Research Article

Improved Results on Delay dependent Stability Criteria of Neural Networks with Interval Time Varying Delay

R. Jeetendra[a], and G. Uma[b]
[a] Assistant Professor, Dept. of Mathematics, Kongu Engineering College, Perundurai, Erode. jee4maths@gmail.com
[b]Assistant Professor, Dept. of Mathematics, Anna University, UCE, Dindigul

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Abstract: This paper examines the stability issue of continuous Neural Networks with a time varying delay. A Lyapunov Krasovskii functional consisting of some simple augmented terms and delay dependent terms is constructed. While calculating the derivative of Lyapunov functional, various integral inequalities such as Auxiliary Function Based Integral Inequality, Wirtinger-based integral inequality and an extended Jensen double integral inequality are jointly adopted and hence in terms of linear matrix inequality a new delay dependent stability criterion is obtained. Two numerical examples are taken to show that the derived result is less conservative than some existing ones.

Keywords: Lyapunov Krasovskii Functional (LKF), Linear Matrix Inequality (LMI), Neural Networks and Time Varying Delay.

1. Introduction

Neural networks have numerous applications in the field of associative memory, signal processing, pattern recognition, optimization problem and other engineering and scientific arena [1, 2]. Time delays are inevitable in practical applications of neural networks. It leads to the instability and oscillation in the neural networks. Nowadays the stability analysis of neural networks with time-varying delays is one of the important research areas. Generally stability criteria on delayed neural networks are of two types namely delay dependent and delay independent. The delay-dependent stability criteria include the information of time delay. Hence the conservative of these criteria is less than the other one. So researchers mainly focus on deriving delay dependent stability criteria. The foremost objective in stability analysis of neural networks is to obtain less conservatism and larger admissible upper bounds of delays. It can be achieved by constructing suitable LKFs and selecting the appropriate bounding techniques. Some of the important methods used in the construction of generalized Lyapunov functional are delay-partitioning LKF [3], augmented LKF, the matrix-refined-function based LKF [4], multiple integral LKF [5] and other novel LKFs like [6] and so on. The bounding techniques used to estimate the integral terms in the derivatives of LKFs includes Jensen's inequality [7], Wirtinger-based inequality [8], auxiliary function based inequality [9], free-matrix-based integral inequality [10], etc.

Feasibility can be improved by means of the terms of the LKF construction and the estimating approach for the derivative of the LKF. Hence, in this paper, a Lyapunov Krasovskii functional consisting of some simple augmented terms and delay dependent terms is constructed. While calculating the upper bound of the Lyapunov functional derivative, the relationship between time varying delay and its lower and upper bounds are considered. Various bounding techniques to get a tighter upper bound such as Auxillary Function Based Integral Inequality, the Wirtinger-based integral inequality and an extended Jensen double integral inequality are utilized and more information of the activation function is taken into account. Based on Lyapunov stability theory, a novel delay-dependent stability criterion is derived which has less conservatism. The effectiveness of the derived criteria is exhibited through numerical examples.

Notations:

In this paper R^n and $R^{m\times n}$ are the n-dimensional Euclidean space and the set of all m × n real matrix respectively. P > 0 denotes that P is a real symmetric positive definite matrix. $*$ indicates the symmetric terms in a symmetric matrix. diag{. . .} means block diagonal matrix and sym{X} = X + X^T where superscript 'T' denotes the transpose of the matrix.

2. Problem formulation

Consider the following neural networks with interval time varying delays:

$$\dot{x}(t) = -Ax(t) + B_1 f(x(t)) + B_2 f(x(t-h(t))) \qquad (1)$$

where $x(t) = \left[x_1(t), x_2(t), ..., x_n(t),\right]^T \in R^n$ is the state neuron vector, n denotes the number of neurons in a neural network. $A = diag\left(\alpha_1, \alpha_2, ..., \alpha_n\right) > 0$ and $B_1, B_2 \in R^{n\times n}$ are the interconnection weight matrices. The time delay h(t) is a continuous differentiable function satisfying $h_1 \le h(t) \le h_2, \quad \dot{h}(t) \le \mu$ where h_1, h_2 *and* μ are known constants. The neuron activation function

$f(x(t)) = \left[f(x_1(t)), f(x_2(t)), ..., f(x_n(t)) \right]^T \in R^n$ is assumed to be continuous, bounded and satisfies the following condition.

$$k_i^- \le \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \le k_i^+, \forall s_1 \ne s_2, i = 1, 2, ..., n \qquad (2)$$

where k_i^- and k_i^+ are constants.

