

Encompassing Learner-Centered Activities through Lesson Study Towards a Constructivist Classroom

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Abstract: This paper aims to raise conversations on designing appropriate learner-centered activities using the approach of Lesson Study to evaluate classroom practices. Through Lesson Study, the researchers carefully designed the lesson in triangle similarity, considering the principles of constructivism and student-centered learning, and executed the lesson in a class. Two emergent perspectives were apparent: (1) constructivism in mathematics classroom could go beyond discovery learning; and (2) anticipating student responses and capitalizing on possible student misconceptions is a powerful tool in designing constructivist lessons. The study suggests that being a constructivist teacher entails a long journey of classroom experiences and being able to reflect on one's practice, especially if this is in relation to the students' learning. Constructivist mathematics classroom activities requires presence of higher-order thinking skills, well-established prior knowledge, and social interaction.

Keywords: Constructivism, Geometry, Lesson Study, Professional Development, Triangle Similarities

1. Introduction

Recent studies claim that in order to understand students' mathematical learning, a constructivist framework may be considered. According to Twomey Fosnot [1], there are four principles that define constructivism. First, learning is developed from schema or from what we already know. Second, new ideas occur as we adapt and change our old ideas. Third, learning involves ideas being created a new rather than through accumulation of facts. Lastly, learning becomes meaningful as old ideas are revisited, new ideas are formulated, and new conclusions which might conflict with old ideas are devised. In practice, a constructivist classroom consists of a learner-centered, active instruction approach. Creating a learner-centered environment has proven to be an effective strategy in enhancing students' learning experiences [2].

In the Philippines, the mindset and discipline towards discovery-based learning (which is one of the aspects of constructivism) has a long journey to take to support discovery-based learning. A study of De Mesa and De Guzman [3] showed that Filipino teachers' pedagogical practices were still verging towards traditional inclination more than constructivism. This only went to show that there are changes that should be done by the teachers in order to contribute better to the teaching-learning process.

Discovery Learning through Learner-centered Activities in Constructivist Classroom

Vintere [4] stated that in a constructivist classroom, the goal of learning is focused on the development of critical thinking personality perceived as a maker of social changes. In planning the lesson, activities are further designed for the students to build their own knowledge and create their own learning.

Student-centered teaching is "the recommended approach to modern day pedagogy especially in the outcomes-based education where the teachers served as the facilitator of learning activities rather than performing the traditional lecture method." [5]. Garner [6] determined that student-centered instruction improved understanding of the mathematical concepts and increased students' ability to communicate their understanding on summative assessments. This type of instruction in mathematics classroom gives avenue for students to strongly contribute, actively explore, think critically using problems and communicate their thinking, asking them to explain the rationale of their answers. [7]

Lesson Study for Teacher Professional Development

Lesson Study is considered as the most crucial educational practice in Japan; and it is done in a very systematic manner [7]. It is a "collective, classroom-based style of professional learning" [8] in which teachers conduct their lessons with considerations on "discussions of subject matter, why they teach, how they teach, and what students can learn" [9]. In addition, it is a mechanism for developing deep pedagogical discourse on mathematics teaching [11] both for in-service and pre-service teachers [12].

Known to Dudley [13], Lesson Study puts emphasis on the process of lesson that typically involves a collaborative effort of a small group of teachers, where one teacher demonstrates the research lesson while other teachers observe the learning and engagement of selected case students. Moreover, the lesson will be updated and revised to further inform other classes or other teaching groups [14].

Regarding the focus of Lesson Study, Stigler and Hiebert [15] described it from their book entitled *The Teaching Gap* as centered to teaching and the students rather than the teacher itself and the students' works. Creating better lessons is a secondary byproduct of the process, but not its primary goal. In terms of the success of a lesson study, it is measured in the learning of students and not in the perfection of a lesson.

This paper aims to raise conversations on designing appropriate learner-centered activities using the approach of Lesson Study to evaluate classroom practices. Through Lesson Study, the researchers carefully design a lesson in geometry (specifically on triangle similarity), considering the principles of constructivism and student-centered learning, and execute the lesson in a class.

