Research Article

Mass – A New Ones Assignment Method for Finding Optimal Solution of Assignment Problems

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Abstract: An assignment problem (AP) is a particular case of a transportation problem, in which the objective is to assign a number of facilities to an equal number of activities at an overall minimum cost, distance, time (or maximum profit). It occupies extremely a major part in the real physical world. The most common method used to solve the APs is the Hungarian method. In this paper, we make an endeavor to introduce a new ones assignment approach namely MASS (Modified Assignment) for finding optimal pattern of assignments to a wide range of APs. 2010 Mathematics Subject Classification: 90C08, 90C10

Keywords: Assignment problems, Hungarian method, Ones assignment method, MASS method.

1. Introduction

The assignment problem is one of the most essential applications in the real world and it is a special class of linear programming in which the objective is to assign n number of jobs to n number of persons or machines at an overall minimum cost or maximum profit. There are various ways to solve the AP. A well known iterative method was developed and published by H.W. Kuhn [1] in 1955 named as Hungarian method (HM). This method is based on subtract or add a constant to every element of a row or column of the cost matrix, in a minimization model and create some zeros in the given cost matrix and then try to find a complete pattern of assignments in terms of zeros. In the recent years considerable numbers of methods have been published by several researchers to find the optimal solution for APs. But, these methods were not able to produce optimal assignment plans to all APs.

In 2012, Hadi Basirzadeh [2] introduced a new approach to APs namely, Ones Assignment Method (OAM) for solving a wide range of such problems. In the procedure of this method, the author first defines the assignment matrix, then by using determinant representation he obtains a reduced matrix which has at least one 1 in each row and in each column. Then by using the OAM, he obtains an optimal solution for AP by assigning 1s to each row and each column. This method is based on creating some 1s in the assignment matrix and then finding a complete assignment in terms of these 1s. This method can be applied only when all the entries of the cost matrix are nonzero.

In 2013, Ghadle K.P. and Muley Y.M. [5] presented a new method namely, Revised Ones Assignment (ROA) method for solving wide range of APs, which is different from the OAM. This method is also based on creating some 1s in the assignment matrix and then tries to find a complete assignment in terms of 1s.

In February 2014, M.D.H. Gamal [6] had brought out some drawbacks in the OAM, due to Hadi Basirzadeh [2], and gave a remedial strategy to overcome the case when some entries of the cost matrix are zeros. The author also gave examples of the APs where the OAM fails to find their optimal solution.

In May 2014, M. Khalid et al. [7] introduced the New Improved Ones Assignment (NIOA) method, which leads to brief computation time comparatively and will attain an exact optimal solution. Besides, this improved version will overcome the drawbacks as indicated by M.D.H. Gamal [6].

In 2015, K. Ghadle and et al.[4] proposed a hybrid method to solve AP which is combination of OAM and Stepping Stone Method. This gives optimal solution within few steps.

In this paper, we have proposed a new OAM, namely MASS (Modified Assignment), to find the optimal pattern of assignment to APs. The new method has been tested for 30 classical benchmark problems from the literature and obtained optimal solution to each problem.

The paper is organized as follows: In Section 1, brief introduction is given. In Section 2, the existing algorithm of the HM is presented. The algorithm for the proposed MASS method is presented in Section 3. In Section 4, one benchmark problem from the literature has been illustrated. Section 5 lists a set of 30 benchmark assignment problems from the literature. The comparisons of results produced by the MASS and HM methods are shown in Section 6. Finally, in Section 7 conclusions are drawn.

2. Algorithm for the Existing Hungarian Assignment Method

Algorithm of this well known method is available in most of the articles published in AP and also in Operations Research textbooks such as J.K. Sharma [3]. Please refer them for the Hungarian procedure.

3. Algorithm for the Proposed MASS Method

The term MASS is coined from the two words 'Modified' and 'Assignment' The MASS method consists of two phases. In the first phase, a complete assignment plan is found out using the ones assignment technique based on the ME rules and in the second phase optimality testing and optimizing of the obtained assignment plan are carried out based on the computed net cost changes of the unassigned cells. The algorithm is as follows:

Phase-I

(Finding a complete assignment plan)

Step 1: Check the Non-zero Entry. If all the entries of the given assignment cost matrix are nonzero, go to Step 2. Otherwise, add 1 with every entry of the row(s) having zero cost entry. This will ensure that all the entries of the cost matrix are nonzero. Go to Step 2.