Lemma 1: (Auxiliary Function Based Integral Inequality [11]) Let x be a differentiable signal in $[a,b] \to R^n$ for a positive definite matrix $R \in R^{n\times n}$, the following inequality holds:

$$(b-a)\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \ge \gamma_1^T R\gamma_1 + 3\gamma_2^T R\gamma_2 + 5\gamma_3^T R\gamma_3$$

where γ_1, γ_2 *and* γ_3 are defined as

$$\gamma_1 = x(b) - x(a);\ \gamma_2 = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds; \gamma_3 = \gamma_1 + \frac{6}{b-a}\int_a^b x(s)ds - \frac{12}{(b-a)^2}\int_a^b\int_u^b x(s)dsdu$$

Lemma 2: (an extended Jensen's double integral inequality [12]) For any constant symmetric positive definite matrix $R \in R^{n\times n}$, real scalars a, b, θ satisfying $a \le s \le b, s \le \theta$, and a vector valued function $x(t) : [a,b] \to R^n$, such that the following integration are well defined, then the following inequality holds

$$\frac{(b-a)(b+a-2\theta)}{2}\int_a^b\int_u^\theta \dot{x}^T(s)R\dot{x}(s)dsdu \le -[\int_a^b\int_u^\theta \dot{x}(s)dsdu]^T R[\int_a^b\int_u^\theta \dot{x}(s)dsdu]$$

Lemma 3: [14]

For a given matrix R>0 and a differentiable function $[a,b] \to R^n$, the following double integral inequality holds:

$$\int_a^b\int_u^b \dot{x}^T(s)R\dot{x}(s)ds \ge 2\psi_1^T R\psi_1 + 4\psi_2^T R\psi_2 + 6\psi_3^T R\psi_3$$

$$\psi_1 = x(b) - \frac{1}{b-a}\int_a^b x(s)ds$$

$$\psi_2 = x(b) + \frac{2}{b-a}\int_a^b x(s)ds - \frac{6}{(b-a)^2}\int_a^b\int_u^b x(s)dsdu$$

$$\psi_3 = x(b) - \frac{3}{b-a}\int_a^b x(s)ds + \frac{24}{(b-a)^2}\int_a^b\int_u^b x(s)dsdu - \frac{60}{(b-a)^3}\int_a^b\int_v^b\int_u^b x(s)dsdudv$$

Lemma 4: [15]

For any vectors β_1 and β_2, a symmetric matrix R, any matrix S satisfying $\begin{bmatrix} R & S \\ * & R \end{bmatrix} \ge 0$ *and* $0 \le \alpha \le 1$, the following inequality holds

$$\frac{1}{\alpha}\beta_1^T R\beta_1 + \frac{1}{1-\alpha}\beta_2^T R\beta_2 \ge \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}^T \begin{bmatrix} R & S \\ * & R \end{bmatrix}\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

Theorem 1 :

For given scalars h_1, h_2 and μ the system (1) is asymptotically stable if there exists positive diagonal matrices $H_i, U_i \in R^{n\times n}(i = 1,2,3,4)$, $\lambda_j = diag\left[\lambda_{1j}, \lambda_{2j}, ..., \lambda_{nj}\right] \in R^{n\times n}(j = 1, 2, ..., 6)$ positive definite matrix $P \in R^{4nx4n}$ and the symmetric positive definite matrices $Q_1, Q_2, Q_3 \in R^{2nx2n}$ $R_1, R_2, T_1, T_2 \in R^{nxn}$ and matrix $S \in R^{3nx3n}$ such that the following LMI hold simultaneously

$$\phi = \begin{bmatrix} \bar{R}_1 & S \\ * & \bar{R}_1 \end{bmatrix} > 0 \qquad (3)$$

$$\Gamma[h(t)=h_1,\dot{h}(t)]<0 \quad (4)$$

$$\Gamma[h(t)=h_2,\dot{h}(t)]<0 \quad (5)$$

where

$$\Gamma[h(t),\dot{h}(t)]=E_1+E_2+E_3+E_4+E_5 \quad (6)$$

$E_1=\pi_1 P\pi_2{}^T+\pi_2 P\pi_1{}^T$

$E_2=2\left[e_5\left[\left(\lambda_1+\lambda_3+\lambda_5\right)-\left(\lambda_2+\lambda_4+\lambda_6\right)\right]e_0{}^T\right]+$

