Nano $g^{*}\alpha$ -Homeomorphisms in nano topological spaces

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Abstract: In This Paper, We Introduce $Ng^{*}A$ -Open And Closed Maps In Nano Topological Spaces And Also We Introduce And Study The Concept Of $Ng^{*}A$ -Homeomorphisms In Nano Topological Spaces. Further We Obtain Certain Charecterizations Of These Maps.

Keywords: $Ng^{*}A$ -Closed Map, $Ng^{*}A$ -Open Map, $Ng^{*}A$ -Continuous Functions, And $Ng^{*}A$ -Homeomorphisms.

Introduction

The notion of homeomorphism plays a very important role in topology. A homeomorphism is a bijective map $f : X \to Y$ when both f and f^{-1} are continuous. Maki et al [7] have introduced and investigated g- homeomorphism and gc -homeomorphism in topological spaces. Lellis Thivagar[6] introduced Nano homeomorphisms in Nano topological spaces. Bhuvaneswari et al.[1] introduced and studied some properties of Nano generalized homeomorphism in Nano topological spaces. In this paper, a new class of homeomorphism called Nano generalized star alpha homeomorphism is introduced and some of its properties are discussed.

Preliminaries

Definition 2.1. [5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

(1) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_{R(x)} = \cup_{x\in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by $x \in U$.

(2) The Upper approximation of X with respect to R is the set of all objects, which can be certain classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_{R(x)} = \cup_{x\in U} \{R(x) : R(x) \cap X\} \neq \emptyset$

(3) The Boundary region of X with respect to R is the set of all objects which can be classified as neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.
That is, $B_R(X) = U_R(X) - L_R(X)$

Proposition 2.2. [5] If (U ,R) is an approximation space and X, $Y \subseteq U$, then

(1) $L_R(X) \subseteq X \subseteq U_R(X)$
(2) $L_R(\phi) = U_R(\phi) = \phi$
(3) $L_R(U) = U_R(U) = U$
(4) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(5) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(6) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(7) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(9) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

(10) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
(11) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3. [5] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:
(i) U and $\phi \in \tau_R(X)$
(ii) The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
That is, $\tau_R(X)$ is a topology on U is called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ is called the nano topological space. Elements of the nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^c$ are called nano closed sets.

Remark 2.4. [5] If $[\tau_R(X)]$ is the nano topology on U with respect to X. Then the set $B = \{U, \tau_R(X), B_R(X)\}$ is the basis for $[\tau_R(X)]$.

Definition 2.5. [1] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano continuous on U, if the inverse image of every nano open set in V is nano open in U.

Definition 2.6. [6] A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called nano open(resp. nano closed) if the image of every nano open set in $(U, \tau_R(X))$ is nano open(resp. nano closed) in $(V, \tau_{R'}(Y))$.

Definition 2.7. [2] A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called Ng –closed (resp. Ng -open) if the image of every nano g-closed set in $(U, \tau_R(X))$ is nano g-closed (resp. nano g-open) in $(V, \tau_{R'}(Y))$.

Definition 2.8. [6] A bijective function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is called Nano homeomorphism if f is both nano continuous and nano open.

Definition 2.9. [2] A bijective function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called Ng -homeomorphism if f is both Ng -continuous and Ng -open.

Definition 2.10. [2] A bijective function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called Ngp -homeomorphism if f is both Ngp -continuous and Ngp -open.

Definition 2.11. Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano generalized star alpha continuous(briefly $Ng^*\alpha$ continuous) functions on U, if the inverse image of every nano open set in V is $Ng^*\alpha$ -open set in U.

3.Nano $g^*\alpha$ -open and closed functions

Definition 3.1. A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be $Ng^*\alpha$ -open (resp. $Ng^*\alpha$ closed) function if the image of every nano open (resp. nano closed) set in $(U, \tau_R(X))$ is $Ng^*\alpha$ -open (resp. $Ng^*\alpha$ -closed) in $(V,\tau_{R'}(Y))$.