2. Methods

This research study employed a qualitative descriptive design through Lesson Study approach as it was the most appropriate research approach for this study because of the nature of the research questions and the intent of the researchers to gain an in-depth understanding of constructivism in a Mathematics classroom. The study was conducted in an all-boys private sectarian school. During the conduct of the lesson, 18 graduate students including the researchers and three teachers from the locale served as the panel observers.

Purposive sampling technique was used to choose a class with forty student-participants in the Grade 9 level since the lesson plan was designed for this age group and grade level.

Data was collected through observation and audio recordings from the group post-conference which were transcribed and analyzed through multiple thematic coding scheme. Supporting evidence were also obtained from documented pictures and answered worksheets.

The researchers followed certain steps in achieving a unified effort for a collaborative lesson study, patterned to the suggested procedure of Stigler and Hiebert [15].

Selection of a Focus or Definition of Problem

Geometry is one of the broadest areas of Mathematics and its inclusion in the mathematics curriculum has been extensively documented. French [16] identified three broad reasons for including Geometry in the mathematics curriculum today: (1) improve visual spatial awareness; (2) develop reasoning skills; and (3) enhance stimulation, challenge and information. To fulfill these aims and better develop the students' mathematical way of thinking, curriculum developers design appropriate teaching approaches and learning activities.

However, there are significant challenges in teaching geometry. In the study of Irsal, Jupri and Prabawanto [17] about the problem-solving skills and understanding of Indonesian Junior High School students on the topics of lines and angles, on average, the results revealed that student's relational skills are still weak. The said students have difficulty in relating one concept to another. One cause is the focus on conducting activities that still focuses on low-level thinking skills which are less challenging and more procedural. Students may understand the concepts themselves (i.e. definitions, postulates, theorems) but they are weak in relational skills, that is, applying concepts in connecting the relations among the components of figures.

The researchers took into consideration the students' lack of relational skills and construction of knowledge in geometry which needs to be addressed. Due to this, the researchers selected a specific geometry topic which centers on triangle similarity. This topic solely focuses on distinguishing congruent and similar triangles, discovering characteristics of similar triangles, and solving real-life problems involving similar triangles. With that being said, the researchers decided to incorporate learner-centered activities for this topic and focus on manifesting cooperation among students.

Planning of the Mathematics lesson or Study lesson

A month before the implementation of the teaching demonstration, the group met several times a week to talk about the further details of the lesson study. Moreover, the decision about the topic was based on consensus in consideration of the main goal of the study. Then, the researchers determined the instructional objectives of the lesson. Furthermore, theories and principles of constructivism and student-centered learning served as the cornerstone of the planned activities which will be given to the pupils.

Lastly, A dry run of the lesson was conducted wherein the assigned teacher executes the lesson to the co-researchers in anticipating possible scenarios and students' responses from the constructed series of questions for some revisions.

Public Teaching and Focused Observation of the Selected Lesson in an Actual Classroom

Tasks were distributed in a way where one member executed the lesson plan on the teaching demonstration, the eight remaining members focused on making the lesson plan, preparing instructional materials such as the

slide decks, triangle cut-outs, protractors, and three sets of students' worksheets. On deciding who will deliver the lesson, the group considered somebody who is teaching Grade 9 Mathematics whereas the selected teacher's experience can serve as a leverage to the group. It was November 29, 2019 (Friday), 9:55-10:55 when the classroom demonstration took place. It lasted for an hour and was videotaped for documentation of the study.

Evidence Based Debriefing and Critiquing while Reflectively Discussing Events Before, During and After the Classroom Observation with Regards to the Group's Goal

Following the teaching demonstration, the class convened in having a round table discussion letting members from each group to express their own opinions and points of improvement they can offer the group. It is in this step where everyone shares their own evaluation, insights and recommendations which centers on events happening before, during, and after the conduct of the research lesson. The researchers then consolidated all the gathered data into this research output, together with theories that supported each theme of the study and highlights to its [lesson study] focus which is incorporating learner-centered activities in teaching triangle similarities. Qualitative data obtained from field notes and audio recordings were transcribed and analyzed through multiple stages of thematic coding analysis in order to highlight the major themes supporting the objectives of the study.