Step 2: Check the Balanced Condition. Check whether the given AP is balanced or not. If the AP is balanced, go to Step 4; otherwise, go to Step 3.

Step 3: Conversion into Balanced AP. If the AP is not balanced, then anyone of the following two cases may arise:

a) If the number of rows exceeds the number of columns, introduce required number of additional dummy columns to the assignment cost matrix to equalize with the rows. The assignment cost for each cell in these dummy column(s) is set to 1. Go to Step 4.

or

b) If the number of columns exceeds the number of rows, introduce required number of additional dummy rows to the assignment cost matrix to equalize with the columns. The assignment cost for each cell in these dummy row(s) is set to 1. Go to Step 4.

Step 4: Construct the Ratio of Costs Matrix (RCM).

a) Perform the Row Minimum Division (RMD) Operation.

Divide each of the costs of every row of the balanced AP by its minimum cost. This will result in a ratio matrix, which will have at least one 1s in each row. If each row and each column of the ratio matrix has at least one 1s, go to Step 5 directly for making the assignments; otherwise, go to Step 4(b).

b) Perform the Column Minimum Division (CMD) Operation.

Divide each of the entries of every column of the ratio matrix obtained in Step 4(a) by its minimum entry. These operations create at least one 1s in each column. Go to Step 5 for making the assignments.

Note: The resultant matrix obtained in Step 4(b) is known as the ratio of costs matrix (RCM). It is noted that there will be at least one 1-entry in each row and in each column of an RCM. The cells having only 1-entry in an RCM are called 1-entry cells.

Step 5: Cover all the 1s with minimum number of lines using ME rules (The word ME is coined from the first letter of the names of the authors Murugesan and Esakkiammal. The readers may refer [8] for the ME rules).

Important point: While solving unbalanced APs, in the obtained RCM, first cover all the 1s entries in the dummy row(s) or column(s) by using the number of lines equals the number of dummy row(s) or column(s) added. Then apply the ME rules for the remaining rows and columns.

Note: There may be three kinds of cells in the assignment matrix obtained via Step 5 namely, the assigned 1-entry cells, the unassigned 1-entry cells and the unassigned > 1 entry cells. The assigned 1-entry cells are called assigned cells or occupied cells or used cells and the remaining cells are called unassigned cells or unoccupied cells or unused cells or empty cells. In a square matrix of order n, there can be only n occupied cells (one in each row and in each column) and n2- n unoccupied cells.

Phase-II

(Optimality testing and optimizing the obtained assignment plan)

Step 1: Compute the net cost change for every unused cell.

In the assignment table with the original assignment costs, trace a loop starting from an unused cell. There may be no loop or one loop or more than one loop from an unused cell. Mark (+) and (-) sign alternatively at each corner of a loop, starting from the unused cell. Compute the effect on cost for the selected unused cell, by adding together the original cost figures found in each cell containing a plus sign and then subtracting the original cost figures found in each cell containing a minus sign. This effect on cost is called the net cost change for the chosen unused cell. If the unused cell has more than one loop, then compute the net cost change for the cell associated with each loop. The maximum among them is considered as the actual net cost change of the unused cell. In the same way, compute the net cost change for every unused cell in the assignment matrix.

Note: The net cost change for an unused cell may be negative or zero or positive. If we make a new assignment only in the unused cell with negative net cost change, then the overall cost may decrease. Do not select the > 1 entry unused cell for new assignment, if a loop cannot be traced from it. However, select the unused 1-entry cell for new assignment, if a loop cannot be traced from it.

Step 2: Test the optimality condition.

If the net cost change for each unused cell is non-negative, then definitely the current pattern of assignments is an optimal one for the given AP. If negative net cost change occurs for certain unused cells, the current pattern of assignments may not be optimal and it has to be improved.

Step 3: Optimize the current assignment plan.

(i) Select an appropriate unused cell for new assignment.

Select the unused cell with the largest negative net cost change to include in the new pattern of assignments. If tie occurs among the unused cells with identical largest negative net cost change, then select each such cell for the new assignment as a separate case. Such a situation may generate an alternative optimal assignment plan to the given AP. The largest negative net cost change of an unused cell indicates the cost decrease that can be achieved by making an assignment in that cell.

