Research Article

Vertex Magic Labeling On V_4 for Cartesian product of two cycles

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Abstract: Let V_4 be an abelian group under multiplication. Let $g: E(G) \rightarrow V_4$. Then the vertex magic labeling on V_4 is induced as $g^*: V(G) \rightarrow V_4$ such that $g^*(v) = \prod_u g(uv)$ where the product is taken over all edges uv of G incident at v is constant. A graph is said to be V_4 - magic if it admits a vertex magic labeling on V_4. In this paper, we prove that $C_m \times C_n$, $m \geq 3, n \geq 3$, Generalized fish graph, Double cone graph and four Leaf Clover graph are all V_4 -magic graphs.

Keyword: Vertex magic labeling on V_4, V_4 -magic graph, Four Leaf Clover Graph.

AMS subject classification (2010): 05C78

1. Introduction

For a non-trivial abelian group V_4 under multiplication a graph G is said to be V_4 -magic graph if there exist a labeling g of the edges of G with non-zero elements of V_4 such that the vertex labeling g^* defined as $g^*(v) = \prod_u g(uv)$ taken over all edges uv incident at v is a constant.

Let $V_4 = \{i, -i, 1, -1\}$ we have proved that the Cartesian product of two graphs,Generalized fish graph, Happy graph,Four Leaf Clover Graph are all
V_4 -magic graphs.

2. Basic Definition

Definition: 2.1Cartesian Product of Two graphs

Cartesian product of two graphs G, H is a new graph GH with the vertex set $V \times V$ and two vertices are adjacent in the new graph if and only if either $u = v$and u' is adjacent to v' in H or $u' = v'$ and u is adjacent to v in G.

Definition: 2.2Generalized Fish Graph

The generalized fish graph is defined as the one point union of any even cycle with C_3. It is denoted by $GF(2n, 3)$. It has $2n + 2$ vertices and $2n + 3$ edges.

Theorem: 2.3 Cartesian product of two cycles $C_m \times C_n$ is a V_4-magic graph with $m, n \geq 3$.

Proof:

Let $V(C_m \times C_n) = \{v_j: 1 \leq j \leq m\} \cup \{v_j' : 1 \leq j \leq m\} \cup$

$$\cup \{v_j'' : 1 \leq j \leq m\} \cup \{v_j''': 1 \leq j \leq m\}$$

$$E(C_m \times C_n) = \{v_j v_{j+1}: 1 \leq j \leq m\} \cup \{v_j' v_{j+1}': 1 \leq j \leq m\} \cup$$

$$\cup \{v_j'' v_{j+1}'' : 1 \leq j \leq m\} \cup \{v_j''' v_{j+1}''' : 1 \leq j \leq m\} \cup$$

$$\cup \{v_j v_j' : 1 \leq j \leq m\} \cup \{v_j' v_j'' : 1 \leq j \leq m\} \cup$$

$$\cup \{v_j'' v_j''' : 1 \leq j \leq m\} \cup \{v_j''' v_j: 1 \leq j \leq m\}$$

$$[v_{m+1} = v_1;\ v_{m+1}' = v_1';\ v_{m+1}'' = v_1'';\ v_{m+1}''' = v_1''';\ v_0 = v_m;\ v_0' = v_m';$$
$$v_0'' = v_m'';\ v_0''' = v_m''']$$

Case 1: Let $m, n \geq 3$ and both are even.

Let us define $g: E(C_m \times C_n) \rightarrow \{i, -i, -1\}$ as

$$g(v_j v_{j+1}) = i when j is odd\ ;\ 1 \leq j \leq m$$
$$g(v_j v_{j+1}) = -i when j is even\ ;\ 1 \leq j \leq m$$
$$g(v_j' v_{j+1}') = i when j is odd\ ;\ 1 \leq j \leq m$$
$$g(v_j' v_{j+1}') = -i when j is even\ ;\ 1 \leq j \leq m$$
$$g(v_j'' v_{j+1}'') = i when j is odd\ ;\ 1 \leq j \leq m$$
$$g(v_j'' v_{j+1}'') = -i when j is even\ ;\ 1 \leq j \leq m$$

$$g(v_j'''v_{j+1}''') = iwhenjisodd\ ;\ 1 \le j \le m$$
$$g(v_j'''v_{j+1}''') = -iwhenjiseven\ ;\ 1 \le j \le m$$
$$g(v_jv_j') = -1\ ;\ 1 \le j \le m$$
$$g(v_j'v_j'') = -1\ ;\ 1 \le j \le m$$
$$g(v_j''v_j''') = -1\ ;\ 1 \le j \le m$$
$$g(v_j'''v_j) = -1\ ;\ 1 \le j \le m$$