The researchers made sure that the participants and the data obtained are treated with utmost confidentiality. The researchers had prepared a letter of consent to the administrators, teachers and students. Permission was granted by authorized personnel of the school.

3. Results & Discussions

Two emergent perspectives that pertain to the purpose of this study were gleaned from the data gathered from the whole cycle of the Lesson Study: (1) constructivism in mathematics classroom could go beyond discovery learning; and (2) anticipating student responses and capitalizing on possible student misconceptions is a powerful tool in designing constructivist lessons.

Constructivism in Mathematics classroom could go beyond discovery learning.

In the first stages of the Lesson Study, the teachers devised activities that involve hands-on learning, considering students' attributes to plan learner-centered activities that would further cater to their needs. However, the following problems emerged such as availability of learning materials and time constraints.

The researchers initially planned to have technology-integrated activities but due to some considerations of facilities in the classroom, the researchers came up with a more traditional kind of technology (i.e. glass board, illustration board, board marker and protractor). For the measuring of the sides of the triangles, the protractor was further maximized by using its ruler side (located at the bottom of the protractor).

Since the class discussion was restricted to an hour, creating too many activities would require a lot more time of preparation and instruction. The teachers find this situation difficult to incorporate hands-on activities.

Teachers are being able to thoughtfully anticipate students' engagement with the activities/tasks being planned for the lesson. This is done via collaboration and brainstorming of ideas of the Lesson Study participants.

This was a conversation during the planning:

Researcher A: "On the activity part, to make the students further establish their knowledge in triangle similarities, maybe we can come up with an activity wherein they will measure the sides and angles of a pair of triangles then they will arrive at the knowledge that the corresponding sides are proportional and the corresponding angles are congruent."

Researcher B: "That's a good idea, but the conflict is that can the students do it within the time frame or will it be more time-consuming? Since let's say there are students need more time to measure those sides and angles"

Researcher C: "Well, we can extend the activity to ten minutes. It will be enough for the students to accomplish the activity."

The class discussion started with a review on the concepts of ratio and proportion. After which, the students had their first activity wherein they determine whether each pair of figures show congruence or similarity. Then, their second activity was determining the characteristics of similar triangles through measuring the sides and angles of a given pair of triangles. After this activity, they are asked on the relation of the corresponding sides and corresponding angles, which will guide them to formulate the concept of similar triangles. Next thing was having a 'Triangle Bank' where they pair similar triangles. Missing lengths or angles were included in the two of the pairs of triangles. Students apply the use of ratio and proportion in solving for the missing side/s of similar triangles. Lastly, they deal with a real-life application of triangle similarities.

The activity on triangle measurement was facilitated in letting the students discover the characteristics of similar triangles. The students used protractors to measure the angles, as well as the sides of similar triangles. To also ensure the validity of the characteristics discovered on the similar triangles, the researchers make use of varied worksheets wherein a different set of similar triangles are assigned from each group of students. Yet, there has been a problem in the process. One problem is the time spent for the students to do the activity. Instead of finishing them in ten minutes, some finished them in fifteen minutes, and some were not able to finish after fifteen minutes. This implies that by engaging in Lesson Study, the teachers solidify the way they anticipate students' engagement via the actual things that happen during the research lesson.

Moreover, during the discussion, the researchers found out gaps of knowledge after their triangle measurement. Figures 1 and 2 are some of the pieces of evidence of the students' activity:

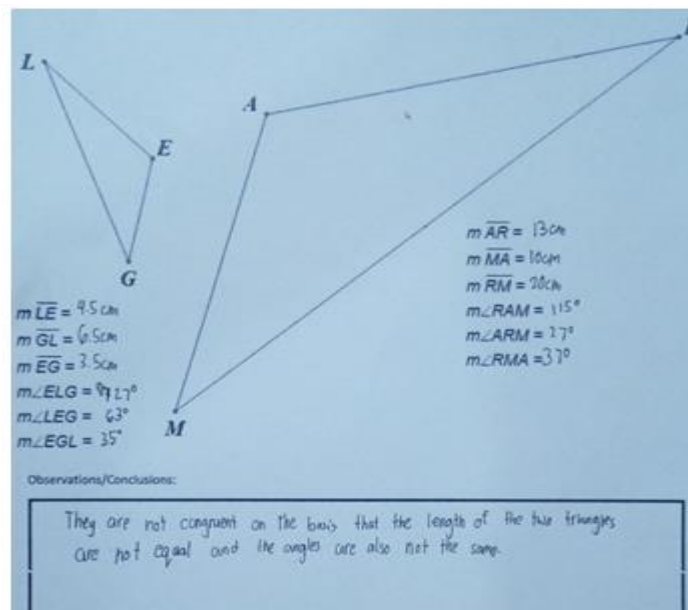


Figure 1. Student A's answer on the first activity.

In Figure 1, Student A was able to determine that the triangles are not congruent. However, he was not able to determine the characteristics of similar triangles (given that the worksheets are already given after introducing the concept of similarity). One observation that leads the student to this kind of answer is the improper use of protractors in measuring the angles and sides that will mislead to the intended observations. In anticipating students' engagements, the actual things that actually happened during the research lesson was able to intimate that it is very important for teachers to also consider the conditions or affordances that would allow the students to successfully engage in the tasks.

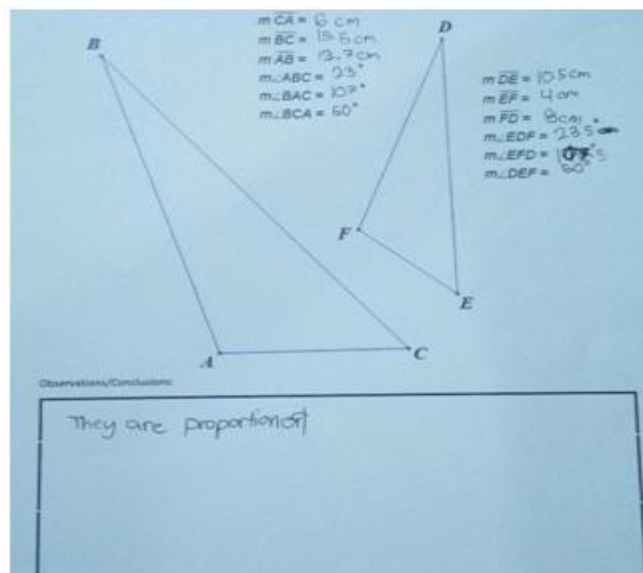


Figure 2. Student B's answer on the first activity

In Figure 2, Student B observed that “they [the corresponding sides of the triangles] are proportional.” He was able to determine the proportionality of the corresponding sides of the triangle (though there are some small discrepancies on the measurements). In terms of the angle’s measurement, he was able to derive at the congruence of the corresponding angles. However, he was not able to conclude that the corresponding angles of similar triangles are congruent.

Furthermore, the class discussion focuses more on corresponding sides which initially, the teacher aims to let the students state that the corresponding sides are proportional.

Teacher: In terms of its corresponding sides, now that we already have the measure of those sides, what is the relation of those sides?

Student C: The measure of the sides of the first triangle is twice of the measure of the sides of the second triangle.

Since the students were not able to mention the term “proportional”, a series of follow-up questions were asked, and examples related to the concept were given until such time that they were able to mention the term “proportional”.

This implies that that discovery of learning is not just having the students saying the exact word or concept in consideration. Discovery learning is not as simple as being able to define terminologies but more on students being able to see relationships on their own, to conjecture and to come up with some sort of generalizations based on testing several cases. In the process, Higher Order Thinking Skills or deep thinking would be necessary.

The researchers’ course professor commented, “The notion of constructivism is a change in the way students think. The teacher had found a way to deal with it. The notion of constructivism is really a big problem, but then, we have to come up with how we are going to deepen such. I was actually looking for higher order thinking skills. My biggest comment is that some tendencies arise when we have the intention to let the students discover something. The thing is, when we say higher order thinking skills, it does not necessarily mean that we have to let the students discover. Sometimes, we want them to create something.”

