Research Article

Mathematical Modelling Of Inventory System With Parabolic Holding Cost, Weibull Distributed Deterioration Backlogging Under Pentagonal Fuzzy Parameters

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ABSTRACT: In the present study, an inventory model with parabolic holding cost, quadratic demand rate, partial backlogging over a time horizon for weibull rate of deteriorating item is proposed. We have supposed the demand rate to be a quadratic function of time. Since the outbreak of pandemic COVID- 19 problem disturbed the political, social, economic, and financial structure of the whole world and both the demand and supply chain management have been affected badly therefore, parabolic holding cost is far better to be taken in account. We explore the inventory system to incorporate three parameters .i.e. purchase cost , backordering cost and cycle time which have been fuzzified using pentagonal fuzzy numbers to obtain total inventory cost. Graded mean integration method and Signed distance method are used to de-fuzzify the total cost. The main aim of the paper is to minimize the total cost per unit time in fuzzy environment. Sensitivity analysis of the optimal solution and its effects have been discussed.

Key Words: quadratic demand, parabolic holding cost, shortages, Partial backlogging, Graded mean integration method, Signed distance method.

1. INTRODUCTION

In the past many decades mathematical ideas have been used in different spheres of real life problems particularly for controlling inventory. The deterioration of items or goods is a realistic feature and plays a pivot role in inventory management. Fuzzy inventory modeling is the closest approach towards reality. Most of the products such as fruits, medicines, fashion goods, alcohol, milk, vegetables, electronic components, photographic films and many more suffer from depletion and start losing their values with passage of time . Due to deterioration, inventory system faces the problem of shortages and loss of goodwill or loss of profit. Moreover, the sales for the product may decline on adding more competitive product in market or change in consumer's behavior or his priority or due to the research with high-tech products .Longer the waiting time, smaller will be backlogging rate which leads to a larger fraction of lost sales and yield less profit. Hence, the factor of partial backlogging becomes significant to be considered.

Chang (1999), Goswami A,etal.(2003), Kazemi (2010) studied inventory models with partial backlogging and fuzzy parameters. Parvathi.P and Gajalakshmi.S(2013) explored a fuzzy inventory model with allowable shortage using trapezoidal fuzzy numbers. Further Nagar H. and Surana. P (2015) presented an Inventory model for deteriorating items using parameters as fuzzy numbers.. Kumar, V., Sharma. A, Gupta. C.B, (2015) developed a deterministic Inventory Model for weibull distributed deteriorating items with selling price dependent demand and parabolic time varying holding cost .In 2017 Rajalakshmi.R.M and Michael. G. has also discussed the inventory model using Fuzzy parameters.

Recently inventory models with variable demand, weibull distributed deterioration rate has been developed by Harish ,Vinod kumar& T. P. Singh (2019,2020) in order to find optimal value of production cycle time minimizing the stock level & total average cost over a time horizon .In this model parabolic holding cost is being added because of drastic unbalanced between supply and demand due to COVID-19 problem. Further the model is fuzzified with pentagonal fuzzy parameters since fuzzy uncertainty justify the real time situation in more effective manner to run various operations of the system. The optimal policy for fuzzy inventory cost of the said model is defuzzified by using signed distance and graded mean integration methods. The sensitivity analysis has been carried out between crisp parameters and fuzzy parameters as well.

Following an introductory part rest of the paper is organized as follows. In section 2, some definitions and properties about fuzzy sets related to this study are presented. In section 3, the notations and assumptions are described in brief for developing the model. Section 4 gives the mathematical formulations of the model. In section 5, the model has been defuzzified with the help of well known formulae. Section 6 illustrates the developed models on a numerical example and sensitive analysis of the model has been carried out to examine the effect of changes in values of the different parameters for optimal inventory policy.. Finally, conclusion are given in last section.

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2. PRELIMINARIES:

PRELIMINARIES

Definition 2.1 Pentagonal Membership Function: A pentagonal membership function is specified by three parameters (a,b,c,d,e) as follows:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & if\ a \le x \le b \\ \frac{x-b}{c-b} & if\ b \le x \le c \\ 1 & if\ x = c \\ \frac{d-x}{d-c} & if\ c \le x \le d \\ \frac{e-x}{e-d} & if\ d \le x \le e \\ 0 & ,otherwise \end{cases}$$

The parameters (a,b,c,d,e) with $a \le b \le c \le d \le e$ determined the x coordinate with five corners of pentagonal membership function.

