4j-3}\ v_{4j-2}) = 2(4j - 2)$; $1 \le j \le m-1$

$f^*(v_{4j-2}\ v_{4j-1}) = 2(4j - 1)$; $1 \le j \le m-1$

$f^*(v_{4j-1}\ v_{4j}) = 8j$; $1 \le j \le m-1$

$f^*(v_{4m-4}\ v_{4m-2}) = 8m - 5$ and

$f^*(v_{4m-3}\ v_{4m-2}) = 8m - 4$

It is clear that, labeling of the edges are distinct by the induced function. Hence G, C_{2m+1} with (path) P_4 which are parallel chords is a vertex odd mean graph.

Example 4: C_9 with (path) P_4 chords as parallel is vertex odd mean graph, illustrated in Fig.5.

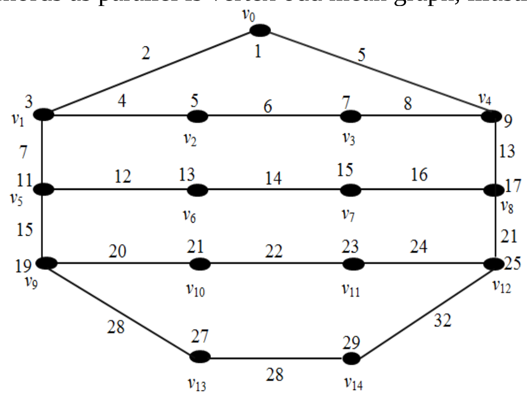

Fig. 5 Cycle C_9 with parallel P_4 chords

Theorem 5: For $m \ge 3$ every cycle C_{2m} with (path) P_4 chords which are parallel is admits square sum labeling.

Proof: Consider G, C_{2m} ($m \ge 3$) with parallel (path) P_4 chords. Let $v_0, v_1, v_2, \ldots, v_{4m-3}$ are vertices of G. Labeling of vertex is defined as $f : V(G) \to \{0, 1, 2, \ldots, 4m-3\}$

$f(v_j) = j$; $0 \le j \le 4m - 3$

Hence, vertices are labeled with above function are distinct.

Let E(G) be the edge set given for C_{2n}, $E(G) = \cup_{i=1}^{7} E_i$ where,

$E_1 = \{(v_{4j-4}\ v_{4j-3})\ ;\ j = 1\}$

$E_2 = \{(v_{4j-3}\ v_{4j+1})\ ;\ 1 \le j \le m-1\}$

$E_3 = \{(v_{4j-4}\ v_{4j})\ ;\ 1 \le j \le m-1\}$

$E_4 = \{(v_{4j-3}\ v_{4j-2})\ ;\ 1 \le j \le m-1\}$

$E_5 = \{(v_{4j-2}\ v_{4j-1})\ ;\ 1 \le j \le m-1\}$

$E_6 = \{(v_{4j-1}\ v_{4j})\ ;\ 1 \le j \le m-1\}$

$E_7 = \{(v_{4m-4}\ v_{4m-3})\}$

Defining the induced edge function $f^* : E(G) \to N$,

$f^*(v_{4j-4}\ v_{4j-3}) = 32j^2-56j+25$; $j = 1$

$f^*(v_{4j-3}\ v_{4j+1}) = 32i^2-16j+10$; $1 \le j \le m-1$

$f^*(v_{4j-4}\ v_{4j}) = 32j^2-32j+16$; $1 \le j \le m-1$

$f^*(v_{4j-3}\ v_{4j-2}) = 32j^2-40j+13$; $1 \le j \le m-1$

$f^*(v_{4j-2}\ v_{4j-1}) = 32j^2-24j+5$; $1 \le j \le m-1$

$f^*(v_{4j-1}\ v_{4j}) = 32j^2-8j+1$; $1 \le j \le m-1$

$f^*(v_{4m-4}\ v_{4m-3}) = 32m^2-56m+25$

It is clear that, labeling of the edges are distinct by the induced function. Hence, Graph G C_{2m} ($m \ge 3$) with parallel (path) P_4 chords is a square sum graph.