Example 3.2. Let U = {a, b, c, d} with U/R = {{a}, {b,c}, {d}} and X = {b, d}. Then $\tau_R(X)$ ={U, ϕ,{d}, {b,c}, {b,c,d}}. Let V = {x,y,z,w} with V/R' = {{y},{z},{x,w}} and Y = {y,w}, then $\tau_{R'}(Y)$ = {V,ϕ,{y},{x,w},{x,y,w}}. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is f (a) = z, f (b) = y, f(c) = x and f (d) = w. Then f is $Ng^*\alpha$ -closed map.

Theorem 3.3. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a function. If f is nano open function, then f is $Ng^*\alpha$ -open function.
Proof. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a nano open function and S be a nano open set in U. Thus f(S) is nano open and hence $Ng^*\alpha$ -open in V. Thus f is $Ng^*\alpha$ -open.

Theorem 3.4. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a function. If f is nano closed function, then f is $Ng^*\alpha$ -closed function.
Proof. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a nano closed function and S be a nano -closed set in U. Thus f(S) is nano closed and hence $Ng^*\alpha$ -closed in V. Thus f is $Ng^*\alpha$ -closed.

Theorem 3.5. A map $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is $Ng^*\alpha$ -closed if and only if for each subset A of V and for each nano open set F containing $f^{-1}(A)$ there is a $Ng^*\alpha$ -open set B of V such that $A \subset V$ and $f^{-1}(V) \subset F$.

Proof. Suppose f is $Ng^*\alpha$ -closed. Let A be a subset of V and F is an open set of U such that $f^{-1}(V) \subset F$. Then $B = V - f(U - F)$ is a $Ng^*\alpha$ -open set containing A such that $f^{-1}(V) \subset F$.

Conversely, suppose that C is a closed set of U. Then $f^{-1}(V - f(C)) \subset (U - C)$ and $U - C$ is Nano open. By hypothesis there is $Ng^*\alpha$ -open set B of V such that $V - f(C) \subset B$ and $f^{-1}(B) \subset U - C$. Therefore $C \subset U - f^{-1}(B)$. Hence $V - B \subset f(C) \subseteq f(U - f^{-1}(B)) \subset V - F$ which implies that $f(C) = V - B$. Since $V - B$ is $Ng^*\alpha$ -closed set in $(V, \tau_R(X))$. Therefore f is $Ng^*\alpha$ -closed function.

Theorem 3.6. A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano-closed and a function $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$ is $Ng^*\alpha$ -closed then their composition $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(Y))$ is $Ng^*\alpha$ -closed.
Proof. Let H be a nano closed set in U. Then f (H) is nano closed in V and $(g \circ f)(H) = g(f(H))$ is $Ng^*\alpha$ -closed, as g is $Ng^*\alpha$ -closed. Hence, $g \circ f$ is $Ng^*\alpha$ -closed.

Theorem 3.8. Every Nano closed map is $Ng^*\alpha$ -closed map but not conversely.
Proof. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a Nano closed map. Let A be a Nano closed set in nano topological space $(U, \tau_R(X))$. Then the image of A under the map f is Nano closed in the Nano topological space $(V, \tau_{R'}(Y))$. Since every Nano closed set is $Ng^*\alpha$ -closed. Hence f is $Ng^*\alpha$ -closed.

The converse of the theorem need not be true as seen from the following example.

Example 3.9. From the example (3.2) f is $Ng^*\alpha$ -closed. But f is not Nano closed, since image {x,y,z},{z,w} are not nano closed in $(V,\tau_{R'}(Y))$.
Theorem 3.10. Every $Ng^*\alpha$ -closed map is Ng -closed map but not conversely.
Proof. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be a Nano $Ng^*\alpha$ -closed map. Let A be a Nano closed set in nano topological space $(U, \tau_R(X))$. Then the image of A under the map f is $Ng^*\alpha$ -closed in the Nano topological space $(V, \tau_{R'}(Y))$. Since every $Ng^*\alpha$ -closed set is Ng -closed. f(A) is Ng -closed set. Hence f is Ng -closed.

The converse of the theorem need not be true as seen from the following example.

