Research Article

Common Fixed Point Theorems Using Rational Contraction In Soft Compact Metric Spaces

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Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 28 April 2021

Abstract: The most important aim of the prevailing paper is searching for a new fixed point theorems using rational Contractions in the setting of soft compact metric spaces which generalizes the results of Sayyed $et\ al,$ [9]. In particular we investigate the soft compact space based rational expression with soft sets and get some new results.

Keywords: Soft sets(ss), soft compact metric space($scms$), rational expression, contraction mappings.

1 introduction

Molodtsov [14] presented the possibility of delicate sets as another numerical gadget for overseeing vulnerabilities and has demonstrated a few utilizations of this hypothesis in fixing numerous down to earth inconveniences in different controls like as economics, engineering, etc. Maji $et\ al.$ $[11, 12]$ examined delicate set thought in detail and gave a utilization of (ss) in determination making issues. Chen $et\ al.$ [2] worked on a fresh out of the plastic new meaning of markdown and expansion of parameters of (ss). Shabir and Naz [16] learned about soft topological territories and clarified the possibility of delicate factor by utilizing various methodologies. A profound conversation of soft set and compact spaces can be seen in ([1,4, 8 and 13]).

Compact metric spaces are one of the maximum vital lessons in popular topological areas [17]. They have many widely recognized homes which may be used in lots of disciplines. Zorlutuma $et\ al.$ [2] introduced compact soft areas round a smooth topology. Sayyed $et\ al.$ [9] defined the concept of $(scms)$ and discuss some results of $(scms)$. In the current work the concept of rational expression will be explore and generalize for $(scms)$ and proved some fixed point results. Throughout this paper we represent Soft sets($\boldsymbol{ss}$), soft compact metric space($\boldsymbol{scms}$), and soft metric space($\boldsymbol{sms}$).

2 preliminaries

Definition 2.1. ([14]) Let V be a universe and A be a set of parameters. Let $P(V)$ denote the power set of V. A pair (F, A) is called a (ss) over V, where F is a mapping given by $F: A \to P(V)$.

Definition 2.2. ([7]) A mapping $\tilde{d}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(A^*)$ is said to be a (sms) on the (ss) $\tilde{X}$ if

1) $\tilde{d}(P_{\Phi}{}^{u}, P_{\psi}{}^{v}) \geq \tilde{0}\ for\ all\ P_{\Phi}{}^{u}, P_{\psi}{}^{v} \in \tilde{X},$
2) $\tilde{d}(P_{\Phi}{}^{u}, P_{\psi}{}^{v}) = \tilde{0}\ if\ and\ only\ if\ P_{\Phi}{}^{u} = P_{\psi}{}^{v},$
3) $\tilde{d}(P_{\Phi}{}^{u}, P_{\psi}{}^{v}) = \tilde{d}(P_{\Phi}{}^{v}, P_{\psi}{}^{u})\ for\ all\ P_{\Phi}{}^{u}, P_{\psi}{}^{v} \in \tilde{X},$
4) $\tilde{d}(P_{\Phi}{}^{u}, P_{\psi}{}^{v}) \leq \tilde{d}(P_{\Phi}{}^{u}, P_{\psi}{}^{w}) + \tilde{d}(P_{\Phi}{}^{w}, P_{\psi}{}^{v})\ for\ all\ P_{\Phi}{}^{u}, P_{\psi}{}^{v}, P_{\psi}{}^{w} \in \tilde{X},$

The (ss) $\tilde{X}$ with the soft metric $\tilde{d}$ on $\tilde{X}$ is called a (sms) and denoted by $(\tilde{X}, \tilde{d}, \tilde{E})$ or $(\tilde{X}, \tilde{d})$.

Definition 2.3. ([7]) Let $(\tilde{X}, \tilde{d}, \tilde{E})$ be a (sms) and $\tilde{r}$ be a non negative soft real number. Then the soft set $B(P_{\Phi}{}^{u}, \tilde{r}) = \{P_{\psi}{}^{v} \in SP(\tilde{X}): \tilde{d}(P_{\Phi}{}^{u}, P_{\psi}{}^{v}) \precsim \tilde{r}\}$ is called soft open ball with center $P_{\Phi}{}^{u}$ and of radius $\tilde{r}$.