“The question is, ‘Is it really necessary to say the exact word?’ Because, during discovery learning, teachers are very eager to hear certain words coming from the students. Sometimes, there is no need for that. To be able for the students to think deeply, to be engaged with an activity, we can ask them to do a problem, thus we address problem-solving. Or give them a puzzle so that they might discover and then it is the time for the teacher to tell them that this is about proportionality.”

Applying and understanding the concept of proportionality is more important than recognizing the terminology.

An enlightening note on constructivism was also followed by this statement from the course professor:

“For this lesson, and for other lessons also, I just noticed that we have to be careful. Constructivism is not only through discovery. Higher order thinking skills can’t only be achieved through discovery. In a way, we might let the students discover something that is not there. We do not let them use their creativity.”

In addition, the observers gave further suggestions in improving the lesson. The observer also commented that the lesson only focused on individual activities. Furthermore, she added, “Why not, instead of individual activities, make it differentiated? Divide the class into groups and let them explain what they have to themselves...”

This supports the idea that an individual’s culture and context play a crucial role in his perception of society and such understanding consequently affects how he constructs his knowledge or how he develops new learning. This idea is known as social constructivism [18,19]. This is where individuals create meaning through their interactions with each other and with the environment they live in.

In addition, many educators had misconceptions on the term “understanding” which means being able to justify procedures and state why a process works. Thus, the key to genuine understanding is being helped to generalize from one’s specific knowledge. [20]

The activity of measuring triangles was meant to let the students discover the concept. However, that activity entailed lack of creating and forming new knowledge. In addition, discovery [in teaching and learning] is a good thing, something that students need, but has its own time and usage. The need that the students should focus into is the process of creating which is a higher order thinking skill.

The notion of learning as an active process that says that passive teaching views the learner as an ‘empty vessel’ to be filled with knowledge, whereas constructivism states that learners construct meaning only through

active engagement with the world and it was also true for teachers' learning on how they could improve the lessons via reflections and realizations based on the actual experiences in doing Lesson Study. Information may be passively received, but understanding cannot be, for it must come from making meaningful connections between prior knowledge, new knowledge, and the process involved in learning. There are other ways to present the other parts of the constructivist lesson aside from discovery if it taps the higher-order thinking skills of the students.

In addition, teachers must know their students' prior knowledge very well. Knowing their prior knowledge will be the basis for planning the activities in the lesson and anticipating students' responses especially if those teachers will have constructivism as their theoretical background in formulating learning activities. This is one thing that must be further emphasized especially Constructivism is one of the underlying principles that supports the framework of the Mathematics K to 12 curriculum in the Philippines. Teachers should also anticipate student responses in relation to their activities and analysis as this would minimize time and discussion struggles. Student-centered activities also provide students opportunities to better strengthen their conceptual understanding. Also, giving students time to interact and share insights with their peers increases their mathematical attitude and widens their mathematical thinking.

Anticipating student responses and capitalizing on possible student misconceptions is a powerful tool in designing constructivist lessons.

In the lesson planning, the teachers collaborate and make anticipations on the following: (1) How the students answer a set of questions during the class discussion and (2) The difficulties they would encounter in the lesson.

From the first anticipation, How the students answer the questions, teachers thought of different correct and incorrect answers that the students are likely to guess. They also described how the teacher responded to each of these incorrect answers. On the first activity, they expected that students could differentiate congruent figures and similar figures. They also anticipated that students would construct the definition of "Similar Triangles" and give its properties based on the given activity. Students found the difference between congruent and similar figures by raising either black side (for congruent) and white side (for similar) of the illustration board (See Figure 3).



Figure 3. Students raising their cardboards on the first activity.

On the part of the discussion wherein the students would define similar triangles, one of the students said the word "Proportional" as one of the properties of similar triangles. Using the follow-up questions given by the teacher, one student completed the definition of congruent triangles. "Triangles are similar if they have congruent corresponding angles and the corresponding parts are proportional." The results showed that teachers successfully anticipated students' possible answers.

