Hmc Labeling Of Certain Types Of Graph

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Abstract:
In this paper we introduce a new graph labeling called HMC labeling. We investigate HMC labeling of Path P_n, Star $K_{1,n}$, Bistar $B_{n,n}$ graphs.

Keywords: Cordial labeling, Cordial graphs, HMC labeling, HMC graphs.

1. Introduction

One of the important area in graph theory is Graph labeling. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Graph labeling problems that appear in graph theory as vast applications. The Graph labelling problem was first introduced by Alex Rosa in 1967. Since Rosa's article many different types of graph labeling problem has been defined and so far the literature survey says about the research papers above 4000 papers.[4] They gave birth to different labeling such as graceful, harmonious, elegant, magic, antimagic, prime labeling etc. The labeling graphs are applied mostly in the coding theory, X-ray, Crystallography, Radar, Astronomy, Communication network addressing, Data base management and Cryptography etc.Here in this article a new labeling is introduced namely Harmonic Mean Cordial labelling(HMC).

2. Preliminaries:

The concept of cordial labeling was introduced by cahit in the year 1987.[3]

Definition 2.1: For graph $G=(V, E)$, Let $f:V(G)\rightarrow\{0,1\}$ be a function. For each edge *uv* assign the label $|f(u)-f(v)|$, f is called a cordial labeling if $|v_f(i)-v_f(j)|\leq 1$ and $|e_f(i)-e_f(j)|\leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x, x\in\{0,1\}$ respectively. A graph which admits cordial labeling is called a cordial graph.

Mean cordial labeling was introduced by Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram. [7]

Definition2.2: For a graph $G=(V,E)$, Let *f* be a function from $v(G)\rightarrow\{0,1,2\}$. For each edge *uv* of *G* assign the label $\left[\frac{f(u)+f(v)}{2}\right]$, f is called a mean cordial labeling if $|v_f(i)-v_f(j)|\leq 1$ and $|e_f(i)-e_f(j)|\leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x, x\in\{0,1,2\}$ respectively. A graph which admits mean cordial labeling is called a mean cordial graph.

Geometric mean cordial graph was introduced by K. Chitra Lakshmi, K. Nagarajan.[3]

Definition2.3: For graph $G=(V,E)$. Let *f* be a function from $v(G)\rightarrow\{0,1,2\}$. For each edge *uv* of *G* assign the label „-$f(u)f(v)$.., f is called a geometric mean cordial labeling if $|v_f(i)-v_f(j)|\leq 1$ and $|e_f(i)-e_f(j)|\leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x, x\in\{0,1,2\}$ respectively. A graph with a geometric mean cordial labeling is called geometric mean cordial graph.

Definition 2.4: A path graph P is a simple graph with $|V_P|=|E_P|+1$ that can be drawn so that all of its vertices and edges lie on a single straight line. A path graph with n vertices and n - 1 edges is denoted P_n. [5]

Definition 2.5: A Star is the complete bipartite graph $K_{1,n}$, a tree with one internal node and n leaves (but no internal nodes and n+1 leaves when n ≤ 1)

Definition 2.6: Bistar is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$.[1]

3. Main Result

Motivated by the concept of mean cordial labeling and geometric mean cordial labeling, we introduce a new labeling as follows:

Definition 3.1: A simple graph $G = (V, E)$ is said to be HMC (Harmonic Mean Cordial) labeling if there exist a function f:$V \to \{1,2\}$ such that the induced edge function g:E→{1,2}defined by $uv = \left|\frac{2f(u)f(v)}{f(u)+f(v)}\right|$, $f(u), f(v) \neq 0$ for each edge and $|v_f(i) - v_f(j)| \le 1$, $|e_g(i) - e_g(j)| \le 1$ where $v_f(x)$ -denotes the number of vertices labeled with x, $e_g(x) -$ denotes the number of edges labeled with **x,** where $x \in \{1,2\}$ respectively. A graph which admits a HMC (Harmonic Mean Cordial) labeling is called HMC (Harmonic Mean Cordial) graph.

Theorem3.2: Path graph P_n admits HMC labeling.

