Comparison and Analysis of Sub-optimal performance of OFDM/SDMA uplink System that use Conventional Multiuser Detection Techniques

Shahnaz K V[a], Papiya Dutta[b], Anirban Kanungoe[c], Arun Kumar[d]

[a,b,c,d] Department of Electronics and Communications Engineering, Bharat Institute of Engineering and Technology
[a]drshahnaz@biet.ac.in, [b]papiyadutta@biet.ac.in, [c]anirban@biet.ac.in, [d]drarunkumar@biet.ac.in

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Abstract: The major challenge in the practical accomplishment of OFDM/SDMA system depends on the efficient implementation of a multiuser detection (MUD) technique that separates the spatially multiplexed signal streams. Several algorithms for MIMO detection for spatially-multiplexed signals have been developed till today. The MIMO MUDs that are mostly used in previous research works and practically used are minimum mean squared error (MMSE), decorrelator (ZF), Vertical-Bell laboratories layered space-time (V-BLAST), successive inter- ference cancellation (SIC) and maximum likelihood detection (MLD). MMSE and ZF are linear MIMO detection techniques, while the remaining procedures are non linear techniques. All MUD techniques in general can be explained as a solution to a quadratic optimization problem in most cases. In this work, basic principles of ZF, MMSE, ZF-OSIC, MMSE-OSIC and ML have been explained briefly and a comparison of their performances have been done. Although the detection techniques evolved so far are matured in theory, their implementation in a real-time scenario is still challenging. It is analyzed in this paper that the poor BER results of sub-optimal detection method in comparison to MLD is due to bad channel effects.

Keywords: OFDM/SDMA, SIC, V-BLAST, ZF, MMSE, MUD, Bad Channel Effect

1. Introduction

A combination of techniques on orthogonal frequency division multiplexing (OFDM) using smart antenna designs have been developed in recent years [1]. The data model developed possesses parallelism and frequency selectivity that lead to high implementation efficiency. These techniques are used with the principal aim of reducing the multipath fading effects on the desired data signals, thus enhancing the capacity and the performance of wireless communication technologies. Design and development of efficient detection techniques for combined OFDM/SDMA uplink system in severe dispersive environment is highly challenging and is an active research area. Section II elaborates on some commonly used detection methods. Section III analyses the performance of these conventional detectors in a bad channel where sub section III-A discusses the detection scenarios faced by the uplink channel. Section IV gives the simulation parameters used and the performance comparison of the conventional methods of detection. The complexity of the detection methods are elaborated in the section IV-B. Section V concludes the paper.

2. Multiuser Detection Methods

Three major classifications of MUD techniques that can be used in OFDM/SDMA systems like any other multiplexing systems are:

a) Exhaustive search type

b) Linear equalization based techniques

c) Nulling and cancellation(Decision-feedback)

The second and third methods are sub-optimal, and computationally less complex than first type, which is mainly the ML technique..

A. MLD Techniques

MLD is a technique that relates the signal received, with all transmitted signals that are feasible and estimates $\boldsymbol{x}$ as the most probable transmitted sequence.

$$\widehat{\boldsymbol{x}} = \underset{x_i \epsilon \{x_1, x_2, \dots, x_I\}}{argmin} \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{x_i}\|^2 \qquad (1)$$

Consider L number of users, A number of receivers and channel matrix $\boldsymbol{H}$. The search is conducted on all possible $I = M^L$ transmitted signals $\boldsymbol{x_i}$ which are part of ensemble $\{\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_I}\}$ $M = 2^m$ complex constellation points (m as bits per symbol) with a given modulation scheme employed. For MLD, it is not necessary that $L \leq A$. A technique to obtain the most likely transmitted signal vector is by finding vector $\boldsymbol{x_i}$ from the group $\{\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_I}\}$ where the probability $Pr(\boldsymbol{x} = \boldsymbol{x_i}/\boldsymbol{y})$ or otherwise, $Pr(\boldsymbol{x_i}/\boldsymbol{y})$ is maximal and is known as maximum a posteriori probability (MAP). According to [7], this is the minimization of probability of error. By applying Bayes' rule, $Pr(\boldsymbol{x_i}/\boldsymbol{y})$ becomes

$$Pr(\boldsymbol{x}_i/\boldsymbol{y}) = \frac{\mathrm{p}(\mathbf{y}/\mathbf{x_i})\mathrm{Pr}\,(\mathbf{x_i})}{\mathrm{p}(\mathbf{y})} \quad (2)$$

