Hyper Brk-Algebras

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Abstract: In this article, BRK-Algebras presents the idea of hyper structure and some related properties are discussed. In addition, we present the concept of the hyper BRK-ideal of a hyper BRK-algebra. A few characteristics are obtained. Furthermore the concept of homomorphism is generalised to hyper BRK-Algebras.

Index Terms. Hyper BRK-algebra, hyper BRK-ideal, homomorphism of BRK-Algebras.

I. Introduction

Ravi Kumar Bandaru[1] introduced a new algebraic structure which is called BRK-algebra. He introduced the concept of *G*-part, *p*-radical, and medial of a BRK-algebra and studied their properties. Ever since F.Marty [5] initiated the study of hyperstructures (multi algebras or poly algebras) in 1934, the study of hyperalgebras gained much importance as these have many applications in several areas of pure and applied sciences. Hypergroups studied by J. Janstosciak [4] in 1997, hyperrings studied by R.Rosaria [6] in 1996, hyper BCK-algebras studied by Xin Xiao-long [7] in 2001, hyperlattices and hypersemilattices studied by Zhao Bin and others [1] are some important hyperstructures studied so far. Zhao Bin [1] introduced the notions of ideal, hyperorder and observed several properties of these. Since the operations applied here are not the usual n-ary operations in the universal algebra, the results of universal algebra can not be applied directly to these hyperstructures.

In this article, BRK-Algebras presents the idea of hyper structure and some related properties are discussed. In addition, we present the concept of the hyper BRK-ideal of a hyper BRK-algebra. A few characteristics are obtained. Furthermore the concept of homomorphism is generalised to hyper BRK-Algebras.

II. Preliminaries

This section presents the definitions and the results which have been already proven for our use in the next sections.

Definition II.1 [1] Let *S alone a non void set and let P(S) means the power set of S*, $P^*(S) = P(S) \setminus \{\emptyset\}$. *A binary hyperoperation "$\otimes$" on S is a function from* $S \times S \rightarrow P^*(S)$ *satisfying the following conditions:*

$$(1)\ a \otimes B = \bigcup_{b\in B} (a \otimes b),$$

$$(2)\ B \otimes a = \bigcup_{b\in B} (b \otimes a),$$

$$(3)\ A \otimes B = \bigcup_{a\in A, b\in B} (a \otimes b) \quad \text{for } a, b \in S \text{ and } A, B \in P^*(S).$$

Definition II.2 [1] A BRK-algebra is a non void set B with a constant 0 and satisfying axioms of a binary operation $*$:

(1) $m * 0 = m$,

(2) $(m * n) * m = 0 * n$ for any $m, n \in B$.

For curtness, we additionally call B a BRK-algebra. In B, we will characterize a binary relation " $\leq$ by $m \leq n$ if and just if $m * n = 0$.

Definition II.3. A hyper BRK-Algebra $(B, \otimes)$ may be a nonempty set with a constant "0" and a hyperoperation $\otimes$ fulfilling

(HB1) $m \otimes 0 = \{m\}$,

(HB2) $(m \otimes n) \otimes m = 0 \otimes n$, for any $m, n \in B$.

Here $m << n$ is characterized by $0 \in m \otimes n$. $M << N$ is characterized concerning all $m \in M$, there exists $n \in N$ with the end goal that $m << n$.

Example II.3 Let B = {0, p, q , r} and define the hyperoperation $\otimes$ as follows:

$\otimes$	0	P	q	R
0	{0}	{p}	{0}	{p}
p	{p}	{0}	{p}	{0}
q	{q}	{p}	{0}	{p}
r	{r}	{q}	{r}	{0}

Then (B, $\otimes$,0) is a hyper BRK-Algebra.

We refer to {a} simply as a" in this article.

Proposition II.4. On the off chance that (B , $\otimes$, 0) *is a hyper BRK-Algebra, at that point, for various x, y* $\in$ *B, the accompanying circumstances hold:*

(1) $0 \otimes 0 = \{0\}$,

(2) $x \otimes x = \{0\}$,

(3) $x \otimes y = 0 \Rightarrow 0 \otimes x = 0 \otimes y$.

(4) $A \otimes 0 = A$.

(5) $(x \otimes 0) \otimes x = 0$.

(6) $0 \otimes (x \otimes y) = (0 \otimes x) \otimes (0 \otimes y)$

(7) $x << 0$ *imply* $x = 0$.

(8) $A << A$

(9) $A \subseteq B \Rightarrow A << B$.

(10) $A << \{0\} \Rightarrow A = \{0\}$.

(11) $x \otimes \{0\} << \{y\} \Rightarrow x << y$.