Proof: Let G = (V, E) be a path graph where V= $\{v_1, v_2, \ldots, v_n\}$ be the vertices of G.

Define f: V → {1, 2} as follows:

Case (i): If *n* is even

$n \equiv 0(\text{mod } 2)$. Let *n* =2*t*, $f(v_i) = 1, \ 1 \le i \le t$

$f(v_{t+i}) = 2, \ 1 \le i \le t$ .Then $v_f(1) = v_f(2) = t$ and $e_g(1) = t, e_g(2) = t-1$

Case (ii): If *n* is odd

$n \equiv 1(\text{mod } 2)$. Let $n = 2t+1$ $f(v_i) = 1, \ 1 \le i \le t$ $f(v_{t+i}) = 2, \ 1 \le i \le t+1$

Then $v_f(1) = t, \ v_f(2) = t+1$ and $e_g(1) = t, e_g(2) = t$

By using the definition of HMC graph, we observe that $|v_f(i) - v_f(j)| \le 1$ and $|e_g(i) - e_g(j)| \le 1$ for all $i, j \in \{1,2\}$ and hence path graph P_n admits HMC graph.

Theorem 3.3: The star $K_{1,n}$ admits HMC labelling.

Proof: Let G = (V, E) = $K_{1,n}$ be the star graph where V= $\{v, v_1, v_2, \ldots, v_n\}$ be the vertices of G. Let $V(K_{1,n}) = \{v, v_i : 1 \le i \le n\}$ and $E(K_{1,n}) = \{v, v_i : 1 \le i \le n\}$

$K_{1,n}$ has n+1 vertices and n edges.

Let v is the apex vertex of $K_{1,n}$

Define $f : V(K_{1,n}) \to \{1,2\}$ as follows. Let $f(v) = 2$.

Case (i): If n is even

$n \equiv 0(\text{mod } 2)$

Let $n = 2t$

Assign the labels 1,2 to each of the *t* vertices respectively.

Then $v_f(1) = t, v_f(2) = t+1$ and $e_g(1) = e_g(2) = t$

Case (ii): If n is odd

$n \equiv 1(\text{mod } 2)$

Let $n = 2t+1$

Assign the label 1 to t + 1 vertices and the label 2 to the remaining each of t vertices respectively. Then $v_f(1) = v_f(2) = t$ and $e_g(1) = t+1, e_g(2) = t.$

From the above cases, we see that $\left|v_f(i) - v_f(j)\right| \le 1, \left|e_g(i) - e_g(j)\right| \le 1$ for all $i, j \in \{1,2\}$ and hence f is HMC labeling.

Theorem 3.4: The Bistar graph $\mathrm{B}_{n,n}$ admits HMC labeling.

Proof: Let G=(V,E)= $\mathrm{B}_{n,n}$ be a Bistar graph containing two copies of $K_{1,n}$. Let $\{ u_1, u_2, \ldots, u_n,\ v_1, v_2, \ldots, v_n \}$ be the corresponding vertices of each copy of $K_{1,n}$ with apex vertex *u* and *v*.

Let $e_i = uu_i, e_i = vv_i$ and $e = uv$ of bistar graph. Note that then $\left|V(\mathrm{B}_{n,n})\right| = 2n+2$ and $\left|E(\mathrm{B}_{n,n})\right| = 2n+1$.

Define $f : V \to \{1,2\}$ as follows

$f(u) = 1, \quad f(v) = 2$

$f(u_i) = 1, \quad 1 \le i \le n$

$f(v_i) = 2, \quad 1 \le i \le n$

In view of the above defined labeling pattern we have, $v_f(1) = v_f(2) = n+1$ and $e_g(1) = n+1, e_g(2) = n$ Thus we proved that $\left|v_f(1) - v_f(2)\right| \le 1$ and $\left|e_g(1) - e_g(2)\right| \le 1$.

Hence, $\mathrm{B}_{n,n}$ admits a HMC graph.

4. Conclusions:

In this paper we introduced the concept of HMC (Harmonic Mean Cordial) labeling and studied the HMC labeling behavior of few standard graphs. The study of HMC labeling of graph obtained from standard graph using the graph operation shall be quite interesting and also will lead to newer results.

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