Example 5: A cycle C_8 with parallel P_4 chords is square sum graph, illustrated in Fig.6

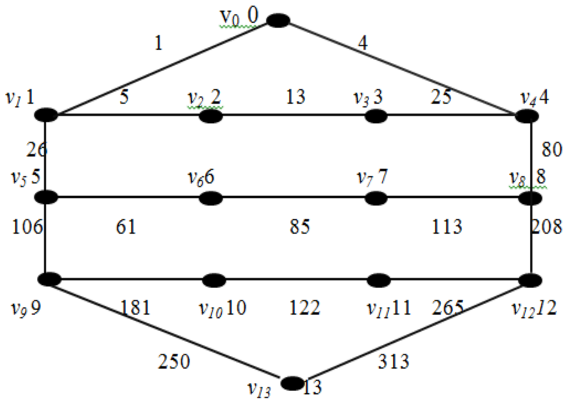

Fig. 6 Cycle C_8 with parallel P_4 chords

Theorem 6: For m ≥ 3 every cycle C_{2m+1} with (path) P_4 which are Parallel chords is admits square sum labeling.

Proof: Consider G, as C_{2m+1} $(m \geq 3)$ with (path) P_4 chords as a parallel. Let $v_0, v_1, v_2, \ldots, v_{4m-2}$ are vertices of G. Labeling of vertex are defined by $f : V(G) \rightarrow \{0, 1, 2, \ldots, 4m-2\}$,

$f(v_j) = j \quad ; 0 \leq j \leq 4m-2$

The above labeling function will label all vertices are distinct.

Let E(G) be the edge set given for C_{2n+1}, $E(G) = \cup_{i=1}^{8} E_i$ where,

$E_1 = \{(v_{4j-4}\, v_{4j-3}) \,;\, j = 1\}$
$E_2 = \{(v_{4j-3}\, v_{4j+1}) \,;\, 1 \leq j \leq m-1\}$
$E_3 = \{(v_{4j-4}\, v_{4j}) \,;\, 1 \leq j \leq m-1\}$
$E_4 = \{(v_{4j-3}\, v_{4j-2}) \,;\, 1 \leq j \leq m-1\}$
$E_5 = \{(v_{4j-2}\, v_{4j-1}) \,;\, 1 \leq j \leq m-1\}$
$E_6 = \{(v_{4j-1}\, v_{4j}) \,;\, 1 \leq j \leq m-1\}$
$E_7 = \{(v_{4m-4}\, v_{4m-2})\}$ and
$E_8 = \{(v_{4m-3}\, v_{4m-2})\}$,

Defining the induced edges by the function $f^* : E(G) \rightarrow N$,

$f^*(v_{4j-4}\, v_{4j-3}) = 32j^2-56j+25 \,;\; j = 1$
$f^*(v_{4j-3}\, v_{4j+1}) = 32j^2-16j+10;\; 1 \leq j \leq m-1$
$f^*(v_{4j-4}\, v_{4j}) = 32j^2-32j+16 \,;\; 1 \leq j \leq m-1$
$f^*(v_{4j-3}\, v_{4j-2}) = 32j^2-40j+13;\; 1 \leq j \leq m-1$
$f^*(v_{4j-2}\, v_{4j-1}) = 32j^2-24j+5;\; 1 \leq j \leq m-1$
$f^*(v_{4j-1}\, v_{4j}) = 32j^2-8j+1 \;;\; 1 \leq j \leq m-1$
$f^*(v_{4m-4}\, v_{4m-2}) = 32m^2-48m+20$ and
$f^*(v_{4m-3}\, v_{4m-2}) = 32m^2-40m+13$

It is clear that, labeling of the edges are distinct by the induced function. Therefore, the Graph G, C_{2m+1} $(m \geq 3)$ with parallel P_4 chords is a square sum graph.

Example 6: A Cycle C_9 with parallel P_4 chords is square sum graph, illustrated in Fig 7.