$\quad 2\left[e_1\left[K_p\left(\lambda_2+\lambda_4+\lambda_6\right)-K_m\left(\lambda_1+\lambda_3+\lambda_5\right)\right]e_0{}^T\right]$

$E_3=\left[e_1 \quad e_5\right]Q_1\left[e_1 \quad e_5\right]^T+\left[e_2 \quad e_6\right](Q_2-Q_1)\left[e_2 \quad e_6\right]^T+(1-\dot{h}(t))\left[e_3 \quad e_7\right](Q_3-Q_2)\left[e_3 \quad e_7\right]^T$

$\quad -\left[e_4 \quad e_8\right]Q_1\left[e_4 \quad e_8\right]^T$

$E_4=h_1^2 e_0 R_1 e_0{}^T+h_{12}{}^2 e_0 R_2 e_0{}^T-\psi\phi\psi^T-\Lambda$

$E_5=e_0[T_1+h_\sigma{}^2 T_2]e_0{}^T-\Delta$

$\bar{R}_1=diag\{R_1,3R_1,5R_1\}$

$\pi_1=\left[e_1 \quad h_1 e_9 \quad \tilde{h}_1 e_{10} \quad \tilde{h}_2 e_{11}\right]$

$\pi_2=\left[e_0 \quad e_1-e_2 \quad e_2-(1-\dot{h}(t))e_3 \quad (1-\dot{h}(t))e_3-e_4\right]$

$\pi_3=\left[e_3-e_4 \quad e_3+e_4-2e_{11} \quad e_3-e_4+6e_{11}-12e_{13}\right]$

$\pi_4=\left[e_2-e_3 \quad e_2+e_3-2e_{10} \quad e_2-e_3+6e_{10}-12e_{14}\right]$

$\psi=\left[\pi_3 \quad \pi_4\right]$

$\Lambda=(e_1-e_2)R_1(e_1-e_2)^T+3(e_1+e_2-2e_9)R_1(e_1+e_2-2e_9)^T+5(e_1-e_2+6e_9-12e_{12})R_1(e_1-e_2+6e_9-12e_{12})^T$

$\Delta=2(e_1-e_9)T_1(e_1-e_9)^T+4(e_1+2e_9-6e_{12})T_1(e_1+2e_9-6e_{12})^T+$
$6(e_1-3e_9+24e_{12}-60e_{15})T_1(e_1-3e_9+24e_{12}-60e_{15})^T+(h_{12}e_1-\tilde{h}_2 e_{11}-$
$\tilde{h}_1 e_{10})T_2(h_{12}e_1-\tilde{h}_2 e_{11}-\tilde{h}_1 e_{10})^T+\sum_{i=1}^{4} sym\{(e_i K_m-e_{i+4})^T H_i(e_{i+4}-K_p e_i)\}+$
$\sum_{i=1}^{3} sym\{[(K_m(e_i-e_{i+1})-(e_{i+4}-e_{i+5})]^T U_i[(e_{i+4}-e_{i+5})-K_p(e_i-e_{i+1})]\}+$
$sym\{[K_m(e_1-e_3)-(e_5-e_7)]^T U_4[(e_5-e_7)-K_p(e_1-e_3)]\}$

$e_0=-e_1A^T+e_5B_1{}^T+e_7B_2{}^T$

$e_i=[0_{n\times(i-1)n} \ I_n \ 0_{n\times(15-i)n}],i=1,2,3,....,15$

$K_m=diag\{k_1^-,k_2^-,.....,k_n^-\};K_p=diag\{k_1^+,k_2^+,.....,k_n^+\}$

where $h_{12}=h_2-h_1;\tilde{h}_2=h_2-h(t);\tilde{h}_1=h(t)-h_1;h_\sigma=\dfrac{(h_2{}^2-h_1{}^2)}{2}$

