Feebly θ-closed sets and its properties

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Abstract: The main goal of this paper is to introduce the concept of feebly θ-open set and investigates the properties of feebly θ-interior, feebly θ-closure, feebly θ-exterior, feebly θ-frontier of a set.

Keyword :s-open, fθ-open, fθ-interior, fθ-closure, fθ-exterior, fθ-frontier.

1. Introduction

In 1970, Levine[4] introduced the concept of generalized closed sets in topological spaces. In the literature, notions of semi-open sets, pre-open sets, α-open sets and semi pre-open sets (= β-open sets) plays an important role in the researches in topological spaces. Since then, these sets have been widely investigated in the literature. Navalagi[8] investigate the concept of α-neighbourhoods in topological spaces. Miguel Caldas et al[7] brings up the some properties of θ-open sets, in 2004. The concept of feebly open and feebly closed sets are introduced by Maheswari and Jain in the year 1982. Bhuvaneswari and Dhana Balan introduced feebly regular closed sets in 2015. In this paper, we introduce feebly θ-open set and investigates the properties of feebly θ-interior, feebly θ-closure, feebly θ-exterior, feebly θ-frontier of a set.

2. Preliminaries

Definition 2.1. [6] Let X be a topological space and A be a subset of X. It is said to be semi regular open if A = sint(scl(A)) and also defined on other hand, it is said to be semi-regular open if both semi open (if A $\subset$ cl(int(A)) [3]) and semi closed (if int(cl(A)) $\subset$ A).

Definition 2.2. [5] A subset A of a topological space X is said to be feebly open (resp. feebly closed) if A $\subset$ scl(int(A)) (resp. sint(cl(A)) $\subset$ A).

Definition 2.3. [9] A map f : X $\rightarrow$ Y is said to be feebly closed (resp. feebly open) if the image of each closed set (resp. open set) in X is feebly closed (resp. feebly open) set in Y .

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Remark 2.4. [1]Every open set (resp. closed set) is feebly open (resp. feebly closed set) .

Definition 2.5.

(1) A subset A of X is said to be feebly regular open(briefly F.reg.open) if A = f.int(f.cl(A)).

(2) A subset A of X is said to be feebly regular closed if A = f.cl(f.int(A)) (briefly F.reg.closed).

(3) A subset A of X is said to be feebly regular clopen if A = f.int(f.cl(f.int(A))). On the other hand, if A is F.reg.open and F.reg.closed.

(4) Let A be subset of X. The feebly regular closure of A (briefly F.reg.cl(A)) is the intersection of all feebly regular closed set containing A and F.reg.int(A) is the union of all feebly regular open set contained in A.

3. Feebly θ-closed sets

In this section we have introduce feebly θ-closed sets and prove some theorems which satisfy the definition.

Definition 3.1. A subset A of X is said to be feebly θ-open if A $\subset$ sθcl(int(A)) and it is denoted by fθ-open set. The complement of fθ-open sets is called fθ-closed sets.

Remark 3.2.

θ-open → open set → pre-open → fθ-open → gsp-open

θg-open ⇔ feebly −open

The converse of the above implications is need not be true is shown in the below example

Example 3.3. If X = {a,b,c} and τ = {X,∅,{c},{b,c}} then fθ-open = {X,∅,{c},{a,c},{b,c}}.

(1) It is clear that the subset {a,c} is fθ-open set but not open set.

(2) The subset {b} is gsp-open but not fθ-open

Example 3.4. If X = {a,b,c} and τ = {X,∅,{b},{c},{b,c}} then fθ-open = {X,∅,{b},{c},{a,b}, {a,c},{b,c}}. The subset {a,b} is fθ-open set but not pre-open set

Remark 3.5. The union of any two fθ-open subset is fθ-open set.

Remark 3.6. The intersection of any two fθ-open subset is also a fθ-open set.

Example 3.7. From Example 3.4, Let the subsets {b} and {b,c} are fθ-open. Then the intersection of {b} and {b,c} is {b} which is also a fθ-open set.