Here, $\mathrm{p}(\mathbf{y}/\mathbf{x_i})$ is the conditional probability density function of the signal vector seen at the receiver provided $\boldsymbol{x_i}$ is transmitted and $\mathrm{Pr}\,(\mathbf{x_i})$ is the probability of i^{th} vector is being sent. When no earlier information is present on the probability that a particular signal is transmitted, it is assumed that *I* signal vectors have the same probability of being sent and $\mathrm{Pr}(\boldsymbol{x_i}) = 1/I$. Assuming this, MUD method is no longer MAP method. Instead it is generally called MLD [7], [9]. The fact that the denominator in (2) is independent of x_i the decision rule based on obtaining the vector that maximizes $Pr(\boldsymbol{x_i}/\boldsymbol{y})$ is equivalent to selecting the x_i that maximizesp$(\mathbf{y}/\mathbf{x_i})$. It can be concluded from [9] that $\mathrm{p}(\mathbf{y}/\mathbf{x_i})$ is a multivariate complex normal distribution. By assuming $\boldsymbol{x_i}$ is transmitted for a specific$\boldsymbol{H}$**,** the mean of $\mathbf{y}$ equals $\boldsymbol{Hx_i}$. This leads to the probability density function (pdf).

$$p(\boldsymbol{y}|\boldsymbol{H}, \boldsymbol{x_i}) = |(\prod \boldsymbol{Q})^{-1}| exp\left(-(\boldsymbol{y} - \boldsymbol{Hx_i})^H \boldsymbol{Q}^{-1}(\boldsymbol{y} - \boldsymbol{Hx_i})\right) \quad (3)$$

Here $\mathbf{Q}$ is the covariance matrix and

$$\boldsymbol{Q} = E[(\boldsymbol{y} - \mu)(\boldsymbol{y} - \mu)^H] = E[(\boldsymbol{y} - \boldsymbol{Hx_i})(\boldsymbol{y} - \boldsymbol{Hx_i})^H] \quad (4)$$

$$Q = E[\boldsymbol{nn}^H] = \sigma_n^2 I_A \quad (5)$$

The resulting conditional pdf is

$$p(\boldsymbol{y}|\boldsymbol{Hx_i}) = \frac{1}{(\prod \sigma_n^2)^A} exp\left(-\frac{1}{\sigma_n^2}(\boldsymbol{y} - \boldsymbol{Hx_i})^H(\boldsymbol{y} - \boldsymbol{Hx_i})\right) \quad (6)$$

Thus finding the maximum of the conditional probability $Pr(\boldsymbol{x_i}/\boldsymbol{y})$ leads to

$$\underset{x_i \epsilon \{\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_I}\}}{argmax} p(\boldsymbol{y}|\boldsymbol{Hx_i}) = \underset{x_i \epsilon \{\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_I}\}}{argmin} \|\boldsymbol{y} - \boldsymbol{Hx_i}\|^2 = \widehat{\boldsymbol{x}} \quad (7)$$

which is same as (1). Since maximum of conditional probability leads to minimization of error probability, MLD can beconsidered as optimal in terms of BER performance.

B. Zero Forcing (ZF) Detection

The estimate of the signal vector transmitted is normally calculated using a particular MUD selected. The data vectors detected then undergo component-wise quantization. Component-wise quantization is sub-optimal in general be- cause the multiplication by equalization matrix **W** introduces the noise component correlation. While detecting the data in a given sub-stream of an OFDM/SDMA signal, a ZF detector considers it as the signal required. The remaining data streams are considered as "interferers". The interferers are nullified by linearly weighing the signals that are received so all the terms that are interfering are cancelled such that $\boldsymbol{w^i h_p} = 0$ and $p \neq i$ and 1 for $p = i$, in which $\boldsymbol{h_p}$represents the p^{th} column vector of $\mathbf{H}$ and $\boldsymbol{w^i}$ be the i^{th} row of a matrix $\mathbf{W}$. Let

$$\boldsymbol{WH} = \boldsymbol{I} \quad (8)$$

Where, the matrix **W** denotes linear processing at the receiver and **I** represents the identity matrix. Each desired element of **x** can be estimated by deliberately turning interferers to zero. If **H** is not a square matrix, W turns out to be the pseudo-inverse of **H:**

$$\boldsymbol{W} = \boldsymbol{H}^{\dagger} = (\boldsymbol{H^H H})^{-\mathbf{1}} \boldsymbol{H^H} \quad (9)$$