Definition 2.2

If $\hat{A}$= (a,b,c,d,e) is a pentagonal fuzzy number then the graded mean integration representation of $\hat{A}$ is defined as

$$P(\hat{A}) = \frac{a+3b+4c+3d+e}{12}$$

Definition 2.3

If $\hat{A}$= (a, b, c,d,e) is a pentagonal fuzzy number then the signed distance of $\hat{A}$ is defined as

$$P(\hat{A}) = \frac{a+2b+2c+2d+e}{8}$$

3. ASSUPTIONS AND NOTATIONS:

To develop the model following assumptions and notations have been considered :

1) Replenishment size is constant and the replenishment rate is infinite.
2) Lead time is zero.
3) T is the length of each production cycle;
4) A be Ordering Cost;
5) $C_1 = f + gt^2$ is the inventory holding cost per unit time, a parabolic function;
6) C_2 is Purchase cost per unit;
7) C_3 is the cost of each deteriorated unit;
8) C_4 is Backordered cost per unit;
9) C_5 is Lost sales cost per unit;
10) C(t) is the total inventory cost ;
11) $\bar{G}_p(\ddot{t})$ is the total inventory cost by Graded Mean Integration Method(Pentagonal) ;
12) $\dot{S}_p(\ddot{t})$ is the total inventory cost by Signed Distance Method(Pentagonal) ;
13) The deterioration rate function θ(t) represents the on-hand inventory deteriorates per unit time and Moreover in the present study the function assumed in the form
    $\theta(t) = \alpha\beta t^{\beta-1}$; $0 < \alpha < 1$, $\beta > 0$, $t > 0$.
    When $\beta = 1$, θ(t) becomes constant a case of exponential decay. When $\beta < 1$, the rate of deterioration is decreasing with t and when $\beta > 1$, the rate of deterioration is increasing with t.
14) The demand rate starts from zero and ends at zero during the inventory period. It is assumed of the form $D(t) = at(T-t)$ where T is the cycle period
15) During stock out period, the backlogging rate is variable and is depends on the length of the waiting time for next replenishment. So that the backlogging rate for negative inventory is,
    $$B(t) = \frac{1}{1+\gamma(T-t)}$$
    γ is backlogging parameter and T-t is waiting time and $t_1 < t < T$.

4. MATHEMATICAL FORMULATIONOF THE MODEL:

Assume an amount S (S>0) as an initial inventory. We find inventory level gradually diminishes due to reasons of market demand and deterioration of the items and ultimately falls to zero at time T . Let I(t) be on hand inventory at any time t. clearly, the differential equations which on hand inventory I(t) must satisfy the following two equations, one due to deterioration and demand, second due to shortage and partial backlogging :

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t) \quad , \quad 0 \le t \le t_1 \quad ....(1)$$

$$\frac{dI(t)}{dt} = -\frac{D(t)}{1+\gamma(T-t)} \quad , \quad t_1 \le t \le T \quad ....(2)$$

Using $D(t) = at(T-t)$, $I(0) = S$ and $I(t_1)=0$ we get

$$S = a\left(\frac{Tt_1^2}{2} + \frac{\alpha T t_1^{\beta+2}}{\beta+2} - \frac{t_1^3}{3} - \frac{\alpha t_1^{\beta+3}}{\beta+3}\right). \quad ......(3)$$

the solution of differential equation (1) can be written as

$$I(t) = -a\left[\frac{Tt^2}{2} - \frac{(\alpha\beta)Tt^{\beta+2}}{2(\beta+2)} - \frac{t^3}{3} + \frac{(\alpha\beta)t^{\beta+3}}{3(\beta+3)} - \frac{Tt_1^2}{2} - \frac{\alpha T t_1^{\beta+2}}{\beta+2} + \frac{t_1^3}{3} + \frac{\alpha t_1^{\beta+3}}{\beta+3}\right] + a\alpha\left[\frac{\alpha T t^{2\beta+2}}{\beta+2} - \frac{\alpha t^{2\beta+3}}{\beta+3} - \frac{Tt^{\beta}t_1^2}{2} - \frac{\alpha T t^{\beta} t_1^{\beta+2}}{\beta+2} + \frac{t^{\beta}t_1^3}{3} + \frac{\alpha t^{\beta} t_1^{\beta+3}}{\beta+3}\right]. \quad ....(4)$$

Using $I(T) = -S_1$ and $I(t_1)=0$ in Equation (2) we get

$$S_1 = \frac{a(T^2-t_1^2)}{2\gamma} + \frac{a(t_1-T)}{\gamma^2} - \frac{a(1+\gamma T)(T-t_1)}{\gamma^2} \quad ……..(5)$$

The solution of Equation (2) be

$$I(t) = \frac{a(t_1^2-t^2)}{2\gamma} + \frac{a(t-t_1)}{\gamma^2} - \frac{a(1+\gamma T)(t_1-t)}{\gamma^2} \quad t_1 \le t \le T \quad ……..(6)$$

Hence total amount of deteriorated units (D)=I(0)−stock loss due to demand

$$= S - \int_0^{t_1} at(T-t)\,dt$$

$$= a\left(\frac{\alpha T t_1^{\beta+2}}{\beta+2} - \frac{\alpha t_1^{\beta+3}}{\beta+3}\right) ....(7)$$