Now $g^*: V(C_m \times C_n) \to i, -i, -1$ is given by

$$g^*(v_j) = g(v_jv_{j+1}) * g(v_jv_j') * g(v_jv_{j-1}) * g(v_jv_j''')$$
$$= (i) * (-i) * (-1) * (-1)$$
$$= 1\ ;\ 1 \le j \le m$$
$$g^*(v_j') = g(v_j'v_{j+1}') * g(v_j'v_{j-1}') * g(v_j'v_j'') * g(v_j'v_j)$$
$$= (-i) * (-i) * (-1) * (-1)$$
$$= 1\ ;\ 1 \le j \le m$$
$$g^*(v_j'') = g(v_j''v_{j+1}'') * g(v_j''v_{j-1}'') * g(v_j''v_j''') * g(v_j''v_j')$$
$$= (i) * (-i) * (-1) * (-1)$$
$$= 1\ ;\ 1 \le j \le m$$
$$g^*(v_j''') = g(v_j'''v_{j+1}''') * g(v_j'''v_{j-1}''') * g(v_j'''v_j) * g(v_j'''v_j'')$$
$$= (i) * (-i) * (-1) * (-1)$$
$$= 1\ ;\ 1 \le j \le m$$

Thus we get $g^*(v_j) = g^*(v_j') = g^*(v_j'') = g^*(v_j''')$= 1; 1≤ j≤ m

Hence when m, n are both even we can conclude that $C_m \times C_n$, satisfy vertex magic labelling on V_4. And Hence its a V_4-magic graph.

Case 2: When both m and n are odd

Let us define $g: E(C_m \times C_n,) \to \{i, -i, -1\}$ as

$$g(v_jv_{j+1}) = -i\ ;\ 1 \le j \le m$$
$$g(v_j'v_{j+1}') = -i\ ;\ 1 \le j \le m$$
$$g(v_j''v_{j+1}'') = -i\ ;\ 1 \le j \le m$$
$$g(v_j''v_{j+1}''') = -i\ ;\ 1 \le j \le m$$
$$g(v_j^{IV}v_{j+1}^{IV}) = -i\ ;\ 1 \le j \le m$$
$$g(v_j^{V}v_{j+1}^{V}) = -i\ ;\ 1 \le j \le m$$
$$g(v_j^{VI}v_{j+1}^{VI}) = -i\ ;\ 1 \le j \le m$$
$$g(v_jv_j') = -i\ ;\ 1 \le j \le m$$
$$g(v_j'v_j'') = -i\ ;\ 1 \le j \le m$$
$$g(v_j''v_j''') = -i\ ;\ 1 \le j \le m$$
$$g(v_j'''v_j^{IV}) = -i\ ;\ 1 \le j \le m$$
$$g(v_j^{IV}v_j^{V}) = -i\ ;\ 1 \le j \le m$$
$$g(v_j^{V}v_j^{VI}) = -i\ ;\ 1 \le j \le m$$

Now $g^*: V(C_m \times C_n,) \to \{i, -i, -1\}$ is given by

$$g^*(v_j) = g(v_jv_{j+1}) * g(v_jv_{j-1}) * g(v_jv_j') * g(v_jv_j^{VI})$$
$$= (-i) * (-i) * (-i) * (-i)$$
$$= 1\ ;\ 1 \le j \le m$$
$$g^*(v_j') = g(v_j'v_{j+1}') * g(v_j'v_{j-1}') * g(v_j'v_j'') * g(v_j'v_j)$$
$$= (-i) * (-i) * (-i) * (-i)$$
$$= 1\ ;\ 1 \le j \le m$$
$$g^*(v_j'') = g(v_j''v_{j+1}'') * g(v_j''v_{j-1}'') * g(v_j''v_j''') * g(v_j''v_j')$$
$$= (-i) * (-i) * (-i) * (-i)$$
$$= 1\ ;\ 1 \le j \le m$$
$$g^*(v_j''') = g(v_j'''v_{j+1}''') * g(v_j'''v_{j-1}'''') * g(v_j'''v_j^{IV}) * g(v_j'''v_j'')$$
$$= (-i) * (-i) * (-i) * (-i)$$
$$= 1\ ;\ 1 \le j \le m$$
$$g^*(v_j^{IV}) = g(v_j^{IV}v_{j+1}^{IV}) * g(v_j^{IV}v_{j-1}^{IV}) * g(v_j^{IV}v_j''') * g(v_j^{IV}v_j^{V})$$
$$= (-i) * (-i) * (-i) * (-i)$$
$$= 1\ ;\ 1 \le j \le m$$
$$g^*(v_j^{V}) = g(v_j^{V}v_{j+1}^{V}) * g(v_j^{V}v_{j-1}^{V}) * g(v_j^{V}v_j^{IV}) * g(v_j^{V}v_j^{VI})$$