(12) $A \otimes \{0\} = \{0\} \Rightarrow A = \{0\}$.

(13) $x \otimes (x \otimes 0) = \{x\}$.

Proof : Let (B, $\otimes$, 0) *is a hyper BRK-Algebra* and *x, y* $\in$ B. At that point

(1) $0 \otimes 0 = \{0\}$.

(2) $x \otimes x = (x \otimes 0) \otimes x = 0 \otimes 0 = 0$.

(3) $x \otimes y = 0$. By HB2, $(x \otimes y) \otimes x = 0 \otimes y \Rightarrow 0 \otimes x = 0 \otimes y$.

(4) $A \otimes 0 = \bigcup_{a \in A} a \otimes 0 = \bigcup_{a \in A} a = A$.

(5) $(x \otimes 0) \otimes x = 0 \otimes 0 = 0$. *(By HB2 and (1)).*

(6) $0 \otimes (x \otimes y) = ((0 \otimes y) \otimes (x \otimes y)) \otimes (0 \otimes y) = (((x \otimes y) \otimes x) \otimes (x \otimes y)) \otimes (0 \otimes y) = (0 \otimes x) \otimes (0 \otimes y)$.

(7) $x << 0 \Rightarrow 0 \in x \otimes 0 = \{x\} \Rightarrow x = 0$.

(8) By (2) $x \otimes x = \{0\}$ that is $0 \in x \otimes x \Rightarrow x << x$. Hence $A << A$.

(9) Since $A \subseteq B \Rightarrow \forall a \in A \exists b = a \in B \ni a << b$. *By(8)* $A \subseteq B \Rightarrow A << B$.

(10) Suppose, $A << \{0\} \Rightarrow \forall a \in A \exists b = 0 \ni a << b. a << 0 \Rightarrow a = 0 \forall a \in A \Rightarrow A = \{0\}$.

(11) By definition.

(12) By definition.

(13) By definition.

Proposition II.5 *Let S alone a subset of a hyper BRK-algebra* (B, $\otimes$,*0*) *and let p, q, r* ∈ B. *In the event that* (p $\otimes$ *q*) $\otimes$ *r<< B, at that point b* $\otimes$ *r << B for all b* ∈ p $\otimes$ *q.*

P roof. Straightforward.

Definition II.6. Let (B, $\otimes$,*0*) be a hyper *BRK*-Algebra and let *S* be a subset of B containing 0. If *S* is a hyper *BRK*-Algebra with respect to the hyper operation "$\otimes$" on B, we state that *S* is a *hyper subalgebra* of B .

Proposition II.7 *Leave S alone a non-void subset of a hyper BRK-Algebra* (B, $\otimes$, *0*). In the event that p $\otimes$ *q* ⊆ *S for all p ,q* ∈ *S, at that point* 0 ∈ *S.*

Proof. Accept that p $\otimes$ q ⊆ *S* for all p , *q* ∈ *S* and let p ∈ *S*. Since p $\otimes$ *p* ⊆ *S*, we have 0 ∈ p $\otimes$ *p* ⊆ *S*. what's more, we are finished.

Theorem II.8 *Leave S alone a non-void subset of a hyper BRK-Algebra* (B, $\otimes$,*0*). *At that time S is a hyper subalgebra of B if and simply if p* $\otimes$ *q* ⊆ *S for all p, q* ∈ *S.*

Proof. Assume *S , a hyper subalgebra of B.* Since *S is a hyper subalgebra, "*$\otimes$*" on S is a function from S × S* → *P*(S*). *Hence p* $\otimes$ q ⊆ *S*. Conversely, suppose that p $\otimes$ q ⊆ *S* for all p, *q* ∈ *S*. Then 0 ∈ *S* by Proposition II.7, for any p , *q* ∈ *S* suggests p, *q* ∈ B *and B is hyper BRK-Algebra then* (*p* $\otimes$ *q)* $\otimes$ *p = 0* $\otimes$ *q*. Therefore *S* is a hyper subalgebra of B.

III.HYPER *BRK*-IDEALS OF HYPER *BRK*-ALGEBRAS

Definition III.1. Leave I alone a non-void subset of a hyper BRK-algebra (B, $\otimes$, 0) .Then I supposed to be a hyper hyper BRK-ideal of B if

(1) 0 ∈ I,

(2) p $\otimes$ q << I and q ∈ I infer p ∈ I for all p, q ∈B.

Definition III.2 Leave I alone a nonempty subset of a hyper BRK-algebra B. At that point I is known as a weak hyper BRK-ideal of B if

(1) 0 ∈ I,

(2) p $\otimes$ q ⊆ I and q ∈ I infer p ∈ I for all p , q ∈ B.