Total Inventory held $(I_1) = \int_0^{t_1}(f + gt^2)I(t)\,dt$

$$I_1 = f\left\{-a\left[-\frac{\alpha(\beta^3+4\beta^2+\beta){t_1}^{\beta+4}}{3(\beta+3)(\beta+1)} + \frac{\alpha(\beta^3+3\beta^2+\beta)T t_1^{\beta+3}}{2(\beta+2)(\beta+1)} + \frac{t_1^4}{4} - \frac{Tt_1^3}{3}\right] + a\alpha\left[-\frac{\alpha T {t_1}^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{\alpha {t_1}^{2\beta+4}}{(\beta+1)(2\beta+4)}\right]\right\} + g\left\{-a\left[\frac{Tt_1^5}{10} + \frac{t_1^6}{18} - \frac{\alpha(5\beta+4)Tt_1^{\beta+5}}{6(\beta+2)(\beta+5)} + \frac{\alpha t_1^{\beta+6}}{3(\beta+3)}\right] + \alpha a\left[-\frac{Tt_1^{\beta+5}}{2(\beta+3)} + \frac{t_1^{\beta+5}}{3(\beta+3)} - \frac{\alpha T t_1^{2\beta+5}}{(\beta+3)(2\beta+5)} + \frac{\alpha t_1^{2\beta+6}}{2(\beta+3)^2}\right]\right\} \quad ....(8)$$

Cost of deteriorated items = C_3 × total amount of deteriorated units

$$= C_3\, a\left(\frac{\alpha T t_1^{\beta+2}}{\beta+2} - \frac{\alpha t_1^{\beta+3}}{\beta+3}\right) \quad ....(9)$$

Backordered cost per cycle $= C_4\int_{t_1}^{T} -I(t)\,dt$

$$= C_4\left\{\frac{a(T^3-t_1^3)}{6\gamma} - \frac{a(T-t_1)t_1^2}{2\gamma} + \frac{a(T-t_1)^2}{2\gamma^2} + \frac{a(1+\gamma T)(t_1-T)^2}{2\gamma^2}\right\} …….(10)$$

Lost sales per cycle = $C_5\int_{t_1}^{T}\left(1 - \frac{1}{1+\gamma(T-t)}\right) at(T-t)\,dt$

$$= C_5\left\{\frac{a(T^3-Tt_1^2)}{2} - \frac{a(T^3-t_1^3)}{3} - \frac{a(T^2-t_1^2)}{2\gamma} + \frac{a(T-t_1)}{\gamma^2} + \frac{a(1+\gamma T)}{\gamma^2}(T-t_1)\right\} \quad …(11)$$

. Purchase cost per cycle = $C_2\, a\left(\frac{Tt_1^2}{2} + \frac{\alpha T t_1^{\beta+2}}{\beta+2} - \frac{t_1^3}{3} - \frac{\alpha t_1^{\beta+3}}{\beta+3}\right)$……(12)

Average total cost per unit time $C(t_1) = \frac{1}{T}$[Total cost per unit time] = $\frac{1}{T}$[Ordering cost +Total Inventory held +Cost of deterioration items+Backordered cost per cycle + Lost sales per cycle + Purchase cost per cycle]

$$= \frac{1}{T}\left\{A + f\left\{-a\left[-\frac{\alpha(\beta^3+4\beta^2+\beta){t_1}^{\beta+4}}{3(\beta+3)(\beta+1)} + \frac{\alpha(\beta^3+3\beta^2+\beta)T t_1^{\beta+3}}{2(\beta+2)(\beta+1)} + \frac{t_1^4}{4} - \frac{Tt_1^3}{3}\right] + a\alpha\left[-\frac{\alpha T {t_1}^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{\alpha {t_1}^{2\beta+4}}{(\beta+1)(2\beta+4)}\right]\right\} + g\left\{-a\left[\frac{Tt_1^5}{10} + \frac{t_1^6}{18} - \frac{\alpha(5\beta+4)Tt_1^{\beta+5}}{6(\beta+2)(\beta+5)} + \frac{\alpha t_1^{\beta+6}}{3(\beta+3)}\right] + \alpha a\left[-\frac{Tt_1^{\beta+5}}{2(\beta+3)} + \frac{t_1^{\beta+5}}{3(\beta+3)} - \frac{\alpha T t_1^{2\beta+5}}{(\beta+3)(2\beta+5)} + \frac{\alpha t_1^{2\beta+6}}{2(\beta+3)^2}\right]\right\} + C_3\, a\left(\frac{\alpha T t_1^{\beta+2}}{\beta+2} - \frac{\alpha t_1^{\beta+3}}{\beta+3}\right) + C_4\left\{\frac{a(T^3-t_1^3)}{6\gamma} - \frac{a(T-t_1)t_1^2}{2\gamma} + \frac{a(T-t_1)^2}{2\gamma^2} + \frac{a(1+\gamma T)(t_1-T)^2}{2\gamma^2}\right\} + C_5\left\{\frac{a(T^3-Tt_1^2)}{2} - \frac{a(T^3-t_1^3)}{3} - \frac{a(T^2-t_1^2)}{2\gamma} + \frac{a(2+\gamma T)}{\gamma^2}(T-t_1)\right\} + C_2 a\left(\frac{Tt_1^2}{2} + \frac{\alpha T t_1^{\beta+2}}{\beta+2} - \frac{t_1^3}{3} - \frac{\alpha t_1^{\beta+3}}{\beta+3}\right)\right]\right\} \quad ....(13)$$