† represents the pseudo-inverse. The estimates of $\mathbf{x}$, $\widehat{\boldsymbol{x}}_{\boldsymbol{ZF}}$is obtained from the above equation as

$$\widehat{\boldsymbol{x}}_{\boldsymbol{ZF}} = \boldsymbol{W_{ZF}x} = \boldsymbol{H}^{\dagger}\boldsymbol{y} \quad (10)$$

where the received vector is denoted as **y**. The decorrelator performs much better in higher SNR regimes as it completely eliminates the interference. On the other side, matched filtering technique tries to increase the desired signal output SNR to the maximum level. The linear detector based on MMSE combines matched filtering and decorrelating techniques in anoptimal manner and it is explained in next section.

C. MMSE Detection Technique

MMSE detection addresses the issue of estimating the vector $\boldsymbol{x}$ based on the observations of $\boldsymbol{y}$ by choosing a function $f(\boldsymbol{y})$ that reduces the mean square error to a minimum. In the case of MMSE-MUD,

$$\widehat{\boldsymbol{x}}_{MMSE} = \boldsymbol{W}_{MMSE}^{H} \times \boldsymbol{y} \qquad (11)$$

where $\boldsymbol{y}$ is the signal vector received. $\boldsymbol{W}_{MMSE}$ is the weight of the matrix given by

$$\boldsymbol{W}_{MMSE} = (\boldsymbol{H}\boldsymbol{H}^{H} + \sigma^{2}\boldsymbol{I})^{-1}\boldsymbol{H}^{H} \qquad (12)$$

Here $\sigma^2$is the noise variance and $\sigma^2 = 1/SNR$, if we consider unit transmit power. The difference from ZF is the added term $\sigma^2\boldsymbol{I}$ and it offers a trade off between noise enhancement and residual interference. When SNR goes on increasing, the MMSE technique converges to the ZF technique. That is $(\boldsymbol{H}\boldsymbol{H}^{H} + \sigma^{2}\boldsymbol{I})^{-1}\boldsymbol{H}^{H} \approx (\boldsymbol{H}^{H}\boldsymbol{H})^{-1}\boldsymbol{H}^{H}$. When SNR is low, MMSE becomes matched filter [10].

D. Successive Interference Cancellation (SIC)

In this case, a stream is decoded and subtracted from the signal vector received by the linear detector. The following stream is then detected and this detection process carries on until all the data streams are detected [2]. The detected vector contains the symbols from each *L* user and as they are using the same modulation technique, the user which generates the least SNR value will become the error performer of the system. The explanation of this technique is given in [3]. The nulling matrix **G**, is initialized with equations of either ZF or MMSE technique by the assumption of perfect channel estimation. The biggest post-detection SNR is calculated for the ordering scheme. This corresponds to selecting the minimum norm row in **G** at every iteration. Initially, the layered signal **G** suppresses signals from all other user antennas. The signal vector obtained after i^{th} layer interference has been cancelled is given as

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_{\boldsymbol{i}} - \widehat{\boldsymbol{x}}_{i}(\boldsymbol{H})_{i} \qquad (13)$$

Where $\widehat{\boldsymbol{x}}_{\boldsymbol{i}}$ represents the decoded symbol on the i^{th} step. $(\boldsymbol{H})_i$ the i^{th} column of channel matrix. The updating of **G** is done by cancelling out the previous pseudo-inverse of **H.** The repetition of this method is done till symbols from each and every transmitting antennas are decoded in the similar manner. It is not necessary for the non-ordered scheme to find out the largest post-detection SNR but it selects the row vector of nulling matrix in a random manner.