(ii) Make a new assignment in the selected cell.

If the cell (i, k) has the largest negative net cost change in the ith row and the cell (i, j) is the currently assigned cell in the ith row, move the assignment from the cell (i, j) to the new cell (i, k). Equivalently, the assignment in the jth column is first moved to the kth column. Due to the unique assignment property in a row and column, this move will induce the current assignment in the kth column, say (m, k) to move to another appropriate column. So, move the allocation from the cell (m. k) to the cell (m, n) having largest negative net cost change or next to the largest negative net cost change. Move the current assignments in this way until to get a new assignment in the jth column from which we have started our first move. Due to these moves a 1-entry cell (from which a loop cannot be traced) and a > 1-entry cell may also get an assignment.

(iii) Write the modified assignment plan.

Write the corresponding modified assignment plan and compute the associated overall cost.

Step 4: Repeat the process.

Repeat the Steps 1 to 3 until there is no negative net cost change for all unused cells or there is no further reduction in the overall cost of an assignment plan. That is, the current assignment plan is an optimal one. Write the optimal assignment plan and compute the associated overall minimum cost of assignment.

Remark: In an optimal assignment table, if an unused 1-entry cell has net cost change zero, it indicates that the given AP will have an alternative optimal assignment plan. Also, if the net cost changes for all the unused cells are strictly > 0, then the given AP has a unique optimal assignment plan only.

4. Numerical Illustrations

Suitable illustrative solution makes the readers to understand the proposed MASS method thoroughly. Bearing in mind, one assignment problem from the literature has been illustrated.

Example-1: Consider the following cost minimizing assignment problem with five jobs and five typists, referred from J. K. Sharma [3], which is shown in Table 1.

Table 1: The given Balanced Minimization AP

	Typists				
Jobs	**1**	**2**	**3**	**4**	**5**
1	85	75	65	125	75
2	90	78	66	132	78
3	75	66	57	114	69
4	80	72	60	120	72
5	76	64	56	112	68

SOLUTION BY THE MASS METHOD

Phase-I: (Finding a complete assignment plan) By applying the steps of Phase-I in the MASS method, one can get the assignment matrix, as shown in Table 2, with a complete assignment plan. The cells with the starred 1s denote the assigned cells.

Table 2: Ratio of costs matrix with assignments

	Typists				
Jobs	**1**	**2**	**3**	**4**	**5**
1	1	1.01	1	1*	1
2	1.03	1.04	1	1.04	1*
3	1*	1.02	1	1.04	1.01
4	1.01	1.05	1*	1.04	1.01
5	1.03	1*	1	1.04	1.04

The obtained complete assignment plan is (1, 4), (2, 5), (3, 1), (4, 3) and (5, 2) with the overall cost Z = 125+78+75+60+64 = $402.

Phase-II: (Optimality testing and optimizing the obtained assignment plan)

In order to perform the optimality test and optimize the obtained assignment plan, consider the assignment table with the original assignment cost figures, as shown in Table 3.

Table 3: The assignment table with the original cost figures

	Typists				
Jobs	**1**	**2**	**3**	**4**	**5**
1	1 85	75	1 65	1* 125	1 75
2	90	78	1 66	132	1* 78

3	1* 75	66	1 57	114	69
4	80	72	1* 60	120	72
5	76	1* 64	1 56	112	68

First iteration: Step 1: Computing the net cost change for every unused cell.

The computation of net cost changes for the unused cells in the assignment table 3 are shown in Table 4.

Table 4: The net cost changes for the unused cells found in Table 3

Unused Cells	Net cost change(s) due the possible loops traced	Maximum net cost change
(1, 1)	2, 4	4
(1, 2)	2, 4	4
(1, 3)	-2, 2	2
(1, 5)	-2, -4	**-2**
(2, 1)	6, 4, 2	6
(2, 2)	4	4
(2, 3)	-2	**-2**
(2, 4)	4, 6, 8	8
(3, 1)	1	1
(3, 3)	2	2
(3, 4)	-3, -1, -5	-1
(3, 5)	2, 4	4
(4, 1)	2, 0	2
(4, 2)	4	4
(4, 4)	0, -2, 2	2
(4, 5)	2, 0, 2	2
(5, 1)	2, 0, -2	2
(5, 3)	---	---
(5, 4)	-4, -2, -6	**-2**
(5, 5)	0, 2, 4, 2	4

Step 2: Testing the optimality condition

As negative net cost change occurs for certain unused cells, the current pattern of assignments may not be optimal and it has to be improved.