Theorem III.3. Let (B, $\otimes$,0) be a hyper BRK-Algebra. At that point every hyper BRK- ideal of B will be a weak hyper BRK-ideal of B.

Definition III.4 Let I be a non-void subset of a hyper BRK-Algebra B. At that point I is known as strong hyper BRK-ideal of B if it satisfies

(1) 0 ∈ I,

(2) (p $\otimes$ q) ∩ I $\neq \phi$ and q ∈ I suggest p ∈ I for all p, q ∈ B.

Note that each ideal I of a hyper BRK-Algebra (B, $\otimes$,0) is a strong hyper BRK-ideal of B.

Theorem III.5 Let I alone a strong hyper BRK-ideal of a hyper BRK-Algebra B. At that point

(i) I is a weak hyper BRK-ideal in B ,

(ii) I is a hyper BRK-ideal in B.

Proof: Since every hyper BRK- ideal of B is a weak hyper BRK-ideal of B. It is sufficient to show (ii). Let p, q∈ B be with the end goal that $p \otimes q << I$ and q ∈ I. For each m∈$p \otimes q$ there exists n∈I sich that m<<n.That is 0 in $m \otimes n$. $\Rightarrow 0 \in (m \otimes n) \cap I \neq \phi$. Then by the definition of strong hyper BRK-ideal m∈I. $. \Rightarrow p \otimes q \subseteq I and (p \otimes q) \cap I \neq \phi$. Then by the definition of strong hyper BRK-ideal p∈I.

Definition III.6. A hyper BRK-ideal I of B is supposed to be reflexive if $p \otimes p \subseteq I$ for all p ∈B.
Lemma III.7. Let P, Q R and J be subsets of B.

(i) If P ⊆Q<<R, then P<<R.

(ii) If $P \otimes x$<< J for x ∈ B, then $a \otimes x$<<J for all a ∈P.

(iii) If J is a hyper BRK-ideal of B and if $P \otimes x$<<J for x ∈ J, then P<<J.

Theorem III.8. Leave J alone a reflexive hyper BRK-ideal of a hyper BRK-algebra B. At that point $(x \otimes y) \cap I \neq \phi$ implies $p \otimes q$<< J for all p, q ∈ B.

Proof. Let p, q ∈B with the end goal that $(p \otimes q) \cap J \neq \phi$. Then t∈ $(p \otimes q) \cap$ I. $\Rightarrow (p \otimes q) \otimes t \subseteq (p \otimes q) \otimes (p \otimes q) << p \otimes p \subseteq J$. Hence $(p \otimes q) \otimes t << J$, By **Lemma III.7,** $p \otimes q << J$.

Theorem III.9. Leave J alone a reflexive hyper BRK-ideal of hyper BRK-algebra B and let H be a subset of B. If H<<J, then H ⊆ J.

Proof. Accept that H<< J and let p∈H. At that point there exists q ∈ J such that p<< q, that is $0 \in p \otimes q$. $0 \in (p \otimes q) \cap J \Rightarrow (p \otimes q) \cap J \neq \phi. \Rightarrow (p \otimes q) << J$.by **Theorem III.8**. It follows from definition of hyper BRK-ideal that p ∈ J so H ⊆ J.

Corollary III.10. Let J be a reflexive hyper BRK-ideal of hyper BRK-algebra B. Then $(p \otimes q) \cap J \neq \phi \Rightarrow p \otimes q \subseteq J \forall p, q \in B$.

Proof. Straightforward.

Theorem III.11. Each reflexive hyper BRK-ideal of hyper BRK-Algebra B is a strong hyper BRK-ideal of B.

Proof. Leave J alone a reflexive hyper BRK-ideal of B and let p, q ∈ B be with the end goal that $(p \otimes q) \cap J \neq \phi$ and q∈ J. By **Theorem III.9** and by definition of hyper BRK-ideal p ∈J. Henceforth J is a strong hyper BRK-ideal of B.

IV. Theorems On Homomorphism

Definition IV.1 *Let(A, $\otimes$,0), (B, $\otimes'$, 0′) be BRK-Algebras. Then a function f: A $\rightarrow$ B is called a hyperhomomorphism if and only if* (1) $f(0) = 0'$

(2)$f(a_1 \otimes a_2) = f(a_1) \otimes' f(a_2)$.

Two hyper BRK-algebras A and B are said to be isomorphic, written as $A \cong B$, if there exists an isomorphism $f : A \rightarrow B$. For any homomorphism $f : A \rightarrow B$ the set { p ∈A : f (p) = 0 } is called kernel of f , denoted by Ker(f) and the set {f (p) : p∈A} is called the image of f, denoted by Imf.