Example 3.11. Let U = {a ,b, c, d} with U/R = {{a},{c},{b,d}} and X = {b,d}. Then $\tau_R(X) = \{U,\phi,\{a\},\{b, d\},\{a, b, d\}\}$. Let V = {x, y, z, w} with $V/R' = \{\{x\},\{w\},\{y,z\}\}$ and Y = {y,w},then $\tau_{R'}(Y) = \{V,\phi,\{w\},\{y,z\},\{y,z,w\}\}$. Define $f : (U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is f(a) = w, f(b) = y, f(c) = x and f(d) = z. Then f is Ng -closed function. But not $Ng^*\alpha$ -closed map, since the image {y,z} are not $Ng^*\alpha$ -closed in $(V, \tau_{R'}(Y))$.

Theorem 3.12. Every $Ng^*\alpha$ -closed map is Ngs -closed map but not conversely.

Proof. Let f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be a Ng*α -closed map. Let A be a Nano closed set in the nano topological space (U, $\tau_R(X)$). Then the image of A under the map f is Ng*α -closed in the Nano topological space (V, $\tau_{R'}(Y)$). Since every Ng*α -closed set is Ngs -closed. f(A) is Ngs -closed set. Hence f is Ngs -closed.

The converse of the theorem need not be true as seen from the following example.

Example 3.13. From the example(3.2) Define f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is f(a) = x, f(b) = y, f (c) = z and f (d) = w, then f is Ngs -closed function. But not Ng*α -closed map, since the image {x,w},{w} are not Ng*α -closed in (V, $\tau_{R'}(Y)$).

Theorem 3.14. Every Ng*α -closed map is Ngp -closed map but not conversely.
Proof. Let f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be a Ng*α -closed map. Let A be a Nano closed set in nano topological space (U, $\tau_R(X)$). Then the image of A under the map f is Ng*α -closed in the Nano topological space (V, $\tau_{R'}(Y)$). Since every Ng*α -closed set is Ngp -closed. f(A) is Ngp -closed set. Hence f is Ngp -closed.

The converse of the theorem need not be true as seen from the following example.

Example 3.15. From the example(3.2) Define f : (U,$\tau_R(X)$) → (V,$\tau_{R'}(Y)$) is f (a) = x, f (b) = y, f (c) = z and f (d) = w, then f is Ngp -closed function. But not Ng*α -closed map, since the image {x}, {x,w} are not Ng*α -closed in (V, $\tau_{R'}(Y)$).

4. Nano g*α -Homeomorphism

In this section, a new form of homeomorphism namely, Ng*α -homeomorphism is introduced and some of the properties are studied.

Definition 4.1. A function f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is said to be Ng*α -homeomorphism if
(i) f is one to one and onto
(ii) f is a Ng*α -continuous
(iii) f is a Ng*α –open

Theorem 4.2. Let f : (U,$\tau_R(X)$) → (V,$\tau_{R'}(Y)$) be one to one and onto mapping. Then f is Ng*α -homeomorphism if and only if f is Ng*α -closed and Ng*α -continuous.

Proof. Let f : (U,$\tau_R(X)$) → (V,$\tau_{R'}(Y)$) be a Ng*α -homeomorphism. Then f is Ng*α -continuous. Let A be Nano closed set in (U, $\tau_R(X)$). Then U − A is Nano open. Since f is Ng*α -open. f(U − A) is Ng*α -open in (V,$\tau_{R'}(Y)$. That is V − f(A) is Ng*α -open in (V, $\tau_{R'}(Y)$). Hence f(A) is Ng*α -closed in (V,$\tau_{R'}(Y)$ for every Nano closed set A is (U, $\tau_R(X)$). Hence f : (U,$\tau_R(X)$) → (V,$\tau_{R'}(Y)$) is Ng*α -closed.

Conversely, let f be Ng*α -closed and Ng*α -continuous function. Let G be a Nano open set in (U, $\tau_R(X)$).Then U −G is Nano closed in (U,$\tau_R(X)$). Since f is Ng*α -closed, f(U −G) is Ng*α−closed in (V, $\tau_{R'}(Y)$. That is f(U − G) = V − F(G) is Ng*α -closed in (V,$\tau_{R'}(Y)$ for every Nano open set G in (U, $\tau_R(X)$). Thus, f is Ng*α -open and hence f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ng*α homeomorphism.