$\frac{dC(t_1)}{dt_1} = \frac{1}{T}$ {f {

$- a\left[-\frac{\alpha(\beta^3+4\beta^2+\beta)(\beta+4)t_1^{\beta+3}}{3(\beta+3)(\beta+1)}+\frac{\alpha(\beta^3+3\beta^2+\beta)(\beta+3)Tt_1^{\beta+2}}{2(\beta+2)(\beta+1)}+\frac{t_1^3}{1}-\frac{Tt_1^2}{1}\right]+a\alpha\left[-\frac{\alpha T t_1^{2\beta+2}}{(\beta+1)}+\frac{\alpha t_1^{2\beta+3}}{(\beta+1)}\right]\}$ + g{-a[$\frac{Tt_1^4}{2}$ + $\frac{t_1^5}{3}$ − $\frac{\alpha(5\beta+4)Tt_1^{\beta+4}}{6(\beta+2)}$ + $\frac{\alpha(\beta+6)t_1^{\beta+5}}{3(\beta+3)}$] + αa[− $\frac{(\beta+5)Tt_1^{\beta+4}}{2(\beta+3)}$ + $\frac{(\beta+5)t_1^{\beta+4}}{3(\beta+3)}$ − $\frac{\alpha T t_1^{2\beta+4}}{(\beta+3)}$ + $\frac{\alpha t_1^{2\beta+5}}{(\beta+3)}$]}+ C_3 a($\frac{\alpha T t_1^{\beta+1}}{1}-\frac{\alpha t_1^{\beta+2}}{1}$) + C_4 { $-\frac{a(t_1^2)}{2\gamma}-\frac{a(2T-3t_1)t_1^1}{2\gamma}-\frac{a(T-t_1)^1}{\gamma^2}+\frac{a(1+\gamma T)(t_1-T)^1}{\gamma^2}$ }+C_5{ $-\frac{a(Tt_1^1)}{1}+\frac{a(t_1^2)}{1}+\frac{a(t_1^1)}{\gamma}-\frac{a(2+\gamma T)}{\gamma^2}$ } + C_2 a($\frac{Tt_1^1}{1}+\frac{\alpha T t_1^{\beta+1}}{1}-\frac{t_1^2}{1}-\frac{\alpha t_1^{\beta+2}}{1}$)]} ……….(14)

$\frac{d^2C}{dt_1^2}$ = $\frac{1}{T}$ {f { $- a\left[-\frac{\alpha(\beta^3+4\beta^2+\beta)(\beta+4)t_1^{\beta+2}}{3(\beta+1)}+\frac{\alpha(\beta^3+3\beta^2+\beta)(\beta+3)Tt_1^{\beta+1}}{2(\beta+1)}+3\frac{t_1^2}{1}-\frac{2Tt_1^1}{1}\right]+a\alpha\left[-\frac{2\alpha T t_1^{2\beta+1}}{1}+\frac{\alpha(2\beta+3)t_1^{2\beta+2}}{(\beta+1)}\right]\}$ + g{-a[$\frac{2Tt_1^3}{1}$ + $\frac{5t_1^4}{3}$ − $\frac{\alpha(5\beta+4)(\beta+2)Tt_1^{\beta+3}}{6(\beta+2)}$ + $\frac{\alpha(\beta+6)(\beta+5)t_1^{\beta+4}}{3(\beta+3)}$] + αa[− $\frac{(\beta+5)(\beta+4)Tt_1^{\beta+3}}{2(\beta+3)}$ + $\frac{(\beta+5)(\beta+4)t_1^{\beta+3}}{3(\beta+3)}$ − $\frac{\alpha(2\beta+4)Tt_1^{2\beta+3}}{(\beta+3)}$ + $\frac{\alpha(2\beta+5)t_1^{2\beta+4}}{(\beta+3)}$]}+C_3 a($\frac{\alpha(\beta+1)Tt_1^{\beta}}{1}-\frac{\alpha(\beta+2)t_1^{\beta+1}}{1}$) + C_4 { $-\frac{a(t_1^1)}{\gamma}-\frac{a(T-3t_1)}{\gamma}+\frac{a}{\gamma^2}+\frac{a(1+\gamma T)}{\gamma^2}$ }+C_5{ $-\frac{a(T)}{1}+\frac{a2t_1^1}{1}+\frac{a}{\gamma}$ } + C_2 a($\frac{T}{1}+\frac{\alpha T(\beta+1)t_1^{\beta}}{1}-\frac{2t_1^1}{1}-\frac{\alpha(\beta+2)t_1^{\beta+1}}{1}$)]} …..(15)

For minimum $C(t_1)$, the necessary condition is

$\frac{dC(t_1)}{dt_1}$= 0 After solving, we get an equation of odd degree then there exists $t_1^*\in(0,T)$ can be solved from equation (14) by using MAT Lab. Also $\frac{d^2C}{dt_1^2}>0$ at T= t_1^*