Definition 3.8. Let (X,τ) be a topological space and let A ⊂ X. A point x ∈ X is said to be a fθ-interior point of A if there exist a fθ-open set G such that x ∈ G ⊂ A. The set of all fθ-interior points of A is called the fθ-interior of A and is denoted by fθ-int(A) . Evidently A contains all its interior points, that is, fθ-int(A) ⊂ A.

Definition 3.9. Let (X,τ) be a topological space and let A ⊂ X. A point x ∈ X is said to be a fθ-closure of A where intersection of all fθ-closed sets containing A and it is denoted by fθ-cl(A).

Definition 3.10. Let (X,τ) be a topological space and let A be a subset of X. A point x ∈ X is called a fθ-cluster point of A if [N −{x}]∩fθint(fθcl(A)) ≠ ∅ for every τ-neighbourhood N of x.

Remark 3.11. The point x is not a fθ-cluster point of A if there exists a neighbourhood N of x such that N ∩ fθint(fθcl(A)) = ∅ or N ∩ fθint(fθcl(A)) = {x}.

Theorem 3.12. fθ-int(A) = ∪{G : G is fθ-open, G ⊂ A}.

Proof x ∈ fθ-int(A) ⇔ A is a neighbourhood of x ⇔ there exists a fθ-open set G, such that x∈ G⊂A ⇔ x∈∪{G : G is fθ-open, G⊂A}. Hence fθ-int(A) = ∪{G : G is fθ-open G⊂A}.

Theorem 3.13. Let (X,τ) be a topological space and let A be a subset of X. Then

(1) fθ-int(A) is a fθ-open set

(2) fθ-int(A) is the largest fθ-open set contained in A

(3) A is fθ-open if and only if fθ-int(A) = A

Proof 1). Let x be any arbitrary point of fθ-int(A) . Then x is a fθ-interior point of A. Hence by definition, A is a neighbourhood of x. Then there exists a fθ-open set G such that x ∈ G ⊂ A. Since G is fθ-open, it is a neighbourhood of each of its points and so A is also a neighbourhood of each point of G. It follows that every point of G is a fθ-interior point of A so that G ⊂ fθ-int(A). Thus it is shown that to each x ∈ fθ-int(A), there

exists a fθ-open set G such that $x \in G \subset$ fθ-int(A). Hence fθ-int(A) is a neighbourhood of each of its points and consequently fθ-int(A) is fθ-open.
2). Let G be any subset of A and let $x \in G$ so that $x \in G \subset A$. Since G is fθ-open, A is a neighbourhood of x and consequently x is a fθ-interior point of A. Hence $x \in$ fθ-int(A). Thus we have shown that $x \in G \Rightarrow x \in$ fθ-int(A) and so $G \subset$ fθ-int(A) $\subset A$. Hence fθ-int(A) contains every fθ-open subset of A and it is therefore the largest fθ-open subset of A.

3). Let A = fθ-int(A) , by(i) fθ-int(A) is a fθ-open set and therefore A is also fθ-open. Conversely A be a fθ-open. Then A is surely identical with the largest fθ-open subset of A. But by (iii), fθ-int(A) is the largest fθ-open subset of A. Hence A = fθ-int(A).

Theorem 3.14. For any two subsets A and B of (X,τ)

(1) If $A \subset B$, then fθ-int(A) $\subset$ fθ-int(B).

(2) fθ-int($A \cap B$) = fθ-int(A) $\cap$ fθ-int(B).

(3) fθ-int(A) $\cup$ fθ-int(A) $\subset$ fθ-int($A \cup B$).

(4) fθ-int(X) = X

(5) fθ-int($\emptyset$) = $\emptyset$.

Proof (i) Let A and B be subsets of X such that $A \subset B$. Let $x \in$ fθ-int(A). Then there exists a fθ-open set U such that $x \in U \subset B$ and hence $x \in$ fθ-int(B). Hence, fθ-int(A) $\subset$ fθ-int(B).