$= (-i) * (-i) * (-i) * (-i)$
$= 1 ; 1 \leq j \leq m$

$$g^* (v_j^{VI}) = g(v_j^{VI} v_{j+1}^{VI}) * g(v_j^{VI} v_{j-1}^{VI}) * g(v_j^{VI} v_j^{V}) * g(v_j^{VI} v_j)$$

$= (-i) * (-i) * (-i) * (-i)$
$= 1 ; 1 \leq j \leq m$

Hence We can conclude that $C_m \times C_n$, is a V_4-magic graph when both m and n are odd as it satisfies vertex magic labelling on V_4. We can also prove this case by labelling each vertex of $C_m \times C_n$, with i we get $g^* (v_j) = 1; 1 \leq j \leq m$ throughout the graph in each cycle.

Also we can prove this case by labelling each vertex of $C_m \times C_n$, with -1 we get $g^* (v_j) = 1; 1 \leq j \leq m$ throughout the graph in each cycle.

Case 3: Let m be even and n be odd

Let us define $g: E(C_m \times C_n,) \rightarrow \{i, -i, -1\}$ as

$$g(v_j v_{j+1}) = iwhenjisodd ; 1 \leq j \leq m$$
$$g(v_j v_{j+1}) = -iwhenjiseven ; 1 \leq j \leq m$$
$$g(v_j' v_{j+1}') = iwhenjisodd ; 1 \leq j \leq m$$
$$g(v_j' v_{j+1}') = -iwhenjiseven ; 1 \leq j \leq m$$
$$g(v_j'' v_{j+1}') = iwhenjisodd ; 1 \leq j \leq m$$
$$g(v_j'' v_{j+1}'') = -iwhenjiseven ; 1 \leq j \leq m$$
$$g(v_j v_{j+1}''') = iwhenjisodd ; 1 \leq j \leq m$$
$$g(v_j v_{j+1}''') = -iwhenjiseven ; 1 \leq j \leq m$$
$$g(v_j^{IV} v_{j+1}^{IV}) = iwhenjisodd ; 1 \leq j \leq m$$
$$g(v_j^{IV} v_{j+1}^{IV}) = -iwhenjiseven ; 1 \leq j \leq m$$
$$g(v_j v_j') = -1 ; 1 \leq j \leq m$$
$$g(v_j' v_j'') = -1 ; 1 \leq j \leq m$$
$$g(v_j'' v_j''') = -1 ; 1 \leq j \leq m$$
$$g(v_j''' v_j^{IV}) = -1 ; 1 \leq j \leq m$$
$$g(v_j^{IV} v_j) = -1 ; 1 \leq j \leq m$$