Example 4.3. Let U = {a,b,c,d} with U/R = {{a},{b,c},{d}} and X = {b,d}. Then $\tau_R(X)$ ={U, ϕ, {d}, {b,c}, {b,c,d}}. Let V = {x,y,z,w} with V/R′ = {{y},{z},{x,w}} and Y = {y,w}, then $\tau_{R'}(Y)$ = {V, ϕ, {x,w},{x,y,w}}. Define f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is f (a) = z, f (b) = y, f (c) = x and f (d) = w. Then f is one to one and onto, Ng*α is open and Ng*α -continuous. So that f : (U,$\tau_R(X)$) → (V,$\tau_{R'}(Y)$) is Ng*α -homeomorphism.

Theorem 4.4. Let $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be $Ng^*\alpha$ -continuous function. Then the fallowing statements are equivalent.

(i) f is an $Ng^*\alpha$ -open function
(ii) f is an $Ng^*\alpha$ -homeomorphism
(iii) f is an $Ng^*\alpha$ -closed function

Proof. (i) ⇒ (ii): By the given hypothesis, the function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is bijective, $Ng^*\alpha$ -continuous and $Ng^*\alpha$ -open. Hence the function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is homeomorphism.

(ii) ⇒ (iii) By the given hypothesis, the function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -homeomorphism and $Ng^*\alpha$ -open. Let A be the nano closed set in $(U,\tau_R(X))$. Then A^c is nano open in $(U, \tau_R(X))$. By assumption, $f(A^c)$ is $Ng^*\alpha$ -open in $(V,\tau_{R'}(Y))$. That is $f(A^c) = (f(A))^c$ is $Ng^*\alpha$ -open in $(V, \tau_{R'}(Y))$ and $f(A)$ is $Ng^*\alpha$ -closed in $(V,\tau_{R'}(Y))$. Hence $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -closed function.

(iii) ⇒ (i) Let F be a nano open set in $(U,\tau_R(X))$. Then F^c is nano closed set in $(U,\tau_R(X))$. By the given hypothesis, $f(F^c)$ is $Ng^*\alpha$ -closed in $(V, \tau_{R'}(Y))$. Now, $f(F^c) = (f(F))^c$ is $Ng^*\alpha$ -closed, (i.e) $f(F)$ is $Ng^*\alpha$ -open in $(V, \tau_{R'}(Y))$ for every nano open set F in $(U, \tau_R(X))$. Hence $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -open function.

Theorem 4.5. Every Nano homeomorphism is $Ng^*\alpha$ -homeomorphism but not conversely.
Proof. If $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Nano homeomorphism, by definition (4.1), f is bijective, Nano continuous and $Ng^*\alpha$ -open. Then $f : (U,\tau_R(X)) \to (V, \tau_{R'}(Y))$ is $Ng^*\alpha$ -open respectively. Hence, the function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano homeomorphism. Every nano continuous function is $Ng^*\alpha$ -open. Then f is bijective, $Ng^*\alpha$ -continuous $Ng^*\alpha$ -open. Therefore f is $Ng^*\alpha$ -homeomorphism.
In the converse part through example we have to prove that f is $Ng^*\alpha$ -homeomorphism but not Nano homeomorphism.

Example 4.6. From the example (4.3) the map f is $Ng^*\alpha$ -homeomorphism. Now $f^{-1}(V) = U$, $f^{-1}(\phi) = \phi$, $f^{-1}(\{y\}) = \{b\}$, $f^{-1}(\{x, w\}) = \{c,d\}$, $f^{-1}(\{x, y, w\}) = \{b, c, d\}$. Hence the inverse image of nano open set in $(V, \tau_{R'}(Y))$ are not nano open in $(U, \tau_R(X))$ and hence f is not nano continuous thus the map $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is not nano homeomorphism.