Step 3: Optimizing the current assignment plan

The largest negative net cost change is -2 which corresponds to the unused cells (1, 5), (2, 3) and (5, 4). If one makes assignment in these cells, the overall cost of assignment may reduce. We try a new assignment in these cells one by one, each as a separate case.

Case (1): Select the cell (1, 5) for new assignment. First, a new assignment is placed at the cell (1, 5). Due to this, the induced modified assignments are shown in Table 5.

Table 5: Modified assignments due to new assignment at (1, 5)

Current Assigned cell	New Assigned cell	Net cost Change
(1, 4)	(1, 5)	-2
(2, 5)	(2, 3)	-2
(4, 3)	(4, 1)	2
(3, 1)	(3, 4)	-1
Overall net cost change		-3

As the overall net cost change is negative, definitely there will be a reduction in the overall cost of assignment. The modified assignment plan is (1, 5), (2, 3), (3, 4), (4, 1) and (5, 2) with the overall cost Z = 75+66+114+80+64 = \$399. Note that, due to the new assignment at the cell (1, 5) the overall cost of assignment has been reduced by \$3.

Case (2): Select the cell (2, 3) for new assignment. Next, a new assignment is placed at the cell (2, 3). Due to this, the induced modified assignments are shown in Table 6.

Table 6: Modified assignments due to new assignment at (2, 3)

Current Assigned cell	New Assigned cell	Net cost Change

(2, 5)	(2, 3)	-2
(4, 3)	(4, 1)	2
(3, 1)	(3, 4)	-1
(1, 4)	(1, 5)	-2
Overall net cost change		-3

As the overall net cost change is negative, definitely there will be a reduction in the overall cost of assignment. The modified assignment plan is (1, 5), (2, 3), (3, 4), (4, 1) and (5, 2) with the overall cost Z = 75+66+114+80+64 = \$399. Observe that, the new assignment assigned at the cell (2, 3) have generated the same identical modified assignment plan, as obtained in case (1).

Case (3): Select the cell (5, 4) for new assignment. Next, a new assignment is placed at the cell (5, 4). Due to this, the induced modified assignments are shown in Table 7.

Table 6: Modified assignments due to new assignment at (5, 4)

Current Assigned cell	New Assigned cell	Net cost Change
(5, 2)	(5, 4)	-2
(1, 4)	(1, 5)	-2
(2, 5)	(2, 3)	-2
(4, 3)	(4, 1)	2
(3, 1)	(3, 2)	1
Overall net cost change		-3

As the overall net cost change is negative, definitely there will be a reduction in the overall cost of assignment. The modified assignment plan is (1, 5), (2, 3), (3, 2), (4, 1) and (5, 4) with the overall cost Z = 75+66+66+80+112 = \$399. Observe that, due to the new assignment at the cell (5, 4) the overall cost of assignment has been reduced by \$3. Also observe that, the obtained modified assignment plan is different, compared to that of obtained in case (1) or case (2).

Second iteration: Consider the modified assignment table obtained from case (1) with the original assignment cost figures, as shown in Table 7. The starred cells are the assigned cells.

Table 7: The modified assignment table with the original cost figures

	Typists				
Jobs	**1**	**2**	**3**	**4**	**5**
1	1 85	75	1 65	1 125	1* 75
2	90	78	1* 66	132	1 78
3	1 75	66	1 57	* 114	69
4	* 80	72	1 60	120	* 72
5	76	1* 64	1 56	112	68

Step 1: Computing the net cost change for every unused cell.

The computation of net cost changes for the unused cells found in the assignment table 7 are shown in Table 8.