∴ C(is minimum at $C(t_1)=t_1^*$

Hence, optimum value of t_1 is t_1^*

FUZZIFICATION OF THE MODEL

let us describe cycle time as fuzzy parameter ẗ

the total cost function with fuzzy cycle time be

C(ẗ)= $\frac{1}{T}$ {A + f { $- a\left[-\frac{\alpha(\beta^3+4\beta^2+\beta)t_i^{\beta+4}}{3(\beta+3)(\beta+1)}+\frac{\alpha(\beta^3+3\beta^2+\beta)Tt_i^{\beta+3}}{2(\beta+2)(\beta+1)}+\frac{t_i^4}{4}-\frac{Tt_i^3}{3}\right]+a\alpha\left[-\frac{\alpha T t_i^{2\beta+3}}{(\beta+1)(2\beta+3)}+\frac{\alpha t_i^{2\beta+4}}{(\beta+1)(2\beta+4)}\right]\}$ + g{-a[$\frac{Tt_i^5}{10}$ + $\frac{t_i^6}{18}$ − $\frac{\alpha(5\beta+4)Tt_i^{\beta+5}}{6(\beta+2)(\beta+5)}$ + $\frac{\alpha t_i^{\beta+6}}{3(\beta+3)}$] + αa[− $\frac{Tt_i^{\beta+5}}{2(\beta+3)}$ + $\frac{t_i^{\beta+5}}{3(\beta+3)}$ − $\frac{\alpha T t_i^{2\beta+5}}{(\beta+3)(2\beta+5)}$ + $\frac{\alpha t_i^{2\beta+6}}{2(\beta+3)^2}$]}+C_3 a($\frac{\alpha T t_i^{\beta+2}}{\beta+2}-\frac{\alpha t_i^{\beta+3}}{\beta+3}$) + $C_{4,i}$ { $\frac{a(T^3-t_i^3)}{6\gamma}-\frac{a(T-t_i)t_i^2}{2\gamma}+\frac{a(T-t_i)^2}{2\gamma^2}+\frac{a(1+\gamma T)(t_i-T)^2}{2\gamma^2}$ } +C_5{ $\frac{a(T^3-Tt_i^2)}{2}-\frac{a(T^3-t_i^3)}{3}-\frac{a(T^2-t_i^2)}{2\gamma}$ + $\frac{a(2+\gamma T)}{\gamma^2}(T-t_i)$ }+$C_{2,i}$a($\frac{Tt_i^2}{2}+\frac{\alpha T t_i^{\beta+2}}{\beta+2}-\frac{t_i^3}{3}-\frac{\alpha t_i^{\beta+3}}{\beta+3}$)]} where i =1,2,3,4,5. ……..(16)

For pentagonal fuzzy parameters i = 1,2,3,4,5. and C(ẗ) = (A_1,A_2,A_3,A_4,A_5), the values of A_1,A_2,A_3,A_4,A_5be obtained by putting i = 1,2,3,4,5. in C(ẗ) respectively.

5.DEFUZZIFICATION BY PENTAGONAL METHODS

GRADED MEAN METHOD

Total cost is given by $\bar{G}_p$ (ẗ)= $\frac{1}{12}$ [$A_1+3A_2+4A_3+3A_4+A_5$]

$\bar{G}_p$ (ẗ)= $\frac{A}{T}+\sum_{i=1}^{5}Q_i$ { $\frac{1}{12T}$ { f { $- a\left[-\frac{\alpha(\beta^3+4\beta^2+\beta)t_i^{\beta+4}}{3(\beta+3)(\beta+1)}+\frac{\alpha(\beta^3+3\beta^2+\beta)Tt_i^{\beta+3}}{2(\beta+2)(\beta+1)}+\frac{t_i^4}{4}-\frac{Tt_i^3}{3}\right]+a\alpha\left[-\frac{\alpha T t_i^{2\beta+3}}{(\beta+1)(2\beta+3)}+\frac{\alpha t_i^{2\beta+4}}{(\beta+1)(2\beta+4)}\right]\}$ + g{-a[$\frac{Tt_i^5}{10}$ + $\frac{t_i^6}{18}$ − $\frac{\alpha(5\beta+4)Tt_i^{\beta+5}}{6(\beta+2)(\beta+5)}$ + $\frac{\alpha t_i^{\beta+6}}{3(\beta+3)}$] + αa[− $\frac{Tt_i^{\beta+5}}{2(\beta+3)}$ + $\frac{t_i^{\beta+5}}{3(\beta+3)}$ − $\frac{\alpha T t_i^{2\beta+5}}{(\beta+3)(2\beta+5)}$ + $\frac{\alpha t_i^{2\beta+6}}{2(\beta+3)^2}$]}+C_3 a($\frac{\alpha T t_i^{\beta+2}}{\beta+2}-\frac{\alpha t_i^{\beta+3}}{\beta+3}$) + $C_{4,i}$ { $\frac{a(T^3-t_i^3)}{6\gamma}-\frac{a(T-t_i)t_i^2}{2\gamma}+\frac{a(T-t_i)^2}{2\gamma^2}+\frac{a(1+\gamma T)(t_i-T)^2}{2\gamma^2}$ } +C_5{ $\frac{a(T^3-Tt_i^2)}{2}-\frac{a(T^3-t_i^3)}{3}-\frac{a(T^2-t_i^2)}{2\gamma}$ + $\frac{a(2+\gamma T)}{\gamma^2}(T-t_i)$ }+$C_{2,i}$a($\frac{Tt_i^2}{2}+\frac{\alpha T t_i^{\beta+2}}{\beta+2}-\frac{t_i^3}{3}-\frac{\alpha t_i^{\beta+3}}{\beta+3}$)]}}.