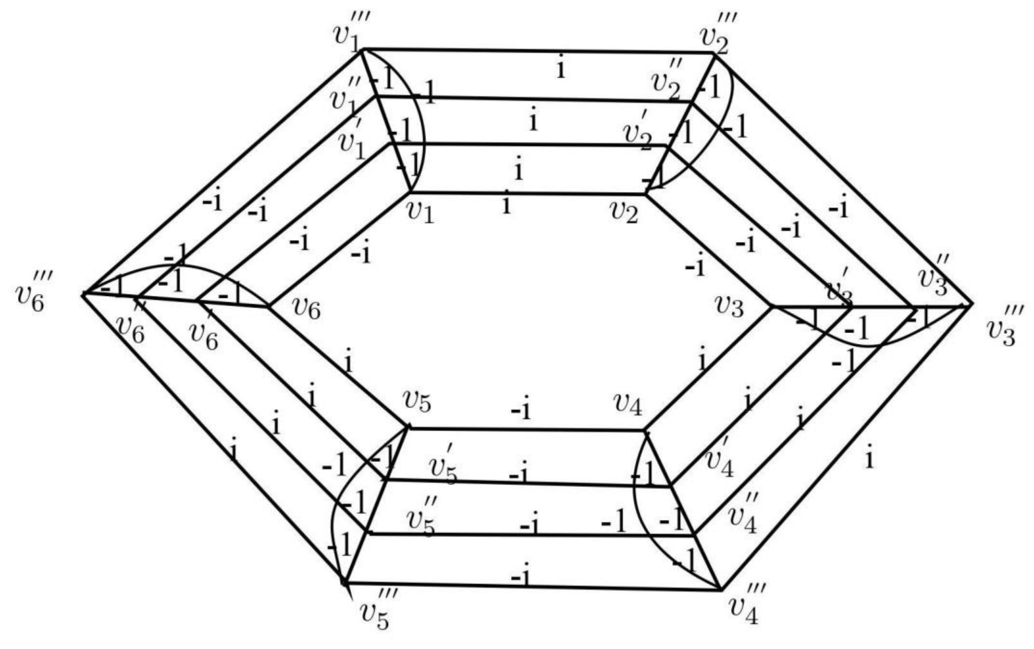

Figure 1$C_6 \times C_4$

Now $g^*: V(C_m \times C_n,) \rightarrow \{i, -i, -1\}$ is given by

$$g^* (v_j) = g(v_j v_{j+1}) * g(v_j v_{j-1}) * g(v_j v_j') * g(v_j v_j^{IV})$$

$= (i) * (-i) * (-1) * (-1)$

$= 1\ ;\ 1 \le j \le m$

$g^*\,(v_j')= g(v_j'v_{j+1}) * g(v_j'v_{j-1}) * g(v_j'v_j'') * g(v_j'v_j)$

$= (i) * (-i) * (-1) * (-1)$

$= 1\ ;\ 1 \le j \le m$

$g^*\,(v_j'')= g(v_j''v_{j+1}'') * g(v_j''v_{j-1}'') * g(v_j''v_j''') * g(v_j''v_j')$

$= (i) * (-i) * (-1) * (-1)$

$= 1\ ;\ 1 \le j \le m$

$g^*\,(v_j''')= g(v_j'''v_{j+1}''') * g(v_j'''v_{j-1}''') * g(v_j'''v_j^{IV}) * g(v_j'''v_j'')$

$= (i) * (-i) * (-1) * (-1)$

$= 1\ ;\ 1 \le j \le m$

$g^*\,(v_j^{IV})= g(v_j^{IV}v_{j+1}^{IV}) * g(v_j^{IV}v_{j-1}^{IV}) * g(v_j\ \hat{}IV v_j'''\ \hat{}''') * g(v_j^{IV}v_j)$

$= (i) * (-i) * (-1) * (-1)$

$= 1\ ;\ 1 \le j \le m$

So we can say that $C_m \times C_n$, is a V_4- magic graph even when m is even and n is odd as it satisfies vertex magic labelling on V_4. Hence from all three cases we can conclude that the Cartesian product $C_m \times C_n$, is a V_4- magic graph by satisfying vertex magic labelling on V_4.

Case (1):
Both $m\&n$ are even ; m=6 and n=4

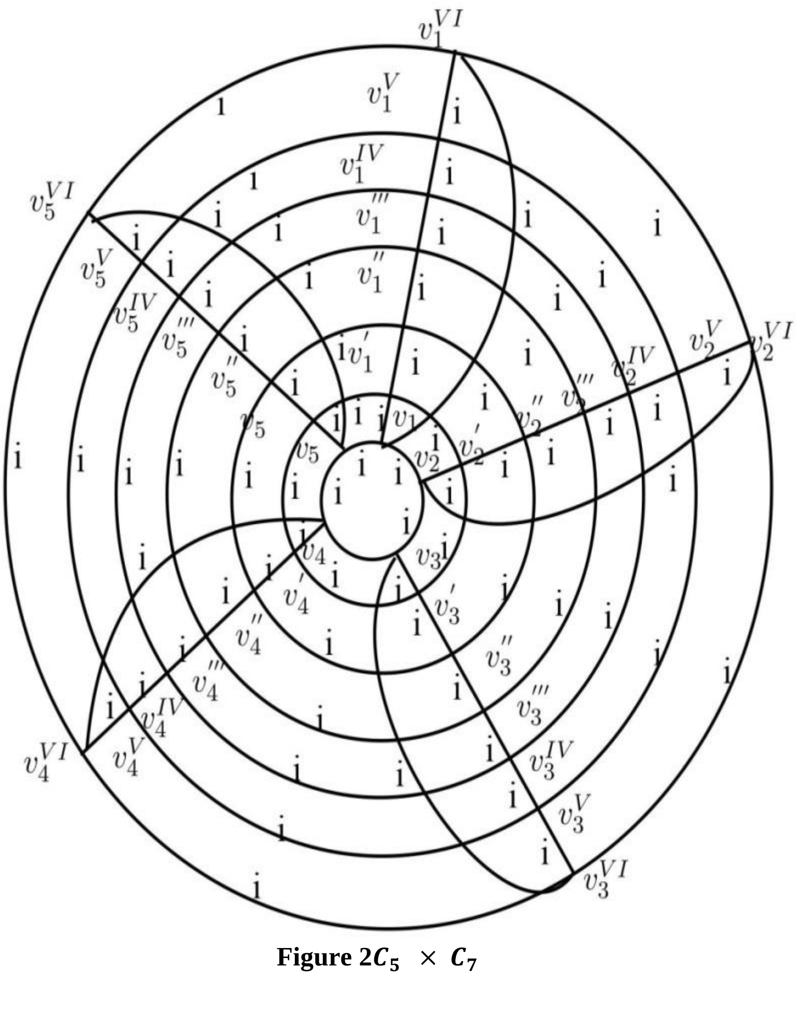

Figure 2C_5 × C_7

It is illustrated in the Figure 1
Case (2):
When both m and n are odd.
Let $m = 5;\ n = 7\ \ (C_5 \times C_7)$
It is illustrated in the Figure 2