Second, anticipating students' difficulties involves predicting the mistakes in doing the activities, challenges, and misconceptions regarding the topic which is Similar Triangles. The teachers predict that the students would find measuring angles using a protractor easy since it is already discussed when they were in 7th grade. They also anticipated that students could finish the activity within 10 minutes. After the lesson, teachers realized most of the students do not know the proper way of using protractor. They just simply put the protractor over the angle without even placing the origin of it in the vertex of the angle. The researchers realized that there are students who completely do not know how to use protractors correctly. While observing the class, there is a conversation between the students:

Student D: How did you do that?

Student E: Di ko alam. Nilagyan ko na lang ng sagot. (I do not know, I just put an answer.)

In addition, an unanticipated answer raised during the class discussion.

Teacher: Based on the measurements of the corresponding angles of similar triangles, what can you conclude?

Student F: The sum of the angles of each of the triangles is 180 degrees.

This conversation suggests that the intended answer is not completely aligned to the students' prior knowledge about triangles.

The students already know the facts about triangles, but they were not able to determine the relationships of those angles. This situation is similar to the situation based on the study of Irsal, Jupri & Prabawanto [17] wherein students may understand the concepts themselves (i.e. definitions, postulates, theorems) but they are weak in relational skills, i.e. applying concepts in connecting the relations among the components of figures.

The prior knowledge of the students in using the protractor has not yet been established so it affected the discussion because they were not able to tell the teacher what is the relationship of the two triangles in terms of their angles. If the students' needed previous knowledge is not yet established, it will really affect the whole discussion. But if what the students encounter is inconsistent with their current understanding, their understanding can change to accommodate new experiences. Furthermore, the idea of constructivism is that learning is a process where prior learning is built up by new learning. The process, however, is also selective as the previous knowledge will have an impact on how a student will adapt to the new learning experiences. [21]

During the post-discussion, there are teachers who gave positive comments about the lesson, especially to the series of given activities and there are comments that gave some areas of improvement. The instructor commented, "It's better to put yourself in the shoes of the students." The teachers must anticipate students' typical thoughts and ideas before planning the lesson and select different learning materials.

4. Conclusion

This paper aims to raise conversations on designing appropriate learner-centered activities using the approach of Lesson Study to evaluate classroom practices. The findings sought from this study helps to augment the benefits of student-centered activities with constructivism by employing Lesson Study to examine its implications on the teaching-learning process in the field of Mathematics, especially in geometry.

Two emergent perspectives were apparent: (1) constructivism in mathematics classroom could go beyond discovery learning; and (2) anticipating student responses and capitalizing on possible student misconceptions is a powerful tool in designing constructivist lessons.

Thoughtful anticipation of students' engagement with the activities/tasks being planned for the lesson is done via collaboration and brainstorming of ideas. Discovery learning is not limited to simple definition of terminologies and concepts, but more on students being able to see relationships on their own, conjecture and come up with some sort of generalizations based on testing several cases.

Being a constructivist teacher entails a long journey of classroom experiences and being able to reflect on one's practice especially if this is in relation to the students and to the entire teaching and learning environment. In terms of the classroom discussion, applying and understanding the concept is more important than recognizing the terminology. Teachers should also anticipate student responses in relation to their activities and analysis as this would minimize time and discussion struggles. Lesson Study supports improvement of lessons towards constructivist classrooms. Lesson Study really helps for the professional development of teachers and the improvement of lessons by giving opportunities to interact from diverse perspectives and discuss some strengths and areas of improvement for the formulation and execution of high-quality mathematics lessons.

In terms of teaching Geometry, the students' lack of relational skills and construction of knowledge in Geometry are still significant gaps evident during the classroom discussion but doing Lesson Study helps to address those gaps in Geometry teaching that could be done one lesson at a time.

This study is limited only to an all-boys school and the intended activities are not fully executed in an hour. The researchers suggest applying the lesson study in other schools. For future research direction, consider differentiated learning activities and problem-based learning as a focus for lesson study that will be conducted in co-educational institutions.

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