where Q_i =1,3,4,3,1 for i=1,2,3,4,5 respectively. ....(17)

$\frac{d\bar{G}_p}{d\ddot{t}} = \sum_{i=1}^{5} Q_i \{ \frac{1}{12T} \{f \{ -a \left[-\frac{\alpha(\beta^3+4\beta^2+\beta)(\beta+4)t_i^{\beta+3}}{3(\beta+3)(\beta+1)} + \frac{\alpha(\beta^3+3\beta^2+\beta)(\beta+3)Tt_i^{\beta+2}}{2(\beta+2)(\beta+1)} + \frac{t_i^3}{1} - \frac{Tt_i^2}{1}\right] + a\alpha \left[- \frac{\alpha T t_i^{2\beta+2}}{(\beta+1)} + \frac{\alpha t_i^{2\beta+3}}{(\beta+1)}\right]\} + g\{-a[\frac{Tt_i^4}{2} + \frac{t_i^5}{3} - \frac{\alpha(5\beta+4)Tt_i^{\beta+4}}{6(\beta+2)} + \frac{\alpha(\beta+6)t_i^{\beta+5}}{3(\beta+3)}] + \alpha a[- \frac{(\beta+5)Tt_i^{\beta+4}}{2(\beta+3)} + \frac{(\beta+5)t_i^{\beta+4}}{3(\beta+3)} - \frac{\alpha T t_i^{2\beta+4}}{(\beta+3)} + \frac{\alpha t_i^{2\beta+5}}{(\beta+3)}]\}+ C_3\ a(\frac{\alpha Tt_i^{\beta+1}}{1} - \frac{\alpha t_i^{\beta+2}}{1}) + C_{4,i} \{ -\frac{a(t_i^2)}{2\gamma} - \frac{a(2T-3t_i)t_i^1}{2\gamma} - \frac{a(T-t_i)^1}{\gamma^2} + \frac{a(1+\gamma T)(t_i-T)^1}{\gamma^2} \}+C_5\{-\frac{a(Tt_i^1)}{i} + \frac{a(t_i^2)}{i} + \frac{a(t_i^1)}{\gamma} - \frac{a(2+\gamma T)}{\gamma^2}\}+C_{2,i}\ a(\frac{Tt_i^1}{1} + \frac{\alpha Tt_i^{\beta+1}}{1} - \frac{t_i^2}{1} - \frac{\alpha t_i^{\beta+2}}{1})]\}$........(18)

$\frac{d^2\bar{G}_p}{d\ddot{t}^2} = \sum_{i=1}^{5} Q_i \{\frac{1}{12T} \{f \{ -a \left[-\frac{\alpha(\beta^3+4\beta^2+\beta)(\beta+4)t_i^{\beta+2}}{3(\beta+1)} + \frac{\alpha(\beta^3+3\beta^2+\beta)(\beta+3)Tt_i^{\beta+1}}{2(\beta+1)} + 3\frac{t_i^2}{1} - \frac{2Tt_i^1}{1}\right] + a\alpha \left[- \frac{2\alpha T t_i^{2\beta+1}}{1} + \frac{\alpha(2\beta+3)t_i^{2\beta+2}}{(\beta+1)}\right]\} + g\{-a[\frac{2Tt_i^3}{1} + \frac{5t_i^4}{3} - \frac{\alpha(5\beta+4)(\beta+2)Tt_i^{\beta+3}}{6(\beta+2)} + \frac{\alpha(\beta+6)(\beta+5)t_i^{\beta+4}}{3(\beta+3)}] + \alpha a[- \frac{(\beta+5)(\beta+4)Tt_i^{\beta+3}}{2(\beta+3)} + \frac{(\beta+5)(\beta+4)t_i^{\beta+3}}{3(\beta+3)} - \frac{\alpha(2\beta+4)Tt_i^{2\beta+3}}{(\beta+3)} + \frac{\alpha(2\beta+5)t_i^{2\beta+4}}{(\beta+3)}]\}+C_3\ a(\frac{\alpha(\beta+i)Tt_i^{\beta}}{i} - \frac{\alpha(\beta+2)t_i^{\beta+1}}{1}) + C_{4,i} \{ -\frac{a(t_i^1)}{\gamma} - \frac{a(T-3t_i)}{\gamma} + \frac{a}{\gamma^2} + \frac{a(1+\gamma T)}{\gamma^2} \}+C_5\{-\frac{a(T)}{1} + \frac{a2t_i^1}{1} + \frac{a}{\gamma}\} + C_{2,i}\ a(\frac{T}{1} + \frac{\alpha T(\beta+1)t_i^{\beta}}{1} - \frac{2t_i^1}{1} - \frac{\alpha(\beta+2)t_i^{\beta+1}}{i})]\}$ ....(19)