Table 8: The net cost changes for the unused cells found in Table 7

Unused Cells	Net cost change(s) due the possible loops traced	Maximum net cost change
(1, 1)	2, 0, -3, 4, 2	4
(1, 2)	2, -1, 4	4
(1, 3)	-2, 0, 4	4
(1, 4)	3, 1	3
(2, 1)	6, 4, 2	6
(2, 2)	4	4
(2, 4)	6, 4, 9	9
(2, 5)	11, 4	11
(3, 1)	-2, 1, -2	1
(3, 2)	1	1
(3, 3)	-2, 0, 3	3
(3, 5)	5, 2, 4	5
(4, 2)	4	4
(4, 3)	-2, 0, -3	**0**
(4, 4)	3, 0, -2	3

(4, 5)	2, 0, 2	2
(5, 1)	0, 2, 0, 3	3
(5, 3)	---	---
(5, 4)	-1, -4, -4, -6	**-1**
(5, 5)	0, 2, 4, 2	4

Step 2: Testing the optimality condition

As negative net cost change occurs at the unused cell (5, 4) the current pattern of assignment may not be optimal and it has to be improved. However, in case (3) of first iteration we have tried a new assignment at this cell (5, 4) and obtained a modified assignment plan (1, 5), (2, 3), (3, 2), (4, 1) and (5, 4) with the overall cost Z = \$399.

Hence, the overall cost of assignment has not reduced further. This indicates that the current modified assignment plan is (1, 5), (2, 3), (3, 4), (4, 1) and (5, 2) with the overall cost Z = 75+66+114+80+64 = \$399 is an optimal one. Also, note that in the optimal assignment table (Table 7), the 1-entry cell (4, 3) has the net cost change zero. This indicates that the given AP has an alternative optimal assignment plan. The alternative assignment plan is (1, 5), (2, 3), (3, 2), (4, 1) and (5, 4) with the overall minimum cost Z = 75+66+66+80+112 = \$399.

In the similar way, the modified assignment table obtained through case (3) with the original assignment cost figures has been tested for its further improvement. But, it also generates the same identical two optimal assignments.

5.Numerical Examples

To justify the efficiency of the proposed MASS method, we have solved 30 numbers of classical benchmark APs in different sizes, from various literature and book, which are listed in Appendix A.

6. Result Analysis

To measure the effectiveness of the proposed MASS method, 30 benchmark problems, listed in Appendix A, have been tested and the results are compared with the results of the existing Hungarian method (HM). The comparison of results is shown in Table 9.

Table 9: Comparison of results obtained by the MASS and HMs

Prob. No. #	**MASS**	**HM**	**Prob. No. #**	**MASS**	**HM**	**Prob. No. #**	**MASS**	**HM**
1.	48	48	**11.**	13	13	**21.**	50	50
2.	14	14	**12.**	900	900	**22.**	214	214
3.	59	59	**13.**	81	81	**23.**	54	54
4.	71	71	**14.**	399	399	**24.**	15	15
5.	09	09	**15.**	67	67	**25.**	54	54
6.	14	14	**16.**	392	392	**26.**	08	08
7.	29	29	**17.**	114	114	**27.**	15	15
8.	21	21	**18.**	99	99	**28.**	73	84*
9.	24	24	**19.**	248	248	**29.**	870	880*
10.	27	27	**20.**	191	191	**30.**	24	43*

From Table 14, we discover that out of 30 benchmark problems tested, for 26 problems the proposed MASS method has produced optimal solution directly in Phase-I itself. For the problems numbered with 10, 14, 15 and 20 only, we have to go to Phase-II in order to improve the complete assignment plant obtained through Phase-I. Further, it is observed that for the problems numbered with 28, 29 and 30, the MASS method has produced optimal assignment plans, whereas the HM has not produced optimal plans. Next, how the proposed MASS technique is different from the existing Hungarian and OAMs and its novelty are given below:

How is the proposed MASS technique different from the existing HM

The HM produces optimal or near optimal solution to the given AP. But, the MASS method produces optimal solution to any given AP. From Table 14, we discover that out

of 30 benchmark problems tested, the MASS method has produced optimal assignment plans for all the 30 problems,

whereas the Hungarian assignment method has produced optimal plans only to 27 problems.

How is the proposed MASS technique different from the existing OAM

The difference between the OAM and the MASS method for solving AP is as follows: In the first one (Step 4), the division by dij is done on each element of the uncovered rows or columns, which dij lies on it. But in the later one (Rule 2 in Step 5 of Phase-I), the division by dij is done only on each uncovered element of the uncovered rows or columns, which dij lies on it. This operation keeps the already existing 1-entry and also creates some new 1-entry in the considered row or column. M.D.H. Gamal [6] has established that the OAM will not produce optimal assignment plan to all the APs. But, the proposed MASS method produces optimal solution to all the APs.