Theorem 4.7. Every $Ng^*\alpha$ -homeomorphism is Ng -homeomorphism but not conversely.
Proof. If $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -homeomorphism, by definition (4.1), f is bijective, $Ng^*\alpha$ -continuous and $Ng^*\alpha$ -open. Then $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ng -continuous and Ng -open respectively. Hence the function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ng -homeomorphism. Therefore every $Ng^*\alpha$ -homeomorphism is Ng -homeomorphism.

Example 4.8. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a,b\},\{c\},\{d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U, \phi,\{a,b\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y,w\}\}$ and $Y = \{y,w\}$, then $\tau_{R'}(Y) = \{V, \phi,\{y,w\},\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $f(a) = x$, $f(b) = y$, $f(c) = z$ and $f(d) = w$. Then f is one to one and onto, Ng is open and Ng -continuous. So that $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ng -homeomorphism. Now $f^{-1}(V) = U$, $f^{-1}(\phi) = \phi$, $f^{-1}(\{y,w\}) = \{b,d\}$. Hence the inverse image of nano open set in $(V,\tau_{R'}(Y))$ is not $Ng^*\alpha$ -open in $(U,\tau_R(X))$ and hence f is not $Ng^*\alpha$ -continuous thus the map $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is not $Ng^*\alpha$ –homeomorphism.

Theorem 4.9. Every $Ng^*\alpha$ -homeomorphism is Ngs -homeomorphism but not conversely.
Proof. If $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -homeomorphism, by definition (4.1), f is bijective, $Ng^*\alpha$ -continuous and $Ng^*\alpha$ -open. Then $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs -continuous and Ngs -open respectively. Hence the function f :

$(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is an Ngs -homeomorphism. Therefore every $Ng^*\alpha$ -homeomorphism is Ngs -homeomorphism.

Example 4.10. From the example (4.3) Define $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is f (a) = w, f (b) = y, f (c) = z, f(d) = x. Then f is one to one and onto, Ngs -open and Ngs -continuous. So that $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs -homeomorphism. Now, $f^{-1}(V) = U$, $f^{-1}(\phi) = \phi$, $f^{-1}(\{y\}) = \{b\}$, $f^{-1}(\{x,w\}) = \{a,d\}$,$f^{-1}(\{x,y,w\}) = \{a,b,d\}$. Hence the inverse image of nano open set in $(V,\tau_{R'}(Y))$ are not $Ng^*\alpha$ -continuous thus the map $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is not $Ng^*\alpha$ homeomorphism.

Theorem 4.11. Every $Ng^*\alpha$ -homeomorphism is Ngp -homeomorphism but not conversely.
Proof. If $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -homeomorphism, by definition (4.1), f is bijective, $Ng^*\alpha$ -continuous and $Ng^*\alpha$ -open. Then $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngp -continuous and Ngp -open respectively. Hence the function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is an Ngp -homeomorphism. Therefore every $Ng^*\alpha$ -homeomorphism is Ngp -homeomorphism.

Example 4.12. Let U = {a,b,c,d} with U/R = {{a},{c},{b,d}} and X = {b,d}. Then $\tau_R(X)$ ={U, ϕ, {a}, {b,d}, {a,b,d}}. Let V = {x,y,z,w} with V/R$'$ = {{x},{w},{y,z}} and Y = {y,w}, then $\tau_{R'}(Y)$ = {V,ϕ,{w},{y,z},{y,z,w}}. Define f : $(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is f (a) = z, f (b) = y, f (c) = x and f(d) = w. Then f is one to one and onto, Ngp is open and Ngp -continuous. Now, $f^{-1}(V) = U$, $f^{-1}(\phi) = \phi$, $f^{-1}(\{w\}) = \{d\}$, $f^{-1}(\{y,z\}) = \{a,b\}$, $f^{-1}(\{y,z,w\}) = \{a,b,d\}$. Hence the inverse image of nano open set in $(V, \tau_{R'}(Y))$ are not $Ng^*\alpha$ -continuous thus the map $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is not $Ng^*\alpha$ -homeomorphism.