Proof:

Consider the following Lyapunov Krasovskii Functional

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t)$$

where $V_1(t)=\delta^T(t)P\delta(t)$

$$V_2(t)=2\sum_{i=1}^{n}\left\{\lambda_{1i}\int_0^{x_i(t)}(f_i(s)-k_i^- s)ds + \lambda_{2i}\int_0^{x_i(t)}(k_i^+ s-f_i(s))ds\right\}$$
$$+2\sum_{i=1}^{n}\left\{\lambda_{3i}\int_0^{x_i(t-h_1)}(f_i(s)-k_i^- s)ds+\lambda_{4i}\int_0^{x_i(t-h_1)}(k_i^+ s-f_i(s))ds\right\}$$
$$+2\sum_{i=1}^{n}\left\{\lambda_{5i}\int_0^{x_i(t-h_2)}(f_i(s)-k_i^- s)ds+\lambda_{6i}\int_0^{x_i(t-h_2)}(k_i^+ s-f_i(s))ds\right\}$$
$$V_3(t)=\int_{t-h_1}^{t}\eta^T(s)Q_1\eta(s)ds+\int_{t-h(t)}^{t-h_1}\eta^T(s)Q_2\eta(s)ds+\int_{t-h_2}^{t-h(t)}\eta^T(s)Q_3\eta(s)ds$$
$$V_4(t)=h_1\int_{t-h_1}^{t}\int_u^t\dot{x}^T(s)R_1\dot{x}(s)dsdu+h_{12}\int_{t-h_2}^{t-h_1}\int_u^t\dot{x}^T(s)R_2\dot{x}(s)dsdu$$
$$V_5(t)=\int_{t-h_1}^{t}\int_v^t\int_u^t\dot{x}^T(s)T_1\dot{x}(s)dsdudv+h_\sigma\int_{t-h_2}^{t-h_1}\int_v^t\int_u^t\dot{x}^T(s)T_2\dot{x}(s)dsdudv$$

with $\delta(t)=col[x(t),\int_{t-h_1}^{t}x(s)ds,\int_{t-h(t)}^{t-h_1}x(s)ds,\int_{t-h_2}^{t-h(t)}x(s)ds];\ \eta(t)=col\left[x(t),\quad f(x(t))\right]$

Calculating the time derivative of V(t) along the given system yields

$$\dot{V}_1(t)=2\delta^T(t)\dot{\delta}(t)=\xi^T(t)E_1\xi(t) \tag{7}$$

$$\dot{V}_2(t)=2\left[f^T(x(t))\left[\left(\lambda_1+\lambda_3+\lambda_5\right)-\left(\lambda_2+\lambda_4+\lambda_6\right)\right]\dot{x}(t)\right]+$$
$$2\left[x^T(t)\left[K_p\left(\lambda_2+\lambda_4+\lambda_6\right)-K_m\left(\lambda_1+\lambda_3+\lambda_5\right)\right]\dot{x}(t)\right]$$
$$=\xi^T(t)E_2\xi(t) \tag{8}$$

$$\dot{V}_3(t)=\eta^T(t)Q_1\eta(t)+\eta^T(t-h_1)(Q_2-Q_1)\eta(t-h_1)+(1-\dot{h}(t))\eta^T(t-h(t))(Q_3-Q_2)\eta(t-h(t))-$$
$$\eta^T(t-h_2)Q_3\eta(t-h_2)$$
$$=\xi^T(t)E_3\xi(t) \tag{9}$$

$$\dot{V}_4(t)=h_1^2\dot{x}^T(t)R_1\dot{x}(t)-h_1\int_{t-h_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds+h_{12}^2\dot{x}^T(t)R_2\dot{x}(t)-h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)ds \tag{10}$$