E-BLAST Technique

The manner by which the detection of the streams take place in the SIC influences the performance since it might lead to error-propagation. The V-BLAST detection technique is similar to that of SIC excluding the fact that at each stage it follows an order for detection of the streams. As a result, it is known as ordered SIC (OSIC). It has been established in [3] that by selecting the best ρ_i at each step in the process of detection it leads to an ordering that is globally optimal. This algorithm is an efficient algorithm on MUD and it provides superior BER performance than ZF and MMSE with some additional computational complexity. The symbols of signals are detected 'vertically' from the same signal vector **y.** The strongest transmitted symbol is detected either usingZF or MMSE as in the case of SIC. The algorithm can be summarized as follows.

1. Initialization: Set i = 1. Find out the first conditioning matrix $\boldsymbol{H}^{\dagger}$ from estimated $\boldsymbol{H}$.

2. Ordering:- i^{th} conditioning matrix $\boldsymbol{W}_{\boldsymbol{i}} = \boldsymbol{H}^{\dagger}$ is set. The smallest norm value over all columns of $\boldsymbol{W}_i$, $m_i = \underset{i}{argmin}\left\|(\boldsymbol{W}_{\boldsymbol{i}})_j\right\|^2$, where m_i denotes the index of the column with the minimum norm. Select this column as $\boldsymbol{g}_{m_i} = (\boldsymbol{W}_i)_{m_i}$. Here the ordering is done for the undetected symbols in decreasing order of the expected ρ_i.

3. Nulling and slicing:- $\widehat{\boldsymbol{x}}_{m_i} = quantize(\boldsymbol{g}_{m_i}\boldsymbol{y}_i)$. Null the interference on symbol m_i from the other $L - i$ undetected symbols and slice$\boldsymbol{g}_{m_i}\boldsymbol{y}_i$ to detect $\widehat{\boldsymbol{x}}_{m_i}$.

4. Interference cancellation: Compute$\boldsymbol{y}_{\boldsymbol{i+1}} = \boldsymbol{y}_{\boldsymbol{i}} - \boldsymbol{h}_{m_i}\widehat{\boldsymbol{x}}_{m_i}$. Remoce the ${m_i}^{th}$ column $\boldsymbol{h}_{m_i}$ from the channel $\boldsymbol{H}$so that the influences due to one transmitting antenna can be removed. Thus the interference due to the detected symbol is predicted and then deducts this interference from all of the transmitted signals.

5. $i = i + 1$. Return to step 2 till all symbols are detected, i.e., $i > L$.

In ZF-VBLAST (also known as ZF-OSIC), ZF scheme is utilized for detection. In MMSE-VBLAST (MMSE-OSIC), MMSE technique is utilized for detection of the streams and the ordering has been done based on the maximum post detection SNR.

Table I Combined Ofdm/Sdma Simulation Parameters

Parameters	**Specifications**
FEC Coding	Half-rate convolutional encoder Generator polynomial
FFT Size	K=64
Data Subcarriers/Pilots	52/4
Cyclic Prefix Length	N=16
Subcarrier Modulation	BPSK
Channel Model	Multipath Rayleigh fading
Number of Multipaths	10
Channel State Information	Perfect
Number of BS Antennas	A=2 and 4
Number of Simultaneous Users	L=4
Channel Bandwidth	20 MHz