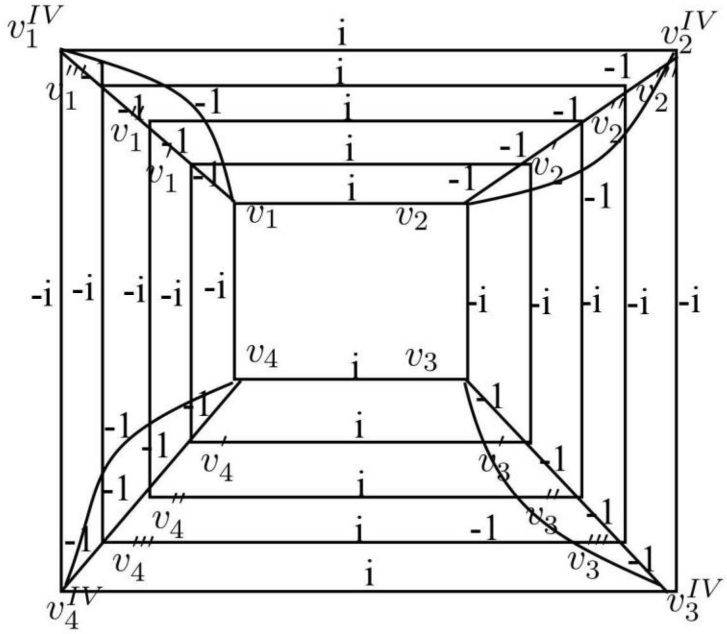

Figure 3: $C_4 \times C_5$

Case (3) :
When m is even and n is odd.
Let m=4; n=5
It is illustrated in the Figure 3

Theorem: 2.4
Generalized fish graph $GF(n,3)$ is a V_4-magic graph for all $n \geq 4$ and n is even.
Proof:
Let $n \geq 4$ and n is even.
Let $V(GF(n,3)) = \{v_j : 1 \leq j \leq n+2\}$ and

$$E(GF(n,3)) = \{v_j v_{j+1} \ : 1 \leq j \leq n \cup v_{\frac{n}{2}+1}v', v_{\frac{n}{2}+1}v^2, v'v^2\}$$
$$[v_{n+1} = v_1;\ v_0 = v_n]$$

Let us define $g: E(GF(n,3)) \rightarrow \{i,-i,-1\}$ as

$$g(v_j v_{j+1}) = iwhenjisodd\ ;\ 1 \leq j \leq n$$
$$g(v_j v_{j+1}) = -iwhenjiseven\ ;\ 1 \leq j \leq n$$

and $g(v_{\frac{n}{2}+1}v') = g(v_{\frac{n}{2}+1}v^2) = g(v'v^2) = -1$
Now $g^*: V((GF(n,3))) \rightarrow \{i,-i,-1\}$ is given by

$$g^*(v_j) = g(v_j v_{j+1}) * g(v_{j-1}v_j);\ 1 \leq j \leq \frac{n}{2}; \frac{n}{2} \leq j \leq n$$
$$= (i) * (-i)$$
$$= 1$$
$$g^*(v_{\frac{n}{2}+1}) = g(v_{\frac{n}{2}}v_{\frac{n}{2}+1}) * g(v_{\frac{n}{2}+1}v_{\frac{n}{2}+2}) * g(v_{\frac{n}{2}+1}v') * g(v_{\frac{n}{2}+1}v^2)$$
$$= (-i) * (i) * (-1) * (-1)$$
$$= 1$$
$$g^*(v') = g(v_{\frac{n}{2}+1}v') * g(v'v^2) = 1$$
$$g^*(v^2) = g(v_{\frac{n}{2}+1}v^2) * g(v'v^2) = 1$$

So throughout $GF(n,3)$ each vertex is equal to the value 1. Hence it admits vertex magic labelling on V_4.
Thus Generalised Fish graph $GF(n,3)$ is said to be a V_4- magic graph.