3 Soft Compact Metric Space

Definition 3.1. ([9]) A (sms) $(\tilde{X}, \tilde{d}, \tilde{E})$ be a $(scms)$. Let $\tilde{\mathbb{C}} = \{(C_i, A_i)\}$ be a family of soft open cover of $\tilde{X}$. Then $\tilde{\mathbb{C}}$ is called a soft open cover of $\tilde{X}$ if each soft point of $\tilde{X}$ is in some (C_i, A_i) in $\tilde{\mathbb{C}}$, that is, $\cup_{(C_i, A_i) \in \tilde{\mathbb{C}}} (C_i, A_i) = \tilde{X}$.

A sub collection of $\tilde{\tilde{\mathbb{C}}}^{\sim}$ of $\tilde{\mathbb{C}}$ whose union is again $\tilde{X}$ then $\tilde{\tilde{\mathbb{C}}}^{\sim}$ is called soft sub cover of $\tilde{X}$ in $\tilde{\mathbb{C}}$. If $\tilde{\tilde{\mathbb{C}}}^{\sim}$ is finite; it is called finite soft sub-cover of $\tilde{X}$ in $\tilde{\mathbb{C}}$.

Definition 3.2. ([9]) Let $(\tilde{X}, \tilde{d}, \tilde{E})$ be a soft metric is called $(scms)$ if every soft cover of $\tilde{X}$ has a finite soft subcover of $\tilde{X}$.

Definition 3.3. ([9]) Let $(\tilde{X}, \tilde{d}, \tilde{E})$ be a $(scms)$, and (Y, A) be a non-empty soft subset of $\tilde{X}$. then $\tilde{Y}$ is said to be soft compact in $\tilde{X}$ if $\tilde{Y}$ is soft compact as a subspace of $\tilde{X}$.

Definition 3.4. A (sms) $(\tilde{X}, \tilde{d}, \tilde{E})$ is called a soft compact if it is soft complete and soft totally bounded.

Proposition 3.5. Let $(\tilde{X}, \tilde{d}, \tilde{E})$ be a $(scms)$. If $(\tilde{X}, \tilde{d}, \tilde{E})$ is a soft sequence(cms), then $(\tilde{X}, \widetilde{d_\alpha})$ is a sequence(cms) for each $\alpha \in A$, where A is a countable set. Here $\widetilde{d_\alpha}$ stands for the soft metric for only parameter α.

Example 3.6. Let $\tilde{A} = \mathbb{N}, X = [0, 1]$ and let soft metric $\tilde{d}$ be defined as follows:

$$\tilde{d}(\tilde{p}_\alpha{}^u, \tilde{q}_{\alpha'}{}^v) = |\alpha - \alpha'| + |\tilde{p} - \tilde{q}| + |u - v|$$

$$\tilde{d}(\tilde{p}_\alpha{}^u, \tilde{q}_{\alpha'}{}^v) = \tilde{d}(\alpha, \alpha') - \left(\tilde{d}(\alpha, \alpha') - \tilde{d}(\tilde{p}, \tilde{q}) - \tilde{d}|u - v|\right)$$

$$\tilde{d}(\tilde{p}_\alpha{}^u, \tilde{q}_{\alpha'}{}^v) = \tilde{d}(\alpha, \alpha') - \tilde{d}(\alpha, \alpha')$$

Where $\mathbb{N}$ is a natural number of set. Therefore $(\tilde{X}, \tilde{d}, \tilde{E})$ be a $(scms)$

4 main results

In this section we use $\tilde{X} \rightarrow (\widetilde{C(X)})$ rational contraction mapping and prove some common fixed point theorems in the framework of $(scms)$.

Theorem 4.1. Let $(\tilde{X}, \tilde{d}, \tilde{E})$ be a $(\boldsymbol{SCMS})$and $\tilde{d}$ be a metric on $\tilde{X}$ such that $(\tilde{X}, \tilde{d})$ is complete soft metric set and let mappings $T, S: \tilde{X} \rightarrow (\widetilde{C(X)})$,satisfy the following conditions;

i) For each $\tilde{\alpha} \in \tilde{X}, T(\tilde{\alpha}), S(\tilde{\alpha}) \in$ closed set$(\tilde{X})$,

ii) $H\big(T(\tilde{\alpha}), S(\tilde{\beta})\big) \leq \psi_1\big[d(\tilde{\alpha}, T(\tilde{\alpha})) + d\big(\tilde{\beta}, S(\tilde{\beta})\big)\big] + \psi_2\big[d\big(\tilde{\alpha}, S(\tilde{\beta})\big) + d\big(\tilde{\beta}, T(\tilde{\alpha})\big)\big]$

Where ψ_1, ψ_2 are non-negative real numbers and $\psi_1 + \psi_2 < 1$. Then T and S has a common fixed point.