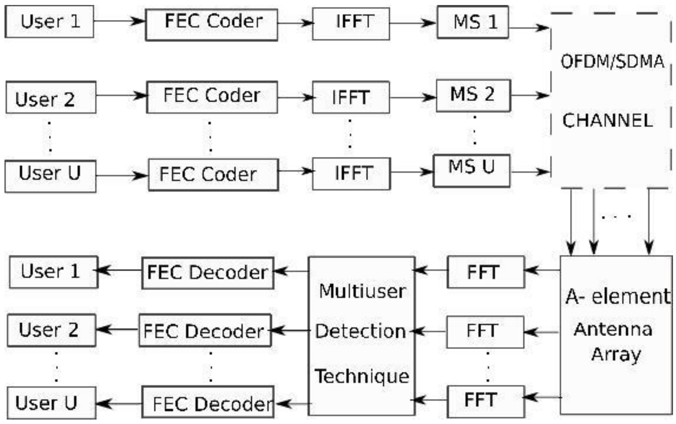

Fig. 1 Combined OFDM/SDMA Uplink System block diagram

3. Detector Performance In Bad Channel

The inferior BER performance of sub-optimal detection method when compared to MLD is due to bad channel effects. Singular Value Decomposition or SVD of $\boldsymbol{H} = \boldsymbol{U} \sum \boldsymbol{V^H}$. Here $\sum$ contains the singular values (σ) of $\boldsymbol{H}$, respectively. The ratio of the largest to the smallest singular value is represented by the condition number $c_H = \sigma_1/\sigma_U \geq 1$. Condition number c_H is very high for a channel that is poorly conditioned.

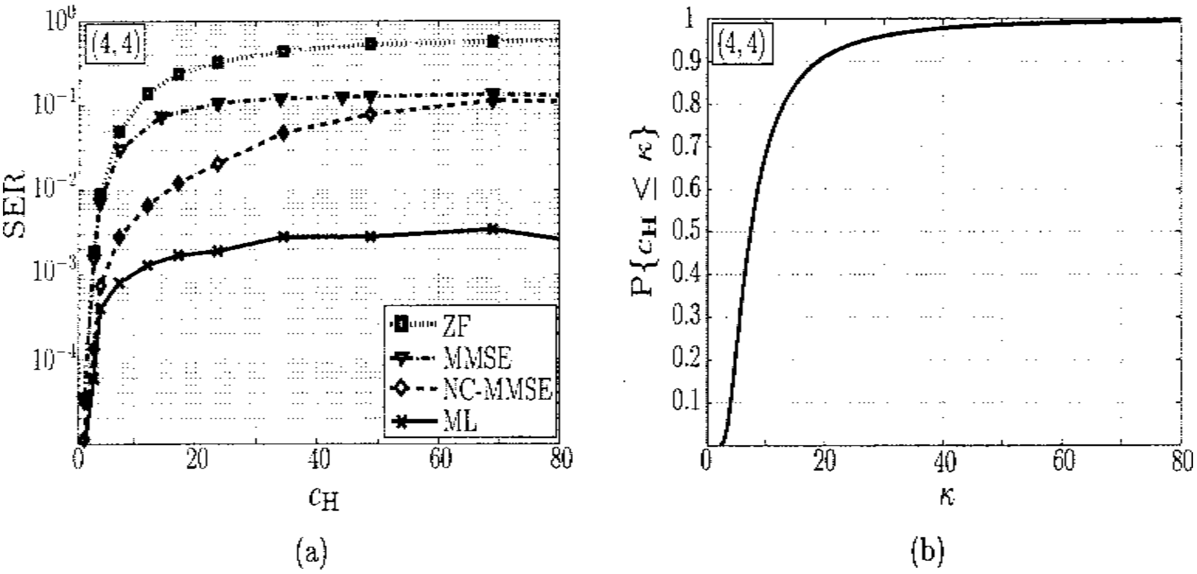

Fig. 2 Performance of the detector and the channel c_H of a 4×4 independent and identically distributed Gaussian channel (a) Performance of SER of different schemes of detection at an SNR of 15 dB (NC-MMSE is similar to SIC) (b) cdf of c_H [6]