□

Example: 2.5 $GF(8,3)$

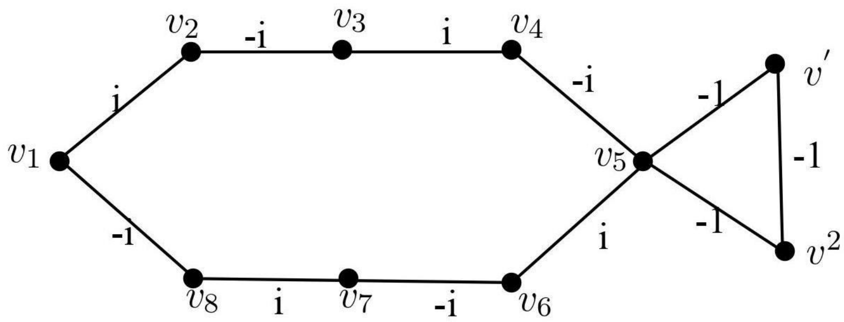

Figure 4 $\boldsymbol{GF(8,3)}$

Four Leaf Clover Graph

Four leaf Clover graph is formed by the combination of a cycle C_8 and a path P_{2n+1} such that the end vertices of the path are attached to a vertex of the cycle.

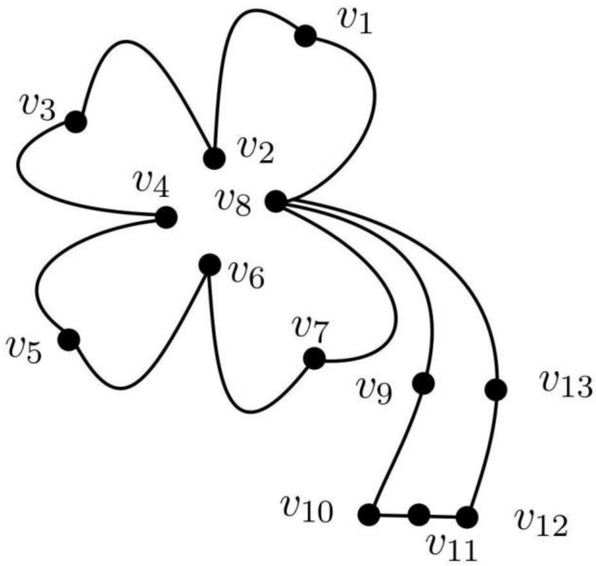

Figure 5

Theorem: 2.6

Four Leaf Clover (FLC) graph is a V_4-magic graph.

Proof:

Let $V(FLC) = \{v_j : 1 \le j \le 8\} \cup \{u_i : 1 \le i \le 2n+1, n \ge 2, n \in N\}$ and

$$E(FLC) = \{v_j v_{j+1} : 1 \le j \le 8\} \cup \{v_8 u_1, v_8 u_{2n} + 1\} \cup \{u_i u_{i+1} : 1 \le i \le 2n1, n \ge 2\}$$

$$[v_0 = v_8 \ ; \ v_9 = v_1 \ ; \ u_{2n+2} = v_8]$$

Let us define $g: E(FLC) \to \{1, -i, -1\}$ as

$$g(v_j v_{j+1}) = i, whenjisodd$$

$$g(v_j v_{j+1}) = -i, whenjiseven$$

$$g(v_8 u_1) = -i$$

$$g(v_8 u_{2n+1}) = i, n \ge 2$$

$$g(u_i u_{i+1}) = i, wheniisodd, i \le 2n+1, n \ge 2$$

$$g(u_i u_{i+1}) = -i, wheniiseven$$

Now $g^*: V(FLC) \to \{i, -i, -1\}$ is given by

$$g^*(v_j) = g(v_j v_{j+1}) * g(v_{j-1} v_j); 1 \le j < 8$$

$$= (i) * (-i) = 1$$

$$g^*(v_8) = g(v_7 v_8) * g(v_8 u_1) * g(v_8 u_{2n+1}) * g(v_8 v_1)$$

$$= (i) * (-i) * (i) * (-i)$$

$= 1$

$$g^*(u_i) = g(u_i u_{i+1}) * g(u_{i-1}u\_i\ ); 2 \le i < 2n$$

$= (-i) * (i) = 1$

$g^*(u_1) = g(u_1 v_8) * g(u_1 u_2)$

$= (-i) * (i) = 1$

$g^*(u_{2n+1}) = g(u_{2n} u_{2n+1}) * g(u_{2n+1} v_8)$

$= (-i) * (i) = 1$

Thus $g^*(v_j) = 1 ; 1 \le j < 8$

$$g^*(u_i) = 1 ; 1 \le i \le 2n + 1$$

Therefore four Leaf Clover graph is a V_4- magic graph as it satisfies vertex magic labeling on V_4.