The proposed MASS Technique in terms of Novelty

For a given AP, the assignment plan obtained by applying any OAM available in the literature, can be tested for its optimality and can be improved towards an optimal assignment plan.

7. Conclusion

In this paper, we have proposed a new method named MASS to find an optimal pattern of assignments to assignment problems. This method finds the optimal assignment plan to a given AP in two phases. In Phase-I, a complete assignment plan is found out using the ones assignment technique based on the ME rules. Optimality testing and optimizing of the obtained assignment plan is carried out in Phase-II based on net cost changes computed for the unassigned cells. The proposed method is tested for 30 classical benchmark APs (balanced minimization and maximization cases, and unbalanced minimization and maximization cases) from the literature. The obtained results substantiate that the proposed MASS method is the most efficient one, which produces optimal solution directly to all 30 instances, whereas the existing HM produces optimal solution directly only to 27 instances. Hence, it is guaranteed that by applying the MASS method one can get an optimal assignment plan to a given AP. Further, for any AP, the complete assignment plan obtained by applying any OAM can also be tested for optimality and can also be improved towards optimal, if not optimal.

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Appendix A: List of classical benchmark APs

Balanced Minimization AP	**Balanced Maximization AP/ Unbalanced Minimization AP**
Problem No.,(Author(s), Year,)	**Problem No.,(Author(s), Year)**
Problem 1 (Anwar Nsaif Jasim, 2017) [C_{ij}] 4×4= [8 20 15 17; 15 16 12 10; 22 19 16 30; 25 15 12 9]	**Problem 16 (R. S. Porchelvi, et al., 2018)** [P_{ij}] 4×4= [140 112 98 154; 90 72 63 99; 110 88 77 121; 80 64 56 88]
Problem 2 (M.D.H. Gamal, 2014) [C_{ij}] 4 ×4= [4 5 2 5; 3 1 1 4; 13 1 7 4; 12 6 5 9]	**Problem 17 (K.P. Ghadle et al., 2013))** [P_{ij}] 4×4= [8 26 17 11; 13 28 4 26; 38 19 18 15; 19 26 24 10]
Problem 3 (K. Ghadle, Y. Muley, 2015) [C_{ij}] 4 ×4= [18 26 17 11; 13 28 14 26; 38 19 18 15; 19 26 24 10]	**Problem 18 (A. Seethalakshmi et al., 2016))** [P_{ij}] 4×4= [42 35 28 21; 30 25 20 15; 30 25 20 15; 24 20 16 12]
Problem 4 (Rajendra B. Patel, 2018) [C_{ij}] 4×4= [10 24 30 15; 16 22 28 12; 12 20 32 10; 9 26 34 16]	**Problem 19 (Aderinto Y.O. et al., 2015)** [P_{ij}] 5×5= [8 26 34 22 16; 13 52 13 52 26; 38 19 36 30 76; 19 26 48 20 38; 46 30 46 22 44]
Problem 5 (H.D. Afroz, M.A. et al., 2017) [C_{ij}] 5×5= [8 4 2 6 1; 0 9 5 5 4; 3 8 9 2 6; 4 3 1 0 3; 9 5 8 9 5]	**Problem 20 (J.K. Sharma, 2017)** [P_{ij}] 5×5= [32 38 40 28 40; 40 24 28 21 36; 41 27 33 30 37; 22 38 41 36 36; 29 33 40 35 39]
Problem 6 (M.D.H. Gamal, 2014) [C_{ij}] 5×5= [12 8 7 0 4; 7 9 1 14 10; 9 0 12 6 7; 7 6 14 6 10; 9 6 12 10 6]	**Problem 21 (Hadi Basirzadeh, 2012)** [P_{ij}] 5×5= [5 11 10 12 4; 2 4 6 3 5; 3 12 5 14 6;; 6 14 4 11 7; 7 9 8 12 5]
Problem 7 (A. Ahamed et al., 2014) [C_{ij}] 5×5= [5 5 7 4 8; 6 5 8 3 7; 6 8 9 5 10; 7 6 6 3 6; 6 7 10 6 11]	**Problem 22 (A. Seethalakshmi, et al., 2017)** [P_{ij}] 5×5= [30 37 40 28 40; 40 24 27 21 36; 40 32 33 30 35; 25 38 40 36 36; 29 62 41 44 39]
Problem 8 (M.D.H. Gamal, 2014) [C_{ij}] 5×5= [7 8 4 15 12; 7 9 1 14 10; 9 1 1 6 7; 7 6 14 6 10; 1 6 12 10 6]	**Problem 23 (N. Sujatha, AVSN Murthy, 2015)** [P_{ij}] 4×3= [11 8 8; 4 33 5; 10 33 5; 1 25 10]
Problem 9 (K.P. Ghadle, et al., 2013)	**Problem 24 (N. Sujatha, AVSN Murthy, 2015)**