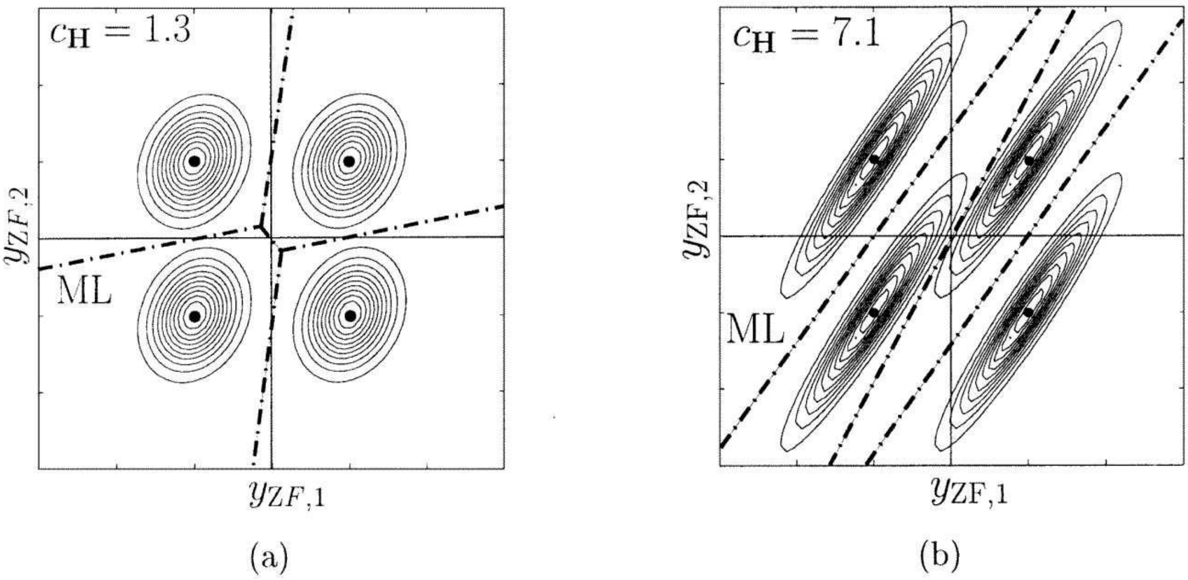

Fig. 3. Diagram of pdf of Zero Force (ZF) and Maximum Likelihood (ML) DR in the ZF-equalized domain using BPSK modulation technique. (a) Realization of a good channel with $c_H = 1.3$ (b) Realization of a bad channel with$c_H = 7.1$. The dash dotted lines indicate the ML DR. [6]

In paper [6], authors conduct some experimental study where SER of detection techniques including ZF, MMSE and ML versus c_H is given (Figure 2a). They have utilized a channel with 4 users and 4 receivers with iid Gaussian channel coefficients and QPSK modulated signal at an SNR of 15dB. The performances of all detectors are same at very lowc_H. The sub-optimal detectors are more affected than MLD. The impact of this behaviour depends on the probability with which the poor channel condition occurs. In Figure 2b, cumulative distribution function (cdf) of c_H estimated is given. The probability that the c_H exceeds a value of 10, 15, and 20 is 32%, 15% and 9% respectively. So it can be concluded from this result that ill-conditioned channel can happen frequently and thus causes notable failure of sub-optimal detection techniques.

Theoretical studies conducted in [8] prove that for a growing number of L users and A receivers, cdf is expanded in accordance with c_H. Thus the effect of bad channel is proportional to the number of antennas. Fig. 3 shows the probability density function of the signal vector received after ZF technique $\boldsymbol{y_{ZF}}$ for a channel with 2 users and 2 receivers, with c_H 1.3 and 7.1. Signal is BPSK modulated. The decision regions (DR) of ZF and ML are shown in the figure. They are similar for a channel with good condition ($c_H = 1.3$) but different for a channel with bad condition ($c_H = 1.3$). Since it is a 2×2 system, there are four DRs. The DR of ML have been compared with the distorted noise pdf, but for ZF they correspond to component wise quantization. The boundary lines denoting the ML-DR are differing mainly by offsets which are orthogonal to the dominant principal axis. This is easily understandable because any shift in the received signal vector in the direction of dominant noise component is caused by noise. It is thus desirable for bad channels that the DR be approximately invariant to shifts in the direction of dominant principal axis. The DR of linear sub-optimal detection techniques do not hold this property since their boundary lines always go through the origin [6].

A. Different Detection Scenarios

In any MIMO system, we confront with three detection scenarios

Under-loaded scenario: In this case number of users, *L* is less than the number of receivers, *A*. The channel matrix **H**is over determined.

Fully-loaded scenario: *L* is equal to *A* and **H** matrix is fully determined.