(ii) We Know that $A\cap B \subset A$ and $A\cap B \subset B$. We have by (i) fθ-int($A\cap B$) $\subset$ fθ-int(A) and fθint($A\cap B$) $\subset$ fθ-int(B). This implies that fθ-int($A\cap B$) $\subset$ fθ-int(A)∩fθ-int(B)−−−−−−−(1). Again, let $x \in$ fθ-int(A)∩fθ-int(B). Then $x \in$ fθ-int(A) and $x \in$ fθ-int(B). Then there exists fθ-open sets U and V such that $x \in U \subset A$ and $x \in U \subset B$. $U \cap V$ is a fθ-open set such that $x \in (U \cap V) \subset (A\cap B)$. Hence $x \in$ fθ-int($A\cap B$). Thus $x \in$ fθ-int(A)∩fθ-int(B) implies that $x \in$ fθ-int($A \cap B$). Therefore, fθ-int(A) $\cap$ fθ-int(B) $\subset$ fθ-int($A \cap B$) −−−−− (2). From (1) and (2) , it follows that fθ-int(A∩B)∩fθ-int(A)∩fθ-int(B). The proofs of (iii), (iv) and (v) are obivious.

Lemma 3.15. Let A be a subset of X

(1) $(\text{f}\theta\text{-int}(A))^c$ = fθ-cl(A^c).

(2) $(\text{f}\theta\text{-cl}(A))^c$ = fθ-int(A^c).

Remark 3.16. (1) fθ-int(A) $\cup$ fθ-int(B) $\neq$ fθ-int($A \cup B$).

(2) fθ-int(fθ-int(A)) = fθ-int(A).

(3) fθ-int(A) $\subset A^c$.

Theorem 3.17. Let (X,τ) be a topological space and let $A \subset X$. Then

(1) fθ-int(A) = $(\text{f}\theta\text{-cl}(A^c))^c$
(2) fθ-cl(A^c) = $(\text{f}\theta\text{-int}(A))^c$

(3) fθ-cl(A) = (fθ-int(A^c))

Proof (i) Obvious.

(ii) Taking complements in (i), (fθ-int(A))^c = (fθ-cl(A^c))^{cc} = fθ-cl(A^c). Taking complements again, (fθ-int(A))^c = (fθ-cl(A^c))^c. That is, fθ-int(A) = (fθ-cl(A^c))^c. Since S^{cc} = S for any set S.
(iii) By (ii) fθ-cl(A^c) = (fθ-int(A))^c. Replacing A by A^c in this, we get fθ-cl(A^c)^c = (fθint(A^c))^c or fθ-cl(A^{cc}) = (fθ-int(A^c))^c. Hence fθ-cl(A) = (fθ-int(A^c))^c.

4. *fθ*-exterior point and *fθ*-frontier

In this section we introduce and investigate the properties of fθ-exterior point and fθ-frontier of the set and prove some of its results satisfying the definition.

Definition 4.1. Let A be a subset of A topological space X. A point x ∈ X is said to be fθ-exterior point of A if there exists a fθ-open set G such that x ∈ G ⊂ A^c where A^c is the complement of A. The set of all fθ-exterior points of A is denoted by fθ-ext(A).

Example 4.2. If X = {a,b,c} and τ = {X,∅,{a},{c},{a,c}} then fθ-open = {X,∅,{a},{c},{a,b},
{a,c},{b,c}}. Let G = {a,c}, x = {c} and A = {b} then the fθ-ext(A) = {a,c}

Remark 4.3. Let A be a subset of A topological space X.

(1) A point x ∈ X is a fθ-interior point of the complement A^c of A.

(2) A point x belongs to fθ-open set G and if G ∩ A = ∅ then it is fθ-exterior point of A.

(3) fθ-ext(A) = fθ-int(A^c).

(4) fθ-ext(A^c) = fθ-int(A^{cc}) = fθ-int(A).

(5) A ∩ fθ-ext(A) = ∅.

Remark 4.4. Since fθ-ext(A) is the fθ-int(A^c), it follows from remark 4.3 that fθ-ext(A) is fθ-open and is the largest fθ-open set contained in A^c.

Theorem 4.5. Let (X,τ) be a topological space and A ⊂ X. Then fθ-ext(A) = ∪{G ∈ τ : G ⊂A^c}.