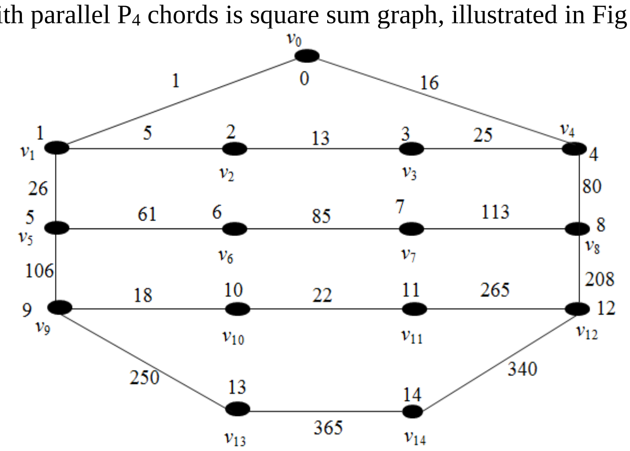

Fig. 7 Cycle C_9 with parallel P_4 chords

Theorem 7: For $m \geq 3$ every cycle C_{2m} with (path) P_4 which are parallel chords is admits square difference labeling.

Proof: Consider G, has $v_0, v_1, v_2, \ldots, v_{4m-3}$ be the vertices. Labeling of vertex is defined by $f : V(G) \rightarrow \{0, 1, 2, \ldots, 4m-3\}$,

$f(v_j) = j \; ; 0 \leq j \leq 4m-3$

Hence vertices labeled are distinct.

Let E(G) be the edge set given for C_{2m}, $E(G) = \cup_{i=1}^{7} E_i$ where,

$E_1 = \{(v_{4j-4}\, v_{4j-3}) \; ; j = 1\}$
$E_2 = \{(v_{4i-3}\, v_{4j+1}) \; ; 1 \leq j \leq m-1\}$
$E_3 = \{(v_{4j-4}\, v_{4j}) \; ; 1 \leq j \leq m-1\}$
$E_4 = \{(v_{4j-3}\, v_{4j-2}) \; ; 1 \leq j \leq m-1\}$
$E_5 = \{(v_{4j-2}\, v_{4j-1}) \; ; 1 \leq j \leq m-1\}$
$E_6 = \{(v_{4j-1}\, v_{4j}) \; ; 1 \leq j \leq m-1\}$
$E_7 = \{(v_{4m-4}\, v_{4m-3})\}$ these edges set C_{2m} $(m \geq 3)$.

Defining the induced edges by the function $f^* : E(G) \rightarrow N$,

$f^*(v_{4j-4}\, v_{4j-3}) = 32j^2-56j+25 \; ; \; j = 1$
$f^*(v_{4j-3}\, v_{4j+1}) = 8(4j-1) \; ; 1 \leq j \leq m-1$
$f^*(v_{4j-4}\, v_{4j}) = 8(4j-2) \; ; 1 \leq j \leq m-1$
$f^*(v_{4j-3}\, v_{4j-2}) = 8j-5 \; ; 1 \leq j \leq m-1$
$f^*(v_{4j-2}\, v_{4j-1}) = 8j-3 ; 1 \leq j \leq m-1$
$f^*(v_{4j-1}\, v_{4j}) = 8j-1 \; ; 1 \leq j \leq m-1$
$f^*(v_{4m-4}\, v_{4m-3}) = 8m-7$

It is clear that, labeling of the edges are distinct by the induced function. Therefore, the Graph G, C_{2m} $(m \geq 3)$ with (path) P_4 chords with parallel is a square Difference graph.

Example 7: C_8 with (path) P_4 chords as a parallel is a square Difference graph, illustrated in Fig 8.

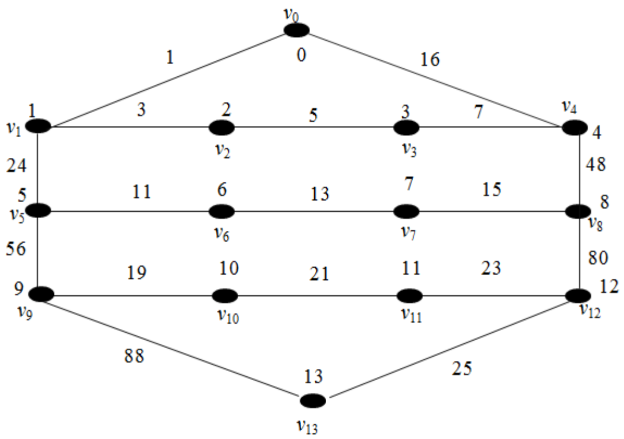

Fig. 8 Cycle C_8 with parallel P_4 chords

Theorem 8: For $m \geq 3$ every cycle C_{2m+1} with (path) P_4 which are parallel chords I admits square difference labeling.