Over-loaded scenario: *L* exceeds *A* and **H** matrix is under determined. In this case the number of users that are moving inside the coverage area of the BS is out of our control. This is more practical case. A sufficient condition to ensure the existence of pseudo inverse in LS detector is that, the *L* number of columns of the **H** matrix are linearly independent. This implies that we have the rank (H) = *L*. The condition that is necessary for this to happen is that the *A* number of rows of **H** is equal to or greater than its *L* number of columns, i.e. $A \geq L$. This means, the maximum number of users or antennas that are transmitting at the same time and getting support from the LS combiner must be lesser in number or equal to the *A* number of receiving antennas which are the first two cases.

4. Results Of Simulation With Discussions

We have analysed the performance of sub-optimal detectors in bad channel condition for a full-load and full-rank MIMO system. In this section, we conduct simulation studies for the performance of these sub-optimal detectors in both full- load and over-load OFDM/SDMA system and compare their average BER performance and computational complexity.

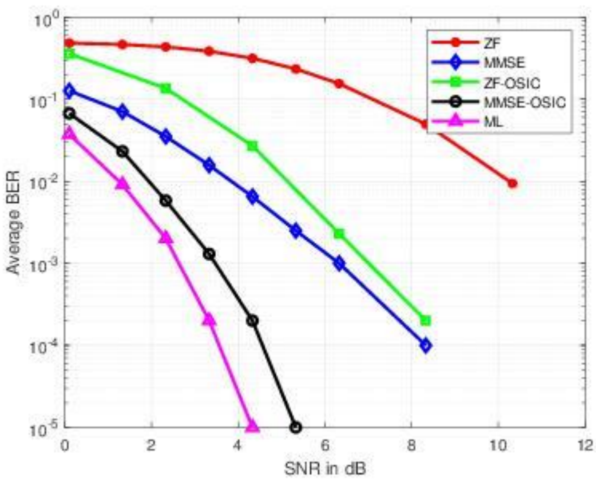

Fig. 4 Average BER performance of OFDM/SDMA when L=4 and A=4

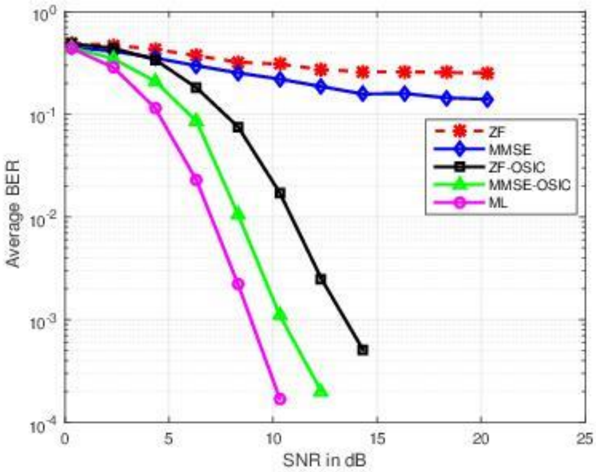

Fig. 5 BER vs SNR performance of OFDM/SDMA when L=4 and A=2

A. Average BER vs SNR Performance Comparisons

The performance of the classical detection methods discussed are evaluated by comparing their average BER for given SNRs. The procedure of simulation for multiuser OFDM/SDMA uplink system can be briefed as in block diagram shown in Fig. 1. The Simulation of the proposed system is done by utilizing MATLAB and refers to IEEE 802.11n along with MU-MIMO supported by the high speed WAN standards of IEEE 802.11ac. The simulation parameters are briefed in TABLE I. It is assumed that there is ideal synchronization between the receiver and the transmitter. Each user transmits using OFDM with 64 subcarriers modulated with BPSK and a cyclic prefix length of 16. The BS antenna array configuration is considered linear with a half-wavelength displacement between the array elements. We have assumed that the receiver perfectly knows the channel state information. In the first case, four users transmitting to a BS equipped with four receiving antennas are considered. This is a full- load scenario. From Fig.4 (and TABLE II), SNR required for achieving average BER 10^{-3} is $3dB$ for ML which is the best performance and $12dB$ for ZF, which is comparatively bad performance but at lesser complexity (as explained in next section). For MMSE it is $6.3dB$ and it is a good performance when compared to its complexity. For MMSE-OSIC, SNR required for the same result is $3.5dB$ which is close to ML but a complexity greater than ZF and MMSE. In the second scenario, four users are transmitting to a BS equipped with two receiving antennas which is an over-loaded scenario for this case. The Fig. 5 gives the performance comparison. Average BER 10^{-3} is achieved at a lesser SNR ($9dB$) compared to the full-load case. From the figure it is clear that ZF and MMSE give the worst performance in this scenario.