For minimum $\bar{G}_p$, the necessary condition is

$\frac{d\bar{G}_p}{d\ddot{t}} = 0$ After solving, we get an equation of odd degree then there exists $t_7^* \in (0,T)$ can be solved from equation (18) by using MAT Lab. also $\frac{d^2\bar{G}_p}{d\ddot{t}^2} > 0$ at $\ddot{t} = t_7^*$

$\therefore \bar{G}_p$ is minimum at $\ddot{t} = t_7^*$

SIGNED DISTANCE METHOD

Total cost is given by $\dot{S}_p(\ddot{t}) = \frac{1}{8}[A_1+2A_2+2A_3+2A_4+A_5]$

$\dot{S}_p(\ddot{t}) = \frac{A}{T} + \sum_{i=1}^{5} z_i \{ \frac{1}{8T} \{ f \{ -a \left[-\frac{\alpha(\beta^3+4\beta^2+\beta)t_i^{\beta+4}}{3(\beta+3)(\beta+1)} + \frac{\alpha(\beta^3+3\beta^2+\beta)Tt_i^{\beta+3}}{2(\beta+2)(\beta+1)} + \frac{t_i^4}{4} - \frac{Tt_i^3}{3}\right] + a\alpha \left[- \frac{\alpha T t_i^{2\beta+3}}{(\beta+1)(2\beta+3)} + \frac{\alpha t_i^{2\beta+4}}{(\beta+1)(2\beta+4)}\right]\} + g\{-a[\frac{Tt_i^5}{10} + \frac{t_i^6}{18} - \frac{\alpha(5\beta+4)Tt_i^{\beta+5}}{6(\beta+2)(\beta+5)} + \frac{\alpha t_i^{\beta+6}}{3(\beta+3)}] + \alpha a[- \frac{Tt_i^{\beta+5}}{2(\beta+3)} + \frac{t_i^{\beta+5}}{3(\beta+3)} - \frac{\alpha T t_i^{2\beta+5}}{(\beta+3)(2\beta+5)} + \frac{\alpha t_i^{2\beta+6}}{2(\beta+3)^2}]\}+C_3\ a(\frac{\alpha Tt_i^{\beta+2}}{\beta+2} - \frac{\alpha t_i^{\beta+3}}{\beta+3}) + C_{4,i} \{ \frac{a(T^3-t_i^3)}{6\gamma} - \frac{a(T-t_i)t_i^2}{2\gamma} + \frac{a(T-t_i)^2}{2\gamma^2} + \frac{a(1+\gamma T)(t_i-T)^2}{2\gamma^2} \} +C_5\{ \frac{a(T^3-Tt_i^2)}{2} - \frac{a(T^3-t_i^3)}{3} - \frac{a(T^2-t_i^2)}{2\gamma} + \frac{a(2+\gamma T)}{\gamma^2}(T-t_i)\}+C_{2,i}a(\frac{Tt_i^2}{2} + \frac{\alpha Tt_i^{\beta+2}}{\beta+2} - \frac{t_i^3}{3} - \frac{\alpha t_i^{\beta+3}}{\beta+3})]\}\}.$

where z_i =1,2,2,2,1 for i=1,2,3,4,5 respectively. ....(20)

$\frac{d\dot{S}p}{d\ddot{t}} = \sum_{i=1}^{5} z_i \{ \frac{1}{8T} \{f \{ -a \left[-\frac{\alpha(\beta^3+4\beta^2+\beta)(\beta+4)t_i^{\beta+3}}{3(\beta+3)(\beta+1)} + \frac{\alpha(\beta^3+3\beta^2+\beta)(\beta+3)Tt_i^{\beta+2}}{2(\beta+2)(\beta+1)} + \frac{t_i^3}{1} - \frac{Tt_i^2}{1}\right] + a\alpha \left[- \frac{\alpha T t_i^{2\beta+2}}{(\beta+1)} + \frac{\alpha t_i^{2\beta+3}}{(\beta+1)}\right]\} + g\{-a[\frac{Tt_i^4}{2} + \frac{t_i^5}{3} - \frac{\alpha(5\beta+4)Tt_i^{\beta+4}}{6(\beta+2)} + \frac{\alpha(\beta+6)t_i^{\beta+5}}{3(\beta+3)}] + \alpha a[- \frac{(\beta+5)Tt_i^{\beta+4}}{2(\beta+3)} + \frac{(\beta+5)t_i^{\beta+4}}{3(\beta+3)} - \frac{\alpha T t_i^{2\beta+4}}{(\beta+3)} + \frac{\alpha t_i^{2\beta+5}}{(\beta+3)}]\}+ C_3\ a(\frac{\alpha Tt_i^{\beta+1}}{1} - \frac{\alpha t_i^{\beta+2}}{1}) + C_{4,i} \{ -\frac{a(t_i^2)}{2\gamma} - \frac{a(2T-3t_i)t_i^1}{2\gamma} - \frac{a(T-t_i)^1}{\gamma^2} + \frac{a(1+\gamma T)(t_i-T)^1}{\gamma^2} \}+C_5\{-\frac{a(Tt_i^1)}{i} + \frac{a(t_i^2)}{i} + \frac{a(t_i^1)}{\gamma} - \frac{a(2+\gamma T)}{\gamma^2}\} + C_{2,i}\ a(\frac{Tt_i^1}{1} + \frac{\alpha Tt_i^{\beta+1}}{1} - \frac{t_i^2}{1} - \frac{\alpha t_i^{\beta+2}}{1})]\}$ ....(21)