Proof By definition, fθ-ext(A) = fθ-int(A^c). But by remark 4.3, fθ-int(A^c) = ∪{G ∈ τ : G ⊂ A^c}. Hence fθ-ext(A) = ∪{G ∈ τ : G ⊂ A^c}.

Theorem 4.6. Let A be a subset of A topological space X. Then a point x ∈ X is a fθ-exterior point of A if and only if x is not a fθ-adherent point of A, that is, if and only if x ∈ A^c.

Proof Let x be a fθ-exterior point of A. Then x is a fθ-interior point of A^c so that A^c is a neighbourhood of x containing no points of A. It follows that x is not a fθ-adherent point of A, that is, x ∈ fθ-cl(A^c).
Conversely, suppose that x is not a fθ-adherent point of A. Then there exists a neighbourhood N of x which contains no points of A. This implies that x ∈ N ⊂ A^c. It follows that A^c is a neighbourhood of x and consequently x is a fθ-interior point of A^c. That is, x is a fθ-exterior point of A.

Corollary 4.7. It follows from the above theorem that fθ-ext(A) = (fθ-cl(A))^c. From this, we conclude that fθ-int(A) = fθ-ext(A^c) = (fθ-cl(A^c))^c.

Definition 4.8. A point x of A topological space X is said to be a fθ-frontier point of a subset A of X if it is neither a fθ-interior nor a fθ-exterior point of A. The set of all fθ-frontier points of A is called the fθ-frontier of A and is denoted by fθ-Fr(A). Simply fθ-Fr(A) = fθ-cl(A) − fθ-int(A)

Theorem 4.9. Let X be a topological space and A ⊂ X. Then a point x in X is a fθ-frontier point of A if and only if every neighbourhood of x intersects both A and A^c.

Proof We have x ∈ fθ-Fr(A) ⇔ x 6∈ fθ-int(A) and x ∈6 fθ-ext(A) = fθ-int(A) ⇔ neither A nor A^c is a neighbourhood of x ⇔ no neighbourhood of x can be contained in A or in A^c ⇔ every neighbourhood of x intersects both A and A^c.

Corollary 4.10. fθ-Fr(A) = fθ-Fr(A^c).

Proof x ∈ fθ-Fr(A) ⇔ every neighbourhood of x intersects both A and A^c ⇔ every neighbourhood of x intersects both $(A^c)^c$ and A^c. Since $(A^c)^c = A^c$ ⇔ x ∈ fθ-Fr(A^c).

Theorem 4.11. Let (X,τ) be a topological space and let A,B be subsets of X. Then

(1) fθ-ext(X) = ∅, fθ-ext(∅) = X

(2) fθ-ext(A) ⊂ A^c

(3) fθ-ext(A) = fθ-ext((fθ-ext(A))^c)

(4) A ⊂ Bτfθ-ext(B) ⊂ fθ-ext(A)

(5) fθ-int(A) ⊂ fθ-ext(fθ-ext(A))

(6) fθ-ext(A ∪ B) = fθ-ext(A) ∩ fθ-ext(B)

Proof (i) fθ-ext(X) = fθ-int(X^c) = fθ-int(∅) = ∅fθ-ext(∅) = fθ-int($∅^c$) = fθ-int(X) = X.

(ii) fθ-ext(A) = fθ-ext(A^c) ⊂ A^c, by (iii) of remark 3.18.

(iii)fθ-ext(fθ-ext(A))^c) = fθ-ext((fθ-int(A^c))^c) = fθ-ext(fθ-int(A^c)^c) = fθ-int(((fθ-int(A^c)^c)^c) = fθ-int(fθ-int(A^c)^{cc}) = fθ-int(fθ-int(A^c)) = fθ-int(A^c) = fθ-ext(A). since A^{cc} = A for any set A.

(iv) A ⊂ B ⇒ $B^c \subset A^c$ ⇒ fθ-int(B^c) ⊂ (fθ-int(A^c)) ⇒ fθ-ext(B) ⊂ fθ-ext(A).