Table II Performance Comparison Of Detectors When Average Ber Is 10-3

MUD	**SNR(dB) L=4, A=4**	**SNR(dB) L=4, A=2**
ZF	12	-
MMSE	6.3	-
ZF-OSIC	7	13.4
MMSE-OSIC	3.5	10.4
ML	3	9

Table III complexity comparison

MUD	**COMPLEX OPERATIONS**
ZF	$K\left((14/3)L^3 + 5L^2 - (8/3)L\right)$ [4]
MMSE	$K\left((14/3)L^3 + 5L^2 - \left(\frac{2}{3}\right)L + 1\right)$ [4]
MMSE-OSIC	$K\left(\sum_{l=1}^{L}(14/3)i^3L + 5i^2L - \left(\frac{2}{3}\right)iL + 1\right)$ [5]
ML	$K(2^{mL}C_o)$

B. Complexity Comparisons

The computational complexity of the OFDM/SDMA MUD is a major issue that influence overall speed of the system. When $L = A,$ maximum complexity conditions appear. The computational complexity (C_o) of measuring $\|\boldsymbol{y} - \boldsymbol{Hx}\|^2$ can be determined as [4]

$$C_0 = 2L^2 + 2L - 1 \qquad (14)$$

Thus the computational complexity of ML detector is$2^{mU}C_0$. Significantly lower computational complexity is achieved by equalization-based detection methods like ZF and MMSE. Gauss-Jordan elimination procedure is used by MATLAB to obtain matrix inversion. This is why the ZF and MMSE complexity per OFDM subcarrier given in [4] uses the above procedure to evaluate the total metric evaluations. MMSE-OSIC complexity is dependent upon summation over iteration, *i* that varies from 1 to L and number of receivers [5]. Fig.6 demonstrates the complexity evaluated for four users and eight users based on TABLE III. A drastic increase in complex operations is observable when the numbers of users are doubled. This phenomenon is visible to the maximum extentin MMSE-OSIC and ML detection procedures.

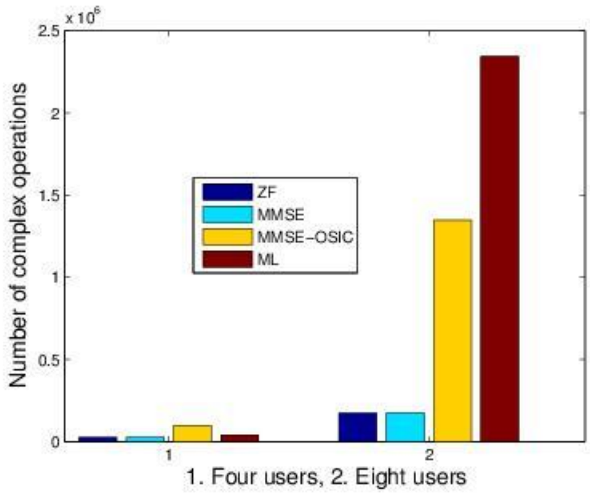

Fig. 6 Complexity comparison of different detectors

5. Conclusion

A detailed study on different conventional MUD detection methods applicable to MIMO systems and their performance in bad channels were conducted. Performed simulation and complexity comparison of these detection methods such as ML, ZF, MMSE, MMSE-OSIC and ML as applicable to combined OFDM/SDMA system. The performance assessment has been done in terms of average BER for two practical detection scenarios namely full-load and over-load. ML detection method demonstrates the optimum performance over methods like ZF, MMSE and MMSE-OSIC but the computational complexity rises exponentially with the increase in the number of users. It became obvious from the result that the performance of popular detectors such as ZF and MMSE fail completely when the channel is undetermined even if they are computationally efficient. MMSE-OSIC gives comparatively better performance but with a high computational complexity compared to ZF and MMSE but less than ML.

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