$\frac{d^2\dot{S}p}{d\ddot{t}^2} = \sum_{i=1}^{5} z_i \{\frac{1}{8T} \{f \{ -a \left[-\frac{\alpha(\beta^3+4\beta^2+\beta)(\beta+4)t_i^{\beta+2}}{3(\beta+1)} + \frac{\alpha(\beta^3+3\beta^2+\beta)(\beta+3)Tt_i^{\beta+1}}{2(\beta+1)} + 3\frac{t_i^2}{1} - \frac{2Tt_i^1}{1}\right] + a\alpha \left[- \frac{2\alpha T t_i^{2\beta+1}}{1} + \frac{\alpha(2\beta+3)t_i^{2\beta+2}}{(\beta+1)}\right]\} + g\{-a[\frac{2Tt_i^3}{1} + \frac{5t_i^4}{3} - \frac{\alpha(5\beta+4)(\beta+2)Tt_i^{\beta+3}}{6(\beta+2)} + \frac{\alpha(\beta+6)(\beta+5)t_i^{\beta+4}}{3(\beta+3)}] + \alpha a[- \frac{(\beta+5)(\beta+4)Tt_i^{\beta+3}}{2(\beta+3)} + \frac{(\beta+5)(\beta+4)t_i^{\beta+3}}{3(\beta+3)} - \frac{\alpha(2\beta+4)Tt_i^{2\beta+3}}{(\beta+3)} + \frac{\alpha(2\beta+5)t_i^{2\beta+4}}{(\beta+3)}]\}+C_3\ a(\frac{\alpha(\beta+i)Tt_i^{\beta}}{i} -$

$\frac{\alpha(\beta+2)t_i^{\beta+1}}{1}) + C_{4,i} \{ -\frac{a(t_i^1)}{\gamma} - \frac{a(T-3t_i)}{\gamma} + \frac{a}{\gamma^2} + \frac{a(1+\gamma T)}{\gamma^2} \} + C_5\{-\frac{a(T)}{1} + \frac{a2t_i^1}{1} + \frac{a}{\gamma}\} + C_{2,i}\ a(\frac{T}{1} + \frac{\alpha T(\beta+1)t_i^{\beta}}{1} - \frac{2t_i^1}{1} - \frac{\alpha(\beta+2)t_i^{\beta+1}}{i})]\}$ ....(22)

For minimum $\dot{S}p$, the necessary condition is

$\frac{d\dot{S}p}{d\ddot{t}} = 0$ After solving, we get an equation of odd degree then there exists $t_1^* \in (0,T)$ can be solved from equation (21) by using MAT Lab also $\frac{d^2\dot{S}p}{d\ddot{t}^2} > 0$ at $\ddot{t} = t_8^*$

$\therefore \dot{S}_p$ is minimum at $\ddot{t} = t_8^*$

6.NUMERICAL ILLUSTRATION AND SENSTIVITY ANALYSIS

To illustrate the model we consider following numerical values of the parameters.
A= 100, a = 1000, α = 0.01, β =1, f= 2, g=3, C_1 = 5₹ ,C_2 = 30₹ , C_3 = 7₹, C_4 = 1₹ , C_5 = 0.6 ₹ , γ = 0.1 ,
For the crisp model Total Cost = 56493.79₹ and cycle time t = 0.67years

For Pentagonal fuzzy model $\tilde{C}_2$= (22,25,28,35,39),$\tilde{C}_4$ = (.5,0.8,1.1,1.3,1.5)

We Obtain following results;

Model	Method	Cycle Time	Total Inventory Cost
Crisp	Crisp method	0.67	56493.79
	Graded Mean Integration Method	0.71	49687.1
	Signed Distance Method	0.7	51512.31

7. CONCLUSION:

An Inventory model of deteriorating products in supply chain process is a very uncertain situation has been developed in which demand rate and holding cost is quadratic function of time while the deterioration follows Weibull Deterioration. Model has fuzzified and then defuzzified with the help of Graded Mean Integration Method and Signed Distance Method. In each case, Graded Mean Method provides minimum total Cost also in this case. The expansion of CORONA Virus affected both the demand and supply badly correspondingly the demand rate and customers needs and priority had changed vigorously. Since the holding cost depends on time horizon and behavior like a parabola is more relevant. The proposed model deals some realistic features likely to be associated with some kind of inventory. The model finds its application in retail business such as of fashionable cloths, domestic goods & electronic component etc.

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