(v) By (ii), we have fθ-ext(A) ⊂ A^c. Then (iv) gives fθ-ext(A^c) ⊂ fθ-ext(fθ-ext(A)). But fθ-int(A) = fθ-ext(A^c). Hence fθ-int(A) ⊂ fθ-ext(fθ-ext(A)).

(vi) fθ-ext(A ∪ B) = fθ-int(A ∪ B)^c = fθ-int($A^c \cap B^c$) = fθ-ext(A) ∩ fθ-ext(B).

Theorem 4.12. Let A be any subset of A topological space X. Then fθ-int(A), fθ-ext(A) and fθ-Fr(A) are disjoint and X = fθ-(A) ∪ fθ-ext(A) ∪ fθ-Fr(A). Further fθ-Fr(A) is a fθ-closed set.

Proof By definition, fθ-ext(A) = fθ-int(A^c). Also fθ-int(A) ⊂ A and fθ-int(A^c) = A^c. Since $A \cap A^c = \emptyset$, it follows that fθ-int(A)∩fθ-ext(A) = fθ-int(A)∩fθ-int(A^c) = ∅. Again by definition of fθ-frontier, we have x ∈ fθ-Fr(A) ⇔ x 6∈ fθ-int(A) and x ∈6 fθ-ext(A) ⇔ x 6∈ fθint(A) ∪ fθ-ext(A) ⇔ x ∈ (fθ-int(A) ∪ fθ-ext(A))^c. Thus fθ-Fr(A) ⇔ (fθ-int(A) ∪ fθext(A))^c −→ (1). It follows that fθ-Fr(A) ∩ fθ-int(A) = ∅ and fθ-Fr(A) ∩ fθ-ext(A) = ∅ and X = fθ-int(A) ∪ fθ-ext(A) ∪ fθ-Fr(A). Since fθ-int(A) and fθ-ext(A) are open sets, we see from (1) that fθ-Fr(A) is a fθ-closed set.

Theorem 4.13. Let (X,τ) be a topological space and let B ⊂ X. Then fθ-cl(A) = fθ-int(A)∪ fθ-Fr(A).

Proof By definition, of fθ-cl(A), we have fθ-cl(A) = ∩{F : F is fθ-closed and F ⊃ A}. Then by De-Morgan law. (fθ-cl(A))^c = ∪{F^c : F^c is fθ-open and $F^c \subset A^c$} = fθ-ext(A). Taking complements, we get (fθ-cl(A))^{cc} = (fθ-ext(A))^c = fθ-int(A) ∪ fθ-Fr(A). Hence fθcl(A) = fθ-int(A) ∪ fθ-Fr(A).

Corollary 4.14. fθ-Fr(fθ-cl(A)) ⊂ A.

Corollary 4.15. fθ-cl(A) = A ∪ fθ-Fr(A).

Proof Since A ⊂ fθ-cl(A) and fθ-Fr(A) ⊂ fθ-cl(A), we have A ∪ fθ-Fr(A) ⊂ fθ-cl(A) −→ (1). Also fθ-Fr(A) = (fθ-int(A) ∪ fθ-ext(A))^c = (fθ-int(A))^c ∩ (fθ-ext(A))^c. Again since fθ-int(A) ⊂ A and fθ-cl(A) = fθ-int(A) ∪ fθ-Fr(A), it follows that fθ-cl(A) ⊂ A ∪ fθFr(A) −→ (2). From (1) and (2), we get fθ-cl(A) = A ∪ fθ-Fr(A).

Theorem 4.16. Every fθ-closed subset of A topological space is the disjoint union of its fθinterior and fθ-frontier.

Proof Let A be a fθ-closed subset of A topological space X, so that fθ-cl(A) = A. A = fθint(A) ∪ fθ-Fr(A). Also we get fθ-int(A) ∩ fθ-Fr(A) = ∅.

Theorem 4.17. Let (X,τ) be a topological space and let A,B be subset of X. Then

(1) fθ-Fr(A) = fθ-cl(A) ∩ A^c −fθ-int(A).