$[C_{ij}]$ 5×5= [12 8 7 15 4; 7 9 1 14 10; 9 6 12 6 7; 7 6 14 6 10; 9 6 12 10 6]	$[C_{ij}]$ 4×5= [5 7 11 6 5; 8 5 5 6 5; 6 7 10 7 3; 10 4 8 2 4]
Problem 10 (Anuj Khandelwal, 2014) $[C_{ij}]$ 5×5= [4 6 7 5 11; 7 3 6 9 5; 8 5 4 6 9; 9 12 7 11 10; 7 5 9 8 11]	**Problem 25 (J.K. Sharma, 2017)** $[C_{ij}]$ 5×4= [9 14 19 15; 7 17 20 19; 9 18 21 18; 10 12 18 19; 10 15 21 16]
Problem 11 (Anuj Khandelwal, 2014) $[C_{ij}]$ 5×5= [2 9 2 7 1; 6 8 7 6 1; 4 6 5 3 1; 4 2 7 3 1; 5 3 5 9 1]	**Problem 26 (Abdur Rashid, 2017)** $[C_{ij}]$ 6×4= [3 6 2 6; 7 1 4 4; 3 8 5 8; 6 4 3 7; 5 2 4 4; 5 7 6 2]
Problem 12 (Aderinto Y.O., Oke M.O., Raji R.A, 2015) $[C_{ij}]$ 5×5= [280 220 310 340 360; 230 200 220 280 310; 240 200 270 300 310 ; 60 130 60 130 180; 70 100 30 120 170]	**Problem 27 (Anju Khandelwal1, 2018)** $[C_{ij}]$ 6×4= [6 5 1 6; 2 5 3 7; 3 7 2 8; 7 7 5 9; 12 8 8 6; 6 9 5 10]
Problem 13 (A. Thirupathy, et al., 2015) $[C_{ij}]$ 5×5= [20 30 25 15 35; 25 10 40 12 28; 15 18 22 32 24; 29 8 43 10 40; 35 23 17 26 45]	**Problem 28 (J.G. Kotwal,T.S. Dhope, 2015)** $[C_{ij}]$ 8×4= [53 62 42 89; 18 35 39 55; 93 80 91 83; 79 23 96 56; 43 16 12 20; 87 70 87 31; 35 79 25 59; 27 16 12 20]
Problem 14 (J.K. Sharma, 2017) $[C_{ij}]$ 5×5= [85 75 65 125 75; 90 78 66 132 78; 75 66 57 114 69; 80 72 60 120 72; 76 64 56 112 68]	**Problem 29 (V. Yadaiah, et al., 2016)** $[C_{ij}]$ 8×5= [300 290 280 290 210; 250 310 290 300 200; 180 190 300 190 180; 320 180 190 240 170; 270 210 190 250 160; 190 200 220 190 140; 220 300 230 180 160; 260 190 260 210 180]
Problem 15 (M. Khalid, M. Sultana, F. Zaidi, 2014) $[C_{ij}]$ 6×6= [20 23 18 10 16 20; 50 20 17 16 15 11; 60 30 40 55 8 7; 6 7 10 20 25 9; 18 19 28 17 60 70; 9 10 20 30 40 55]	**Problem 30 (J.G.Kotwal, T.S. Dhope, 2015)** $[C_{ij}]$ 10×4= [11 8 9 8; 4 5 29 33; 10 5 29 33; 1 18 25 31; 23 22 33 30; 3 9 13 19; 6 8 27 32; 32 30 39 38; 36 35 31 21; 15 11 10 28]

Note: Problems 1 – 15 are balanced minimization case, 16 – 22 are balanced maximization case, 23 is unbalanced maximization case and 24 – 30 are unbalanced minimization case.