□

Example: FLC

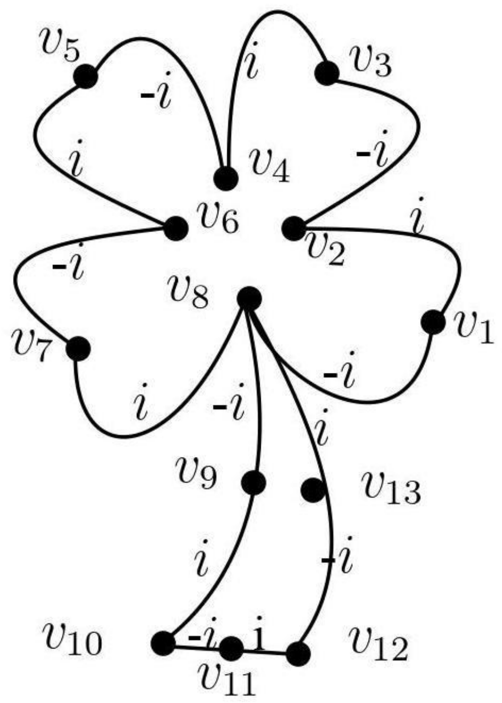

Figure 5

Theorem: 2.6 Double Cone DC_n; $n \ge 3$ is a V_4-magic graph.

Proof: Let $n \ge 3$

Case (i): n is even

Let $V(DC_n) = \{v_j: 1 \le j \le n\} \cup \{v^1, v^2\}$ and

$$E(DC_n) = \{v_j v_{j+1} : 1 \le j \le n\} \cup \{v^1 v_j: 1 \le j \le n\} \cup \{v^2 v_j: 1 \le j \le n\}$$