(2) fθ-int(A) = A −fθ-Fr(A).

(3) (fθ-Fr(A))^c = fθ-int(A) ∪ fθ-int(A^c).

(4) fθ-Fr(fθ-int(A)) ⊂ fθ-Fr(A).

(5) fθ-Fr(fθ-cl(A)) ⊂ fθ-Fr(A).

(6) fθ-Fr(A ∪ B) ⊂ fθ-Fr(A) ∪ fθ-Fr(B).

(7) fθ-Fr(A ∩ B) ⊂ fθ-Fr(A) ∪ fθ-Fr(B).

Proof (i) We have fθ-Fr(A)=(fθ-int(A) ∪ fθ-ext(A))^c

= (fθ-int(A))^c ∩ (fθ-ext(A))^c by De-Morgan law

= (fθ-cl(A^{c}1))^{cc} ∩ (fθ-cl(A))^{cc}

= (fθ-cl(A^c))^{cc} ∩ (fθ-cl(A)), by 4.6. Now fθ-cl(A) ∩ fθ-cl(A^c)

= fθ-cl(A)−(fθ-cl(A^c))^c = fθ-cl(A)−fθ-int(A), by 4.6. Hence fθ-Fr(A) = fθ-cl(A)∩A = fθcl(A) − fθ-int(A).
(ii) sA −fθ-Fr(A) = A − (fθ-cl(A) − fθ-int(A)), by (i)

= fθ-int(A) since fθ-int(A) ⊂ A

(iii) We have (fθ-Fr(A))^c = (fθ-cl(A) − fθ-int(A^c)) by (i)

= fθ-cl(A^c) ∪ (fθ-cl(A^c)^c using De-Morgan law and by corollary 4.6, (fθ-cl(A^c))^c = fθ-int(A) and so fθ-int(A^c) = (fθ-cl(A^c))^c = (fθ-cl(A^{cc}))^c = (fθ-cl(A))^c since A^{cc} = A. Therefore (fθ-Fr(A))^c = fθ-int(A^c) ∪ fθ-int(A) = fθ-int(A) ∪ fθ-int(A^c).
(iv) fθ-Fr((fθ-int(A)) = fθ-cl(fθ-int(A)) ∩ fθ-cl(fθ-int(A^c)), by (i)

= fθ-cl(fθ-int(A))∩fθ-cl(fθ-cl(A^c))^c = fθ-cl(fθ-int(A))∩fθ-cl(A^c) ⊂ fθ-cl(A)∩fθ-cl(A^c) = fθ-Fr(A) by (i). Thus fθ-Fr(fθ-int(A)) ⊂ fθ-Fr(A).

(v) fθ-Fr(fθ-cl(A)) = fθ-cl(A) ∩ fθ-cl(fθ-cl(A^c)), by (i)

= fθ-cl(fθ-cl(A)) ∩ fθ-cl(fθ-cl(A^c)). Now A ⊂ fθ-cl(A) ⇒ fθ-cl(fθ-cl(A^c)) ⊂ fθ-cl(A^c).

Hence fθ-Fr(A) ⊂ fθ-cl(A) ∩ fθ-cl(A^c) = fθ-Fr(A).

(vi) fθ-Fr(A ∩ B) = fθ-cl(A ∪ B) ∩ fθ-cl(A ∪ B)^c, by (i)

= (fθ-cl(A) ∪ fθ-cl(B)) ∩ (A^c ∩ BA^c), by using De-Morgan law

= (fθ-int(A))^c∩(fθ-ext(A))^c. Again since fθ-int(A) ⊂ A and fθ-cl(A) = fθ-int(A)∪fθ-Fr(A), it follows that fθ-cl(A) ⊂ A ∪ fθ-Fr(A) ⇒ (2). From (1) and (2), we get fθ-cl(A) = A ∪ fθFr(A).

5. Conclusion

In this paper, a new form of feebly θ- closed sets is introduced and the concepts of f-open, fθ- open, fθ-interior, fθ- closure, fθ- exterior and fθ- frontier are studied and various properties of feebly θ-closed sets are investigated.

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