Proof: Consider, G has $v_0, v_1, v_2, \ldots, v_{4m-2}$ are the vertices of *G*. The vertex labeling is defined by $f : V(G) \rightarrow \{0, 1, 2, \ldots, 4m-2\}$,

$f(v_j) = j \; ; 0 \leq j \leq 4m-2$

Hence vertices are labeled distinctly.

Let E(G) be the edge set given for C_{2m+1}, $E(G) = \cup_{i=1}^{8} E_i$ where,

$E_1 = \{(v_{4j-4}\, v_{4j-3}) \; ; j = 1\}$
$E_2 = \{(v_{4j-3}\, v_{4j+1}) \; ; 1 \leq j \leq m-1\}$
$E_3 = \{(v_{4j-4}\, v_{4j}) \; ; 1 \leq j \leq m-1\}$
$E_4 = \{(v_{4j-3}\, v_{4j-2}) \; ; 1 \leq j \leq m-1\}$
$E_5 = \{(v_{4j-2}\, v_{4j-1}) \; ; 1 \leq j \leq m-1\}$
$E_6 = \{(v_{4j-1}\, v_{4j}) \; ; 1 \leq j \leq m-1\}$
$E_7 = \{(v_{4m-4}\, v_{4m-2})\}$ and
$E_8 = \{(v_{4m-3}\, v_{4m-2})\}$

Defining the induced edges by the function $f^* : E(G) \rightarrow N$,

$f^*(v_{4j-4}\, v_{4j-3}) = 32j^2 - 56j + 25 \; ; \; j = 1$

$f^*(v_{4j-3}\, v_{4j+1}) = 8(4j - 1)\ ;\ 1 \le j \le m-11$
$f^*(v_{4j-4}\, v_{4j}) = 8(4j - 2)\ ;\ 1 \le j \le m-1$
$f^*(v_{4j-3}\, v_{4j-2}) = 8j - 5\ ;\ 1 \le j \le m-1$
$f^*(v_{4j-2}\, v_{4j-1}) = 8j - 3;\ 1 \le j \le m-1$
$f^*(v_{4j-1}\, v_{4j}) = 8j - 1\ ;\ 1 \le j \le m-1$
$f^*(v_{4m-4}\, v_{4m-2}) = 4(4m-3)$ &
$f^*(v_{4m-3}\, v_{4m-2}) = 8m-5$

It is clear that, labeling of the edges are distinct by the induced function. Hence, graph admits the square difference labeling.

Example 8: C_8 with (path) P_4 chords are parallel, is a square Difference graph, illustrated in Fig 9.

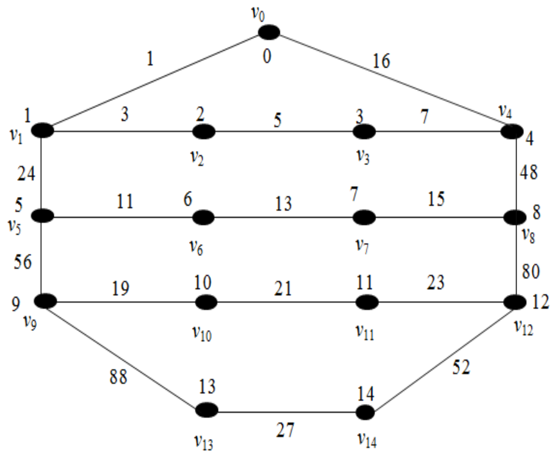

Fig. 9 Cycle C_9 with parallel P_4 chords

Conclusion:

Here, we have proposed the certain results, which obtains the labeling on Cycle with Parallel (path) P_4 Chord; We have proved that the graphs C_{2m} ($m \ge 3$) with Parallel P_4 Chord and C_{2m+1} ($m \ge 3$) with Parallel (path) P_4 Chord permits vertex even mean, vertex odd mean labeling. In addition to this, we also proved results for Square sum and difference labeling.

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