Applying Lemma (1) and Lemma (4) we get

$$-h_1\int_{t-h_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds\le-\xi^T(t)\{(e_1-e_2)R_1(e_1-e_2)^T+3(e_1-e_2-2e_9)R_1(e_1-e_2-2e_9)^T+ \tag{11}$$
$$5(e_1-e_2+6e_9-12e_{12})R_1(e_1-e_2+6e_9-12e_{12})^T\}\xi(t)$$

$$-h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)ds\le-h_{12}\int_{t-h_2}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)ds-h_{12}\int_{t-h(t)}^{t-h_1}\dot{x}^T(s)R_2\dot{x}(s)ds$$

$$\leq -\frac{h_{12}}{h_2-h(t)}\xi^T(t)\{(e_3-e_4)R_1(e_3-e_4)^T+3(e_3+e_4-2e_{11})R_1(e_3+e_4-2e_{11})^T+$$
$$5(e_3-e_4+6e_{11}-12e_{13})R_1(e_1-e_2+6e_{11}-12e_{13})^T\}\xi(t)$$
$$-\frac{h_{12}}{h(t)-h_1}\xi^T(t)\{(e_2-e_3)R_1(e_2-e_3)^T+3(e_2+e_3-2e_{10})R_1(e_2+e_3-2e_{10})^T+$$
$$5(e_2-e_3+6e_{10}-12e_{14})R_1(e_2-e_3+6e_{10}-12e_{14})^T\}\xi(t)$$
$$\leq -\xi^T(t)\{\psi\phi\psi^T\}\xi(t) \qquad (12)$$
$$\dot{V}_5(t)=\dot{x}^T(t)[T_1+h_\sigma{}^2T_2]\dot{x}(t)-\int_{t-h_1}^{t}\int_{u}^{t}\dot{x}^T(s)T_1\dot{x}(s)dsdu-h_\sigma\int_{t-h_2}^{t-h_1}\int_{u}^{t}\dot{x}^T(s)T_2\dot{x}(s)dsdu$$

By Lemma (3),

$$\int_{t-h_1}^{t}\int_{u}^{t}\dot{x}^T(s)T_1\dot{x}(s)dsdu\leq -\xi^T(t)\{2(e_1-e_9)T_1(e_1-e_9)^T+4(e_1+2e_9-6e_{12})T_1(e_1+e_9-6e_{12})^T+$$
$$6(e_1-3e_9+24e_{12}-60e_{15})T_1(e_1-3e_9+24e_{12}-60e_{15})^T\}\xi(t) \qquad (13)$$

Using Lemma (2),

$$-h_\sigma\int_{t-h_2}^{t-h_1}\int_{u}^{t}\dot{x}^T(s)T_2\dot{x}(s)dsdu\leq -\xi^T(t)\{(h_{12}e_1-\tilde{h}_2e_{11}-\tilde{h}_1e_{10})T_2(h_{12}e_1-\tilde{h}_2e_{11}-\tilde{h}_1e_{10})^T\}\xi(t) \qquad (14)$$

By the assumption of activation function (2) we have

$$a_i(s):2\left[K_mx(s)-f(x(s))\right]^T H_i\left[f(x(s))-K_px(s)\right]\geq 0$$
$$b_i(s_1,s_2):2\left[K_m\left(x(s_1)-x(s_2)\right)-\left(f(x(s_1)-f(x(s_2))\right)\right]^T U_i\left[\left(f(x(s_1)-f(x(s_2))\right)-K_p\left(x(s_1)-x(s_2)\right)\right]\geq 0$$

where $H_i=diag\left[a_{1i},a_{2i},.....,a_{ni}\right]\geq 0, U_i=diag\left[b_{1i},b_{2i},.....,n_{ni}\right]\geq 0, i=1,2,3,4.$

Then the following inequalities hold

$$a_1(t)+a_2(t-h_1)+a_3(t-h(t))+a_4(t-h_2)\geq 0 \qquad (15)$$
$$b_1(t,t-h_1)+b_2(t-h_1,t-h(t))+b_3(t-h(t),t-h_2)+b_4(t,t-h(t))\geq 0 \qquad (16)$$