$$[v_{n+1} = v_1;\ v_{j-1} = v_n]$$

Let us define $g: E(DC_n) \to \{i, -i, -1\}$ as

$$g(v_j v_{j+1}) = i, \quad whenjisodd ,1 \le j \le n$$

$$g(v_j v_{j+1}\ ) = -i, \quad whenjiseven, 1 \le j \le n$$

$$g(v_j v') = i,\ \ 1 \le j \le n$$

$$g(v_j v^2\ ) = -i, \quad 1 \le j \le n$$

Now $g^*: V(DC_n) \to \{i, -i, -1\}$ is given by

$$g^*(\ v_j) = g(v_j v_{j+1}\ ) * g(v_{j-1} v_j\ ) * g(v_j v_j') * g(v_j v^2)$$

$= (i) * (-i) * (i) * (-i)$

$= 1; 1 \le\ j \le\ n$

$$g^*(v') = g(v_1 v') * g(v_2 v') * g(v_3 v') * \cdots * g(v_n v')$$

$= (i) * \cdots * (i)$

$= 1$

$$g^*(v^2) = g(v_1v^2) * g(v_2v^2) * g(v_3v^2) * \cdots * g(v_nv^2)$$

$= (-i) * (-i) * \cdots * (-i)$

$= 1$

□

Example: DC_8

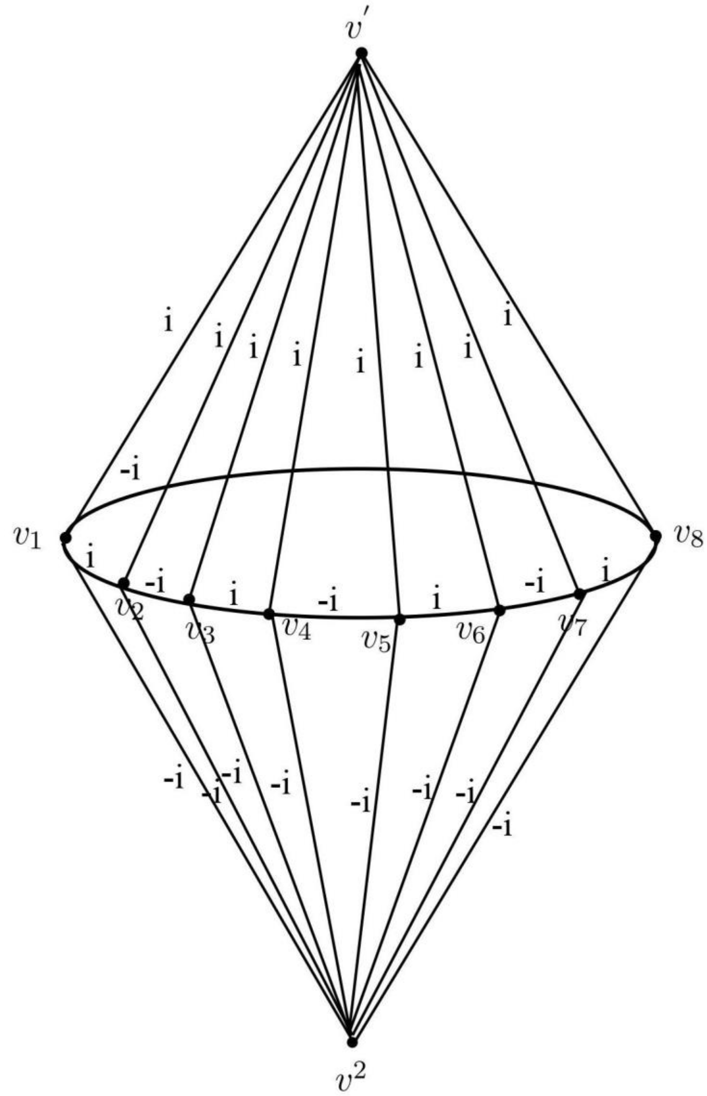

Figure 6: DC_8

Case (ii): $n$is odd

Let $V(DC_n) = \{v_j : 1 \le j \le n\} \cup \{v^1, v^2\}$ and

$$E(DC_n) = \{v_jv_{j+1} : 1 \le j \le n\} \cup \{v^1v_j : 1 \le j \le n\} \cup \{v^2v_j : 1 \le j \le n\}$$

$$[v_{n+1} = v_1 ;\ v_{j-1} = v_n]$$

Let us define $g: E(DC_n) \to \{i, -i, -1\}$ as

$$g(v_jv_{j+1}) = i ;\ 1 \le j \le n$$

$$g(v_jv^1) = -1 ;\ 1 \le j \le n$$

$$g(v_jv^2) = -1 ;\ 1 \le j \le n$$

Now $g^*: V(DC_n) \to \{i, -i, -1\}$ is given by

$$g^*(v_j) = g(v_jv_{j+1}) * g(v_{j-1}v_j) * g(v_jv^1) * g(v_jv_j^2)$$

$= (i) * (i) * (-1) * (-1)$

$= -1 ;\ 1 \le j \le n$

$$g^*(v^1) = g(v_1v^1) * g(v_2v^1) * \cdots * g(v_nv^1)$$
$$= (-1) * (-1) * \cdots * (-1) * (-1)$$
$$= -1$$
$$g^*(v^2) = g(v_1v^2) * g(v_2v^2) * \cdots * g(v_nv^2)$$
$$= (-1) * (-1) * \cdots * (-1) * (-1)$$
$$= -1$$

So when n is even, we get the constant value 1 at each vertex and when n is odd, we get the constant value -1 at each vertex.

Thus $DC_n$is a V_4-magic graph as it admits vertex magic labeling on V_4.

□

Example: DC_9

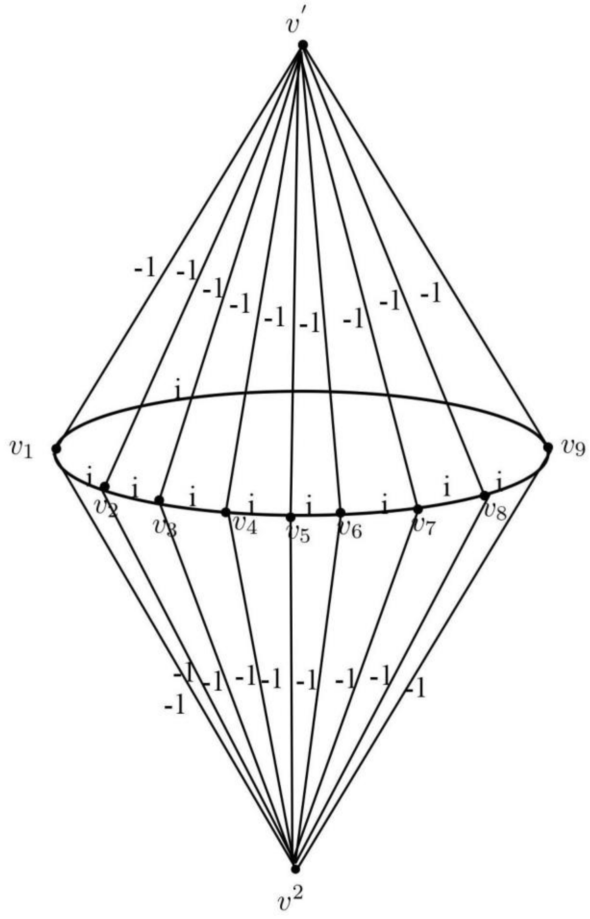

Figure 9: DC_9

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