Theorem 4.13. A one to one map f of $(U,\tau_R(X))$ onto $(V, \tau_{R'}(Y))$ is a $Ng^*\alpha$ -homeomorphism if and only if f ($Ng^*\alpha cl(A)$) = Ncl (f (A)) for every subset A of $(U, \tau_R(X))$.
Proof. If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $Ng^*\alpha$ -homeomorphism then f is $Ng^*\alpha$ -continuous and $Ng^*\alpha$ -closed. If A ⊆ U, it follows that f ($Ng^*\alpha cl(A)$ ⊆ Ncl (f(A)). Since f is $Ng^*\alpha cl(A)$ is nano closed in $(U, \tau_R(X))$ and f is $Ng^*\alpha$ -closed map, f($Ng^*\alpha cl(A)$) = f($Ng^*\alpha cl$(f(A)). Since A ⊆ $Ng^*\alpha cl(A)$, f(A) ⊆ f($Ng^*\alpha cl(A)$) and hence Ncl(f(A)) ⊆ Ncl(f($Ng^*\alpha cl(A)$)) = f($Ng^*\alpha cl(A)$). Thus Ncl(f(A)) ⊆ f($Ng^*\alpha cl(A)$). Therefore, f ($Ng^*\alpha cl(A)$ = Ncl(f(A)) if f is $Ng^*\alpha$ -homeomorphism.

Conversely, If f($Ng^*\alpha cl(A)$) = Ncl(f(A)) for every subset A of $(U,\tau_R(X))$, then f is $Ng^*\alpha$ continuous. If A is nano closed in $(U, \tau_R(X))$, then $Ng^*\alpha cl(A)$ = A which implies f($Ng^*\alpha cl(A)$ = f(A). Hence, by the given hypothesis, it follows that Ncl(A) = f(A). Thus f(A) is nano closed in V and hence $Ng^*\alpha$ -closed in V for every nano closed set A in U. That is f is $Ng^*\alpha$ -closed. Thus $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -homeomorphism.

Remark 4.14. The composition of two $Ng^*\alpha$ -homeomorphism need not be a $Ng^*\alpha$ -homeomorphism as seen from the example.

Example 4.15. Let $(U, \tau_R(X))$, $(V, \tau_{R'}(Y))$ and $(W, \tau_{R''}(Z))$ be three nano topological spaces and U = V = W = {a,b,c,d}, then $\tau_R(X)$ = {U, ϕ,{d},{b,c},{b,c,d}} with U/R = {{a},{b,c},{d}} and X = {b, d}. $\tau_{R'}(Y)$ = {V, ϕ, {b}, {a, d}, {a, b, d}} with V/R$'$ = {{b},{c},{a,d}} and Y = {b,d}. $\tau_{R''}(Z)$={W, ϕ, {a},{c,d},{a,c,d}} with W/R$''$ = {{a},{b},{c,d}} and z = {a, c}. Define two function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ as f (a) = c, f (b) = b, f (c) = a, f(d) = d and g(a) = a, g(b) = d, g(c) = b, and g(d) = c. Here the function f and g are $Ng^*\alpha$ -continuous and bijective. Also the image of every Nano open set in $(U,\tau_R(X))$ is $Ng^*\alpha$ –open $(V, \tau_{R'}(Y))$. That is f{(d)} = {d}, f{(b,c)} = {a,b}, f{(a,b,d)} = {b,c,d}. Thus the function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is $Ng^*\alpha$ -open and thus $Ng^*\alpha$ -homeomorphism. The map $g : (V,\tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is also $Ng^*\alpha$ -continuous ,bijective and open. Hence g is also $Ng^*\alpha$ -homeomorphism. But their composition $g \circ f : (V,\tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is not a $Ng^*\alpha$ -homeomorphism. Because for the Nano open set A = {c,d} in $(W, \tau_{R''}(Z))$, $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}\{c,d\}) = f^{-1}(\{b,d\}) = \{b,d\}$ is not in $Ng^*\alpha$ -open in $(U, \tau_R(X))$. Hence the composition $g \circ f : (U,\tau_R(X)) \to (W, \tau_{R''}(Z))$, is not $Ng^*\alpha$ continuous and thus not a $Ng^*\alpha$ -homeomorphism. Thus the composition of two $Ng^*\alpha$ -homeomorphism need not be a $Ng^*\alpha$ -homeomorphism.

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