Combining the equations (7)-(16) we get

$$\dot{V}(t)\leq \xi^T(t)\Gamma(h(t),\dot{h}(t))\xi(t)$$

where $\Gamma(h(t),\dot{h}(t))$ is defined in (6) and

$$\xi(t)=[x^T(t),x^T(t-h_1),x^T(t-h(t)),x^T(t-h_2),f^T(x(t)),f^T(x(t-h_1)),f^T(x(t-h(t))),$$
$$f^T(x(t-h_2)),\frac{1}{h_1}\int_{t-h_1}^{t}x^T(s)ds,\frac{1}{\tilde{h}_1}\int_{t-h(t)}^{t-h_1}x^T(s)ds,\frac{1}{\tilde{h}_2}\int_{t-h_2}^{t-h(t)}x^T(s)ds,\frac{1}{h_1^2}\int_{t-h_1}^{t}\int_{u}^{t}x^T(s)dsdu,$$
$$\frac{1}{\tilde{h}_2^2}\int_{t-h_2}^{t-h(t)}\int_{u}^{t-h(t)}x^T(s)dsdu,\frac{1}{\tilde{h}_1^2}\int_{t-h(t)}^{t-h_1}\int_{u}^{t-h_1}x^T(s)dsdu,\frac{1}{h_1^3}\int_{t-h_1}^{t}\int_{v}^{t}\int_{u}^{t}x^T(s)dsdudv]^T$$

Therefore, if LMIs (3)-(5) hold, then the following holds for a sufficiently small scalar $\varepsilon>0$

$$\dot{V}(t)\leq -\varepsilon\left\|x(t)\right\|^2$$

which shows the asymptotic stability of the given system (1). This completes the proof.

3. Numerical Examples

Two numerical examples are considered for the analysis of our criteria and some existing works.

Example 1

Consider the system $\dot{x}(t)=-Ax(t)+B_1f(x(t))+B_2f(x(t-h(t))$ where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix}$$

$$K_m = diag\{0,0\} \quad K_p = diag\{0.4, 0.8\} \ .$$

In order to verify the advantages of the proposed method the maximum delay bounds for of the given system with various h_1 and μ are listed in Table1.

Example 2

Consider the system $\dot{x}(t) = -Ax(t) + B_1 f(x(t)) + B_2 f(x(t-h(t))$ where

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix} \ K_m = diag\{0,0\} \quad K_p = diag\{0.4, 0.8\}$$

Table 2 depicts that the results obtained by our method are less conservative than those of [20], [21] and [22].

Table 1. Upper bounds (h_2) for various h_1 and μ

h_1	Method	μ = 0.8	μ = 0.9	Unknown μ
0.5	[15]	0.8262	0.8215	0.8183
	[16]	1.1217	0.9984	0.9037
	[17]	1.4508	1.4042	1.0862
	Theorem 1	1.9609	1.6979	1.6755
0.75	[15]	0.9669	0.9625	0.9592
	[16]	1.2213	1.1021	1.0102
	[18]	1.3990	1.2241	1.0972
	[17]	1.4891	1.4789	1.1838
	Theorem 1	2.1060	1.9107	1.9019
1	[15]	1.1152	1.1108	1.1075
	[16]	1.3432	1.2238	1.1318
	[18]	1.4692	1.2948	1.1774
	[17]	1.6892	1.6880	1.4000
	Theorem 1	2.2709	2.1126	2.1111

Table 2. Upper bounds (h_2) for various h_1 and μ

h_1	Method	μ = 0.8	μ = 0.9	Unknown μ
0	[19]	1.2281	0.8639	0.8298
	[20]	1.6831	1.1493	1.0880
	[21]	2.3534	1.6050	1.5103
	Theorem 1	5.2089	2.2314	1.8360
1	[20]	2.5967	2.0443	1.9621
	[21]	3.2575	2.4769	2.3606
	Theorem 1	6.1369	2.8869	2.7602
100	[20]	101.5946	101.0443	100.9621
	[21]	102.2552	101.4769	101.3606
	Theorem 1	103.6081	101.8528	101.7460

Conclusion

This paper studies the stability issue of continuous Neural Networks with a time varying delay. A novel Lyapunov Krasovskii functional consisting of some simple augmented terms and delay dependent terms is constructed. By employing of various bounding techniques to get larger admissible bounds a new less conservative stability criterion is developed in terms of linear matrix inequality. Finally two numerical examples are discussed to substantiate the efficacy of the proposed theorem.

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