Lemma IV.2. Let $h : A \rightarrow B$ be homomorphism of hyper BRK-Algebras.Then
(1) h(0) = 0
(2) $p \otimes q = 0$ implies $h(p) \otimes h(q) = 0$.

Proof.(1) $h(0) = h(0 \otimes 0) = h(0) \otimes h(0) = 0$.
(2) If $p \otimes q = 0$, then $h(p \otimes q) = h(0)$ which implies $h(p) \otimes h(q) = 0$.

Corollary IV.3. If $h : A \rightarrow B$ is homomorphism of hyper BRK-algebras, then Kerh is a closed ideal of A and Imh is a hyper subalgebra of B.

Theorem IV.4. *Let $h: M \rightarrow N$ be a hyper homomorphism of hyper BRK-algebras. On the off chance that p << q in M at that point h (p) << h(q) in N.*

Proof. Let p, q ∈M *be with the end goal* that p << q. At that point 0 ∈ p$\otimes q$, and so 0´= h(0) ∈h(p $\otimes q$) = h(p) $\otimes h$(q). Consequently, h(p) << h(q) in N.

Theorem IV.5. *Let $h: M \rightarrow N$ be a hyper homomorphism of hyper BRK-algebras. In the event that J is a hyper BRK-ideal of N, At that point h^{-1}(J) is a hyper BRK-ideal of M.*

Proof. 0 ∈h^{-1}(J). Let p,q ∈ M be with the end goal that p $\otimes q$ <<h^{-1}(J) and q∈ h^{-1}(J). At that point h(q) ∈J, and for each r ∈ p $\otimes q$ ∃ s ∈ h^{-1}(J) with the end goal that r << s, that is, 0 ∈ s$\otimes r$. It gives 0´= h(0) ∈h(s$\otimes r$) = h(s) $\otimes h$(r) ⊆ J$\otimes h$(p$\otimes q$) = J$\otimes$ (h(p) $\otimes$ h(q)), so that h(p) $\otimes h$(q) << J. Since J is a hyper BRK-ideal of N, it follows that h(q) ∈ J, that is, q ∈h^{-1}(J) . Henceforth h^{-1}(J) is a hyper BRK-ideal of M.

Theorem IV.6. *If $h: M \rightarrow N$ is a hyper homomorphism of hyper BRK- algebras, then ker*(h) ={p ∈M : h(p) = 0} *, called the kernel of f , is a hyper ideal of M .*

Theorem IV.7. *Let $h: M \rightarrow N$ be an onto hyper homomorphism of hyper BRK-algebras. On the off chance that J is a hyper BRK-ideal of M containing ker*(h)*,at that point h*(J) *is a hyper BRK-ideal of N .*

Proof. Note that 0´= h (0) ∈ h (J). Let r, s ∈ N be such that r $\otimes s$ <<h (J) and s ∈ h (J). Since h is onto, it follows that there exist p, q ∈M such that h(p) = r and h(q) = s. Thus h(p$\otimes q$) = h(p) $\otimes h$(q) = r$\otimes s$ << h (J). Let k∈r $\otimes$ s and for every t∈f(r$\otimes s$) there exist u∈h(J) such that t << u. Then, h(t) << h(u), that is 0 ∈ h(t) $\otimes$ h(u) = h(t $\otimes u$). It follows that t$\otimes u$ ⊆ $kerh$ ⊆J so that t $\otimes u$ << J. Since J is a hyper ideal of M, it follows that t ∈ J . Hence p $\otimes q$ ⊆ J and so p$\otimes q$ << J. Since q∈J, then by definition of p∈J so that r = h(p) in h(J). Hence h(J) is a hyper BRK-ideal of N.

Theorem IV.8. *Let $f: P \rightarrow Q$ and $g: P \rightarrow R$ be two homomorphisms of hyper BRK-algebras with the end goal that f is onto and ker(f) ⊆ ker(g). At that point, there exists a homomorphism $h: Q \rightarrow R$ such that h ∘ f = g.*

Proof. Let q∈Q. Since f is onto, there exists p∈P such that q = f (p). Define $h: Q \rightarrow R$ by h(q) = g(p), for all q ∈Q. Now, we show that h is well-defined. Let q_1, q_2 ∈Q and $q_1 = q_2$. Since f is onto, then there are p_1, p_2 ∈P such that $q_1 = f(p_1)$ and $q_2 = f(p_2)$. Hence $f(p_1) = f(p_2)$ and accordingly $g(p_1) = g(p_2)$. Clearly, $h \circ f = g$. It is easy to show that h is a homomorphism.

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