Proof. Let $\widetilde{\alpha_0} \in X, T(\widetilde{\alpha_0})$ is a non-empty closed set of $\tilde{X}$. We can choose that $\widetilde{\alpha_1} \in T(\widetilde{\alpha_0})$, for this $\widetilde{\alpha_1}$ by the same reason mentioned above $S(\widetilde{\alpha_1})$ is non-empty closed set of $\tilde{X}$.

Since $\widetilde{\alpha_1} \in T(\widetilde{\alpha_0})$ and $S(\widetilde{\alpha_1})$ are closed set of $\tilde{X}$, there exist $\widetilde{\alpha_2} \in S(\widetilde{\alpha_1})$ such that

$$d(\widetilde{\alpha_1}, \widetilde{\alpha_2}) \leq H(T(\widetilde{\alpha_0}), S(\widetilde{\alpha_1})) + \Phi,$$

Where $\Phi = max\left\{\frac{\psi_1 + \psi_2}{1-(\psi_1 + \psi_2)}, \frac{\psi_1 + \psi_2}{1-(\psi_1 + \psi_2)}\right\}$

$$\begin{aligned} d(\widetilde{\alpha_1}, \widetilde{\alpha_2}) &\leq H\big(T(\widetilde{\alpha_0}), S(\widetilde{\alpha_1})\big) + \Phi \\ &\leq \psi_1\big[d(\widetilde{\alpha_0}, T(\widetilde{\alpha_0})) + d\big(\widetilde{\alpha_1}, S(\widetilde{\alpha_1})\big)\big] + \psi_2[d(\widetilde{\alpha_0}, S(\widetilde{\alpha_1})) + d(\widetilde{\alpha_1}, T(\widetilde{\alpha_0}))] + \Phi \\ &\leq \psi_1[d(\widetilde{\alpha_0}, \widetilde{\alpha_1}) + d(\widetilde{\alpha_1}, \widetilde{\alpha_2})] + \psi_2[d(\widetilde{\alpha_0}, \widetilde{\alpha_2}) + d(\widetilde{\alpha_1}, \widetilde{\alpha_1})] + \Phi \\ &\leq \psi_1[d(\widetilde{\alpha_0}, \widetilde{\alpha_1}) + d(\widetilde{\alpha_1}, \widetilde{\alpha_2})] + \psi_2[d(\widetilde{\alpha_0}, \widetilde{\alpha_1}) + d(\widetilde{\alpha_1}, \widetilde{\alpha_2})] + \Phi \end{aligned}$$

$$d(\widetilde{\alpha_1}, \widetilde{\alpha_2}) \leq \frac{\psi_1 + \psi_2}{1-(\psi_1 + \psi_2)} d(\widetilde{\alpha_0}, \widetilde{\alpha_1}) + \Phi$$

$$d(\widetilde{\alpha_1}, \widetilde{\alpha_2}) \leq \Phi d(\widetilde{\alpha_0}, \widetilde{\alpha_1}) + \Phi$$

Thus for this $\widetilde{\alpha_2}$, $T(\widetilde{\alpha_2})$ is a non-empty closed set of $\tilde{X}$.

Since $\widetilde{\alpha_2} \in S(\widetilde{\alpha_1})$ and $S(\widetilde{\alpha_1})$ and $T(\widetilde{\alpha_2})$ are closed set of $\tilde{X}$, there exist $\widetilde{\alpha_3} \in T(\widetilde{\alpha_2})$

Such that

$$\begin{aligned} d(\widetilde{\alpha_2}, \widetilde{\alpha_3}) &\leq H(T(\widetilde{\alpha_2}), S(\widetilde{\alpha_1})) + \Phi^2 \\ &\leq \psi_1\big[d(\widetilde{\alpha_2}, T(\widetilde{\alpha_2})) + d\big(\widetilde{\alpha_1}, S(\widetilde{\alpha_1})\big)\big] \\ &\quad + \psi_2[d(\widetilde{\alpha_2}, S(\widetilde{\alpha_1})) + d(\widetilde{\alpha_1}, T(\widetilde{\alpha_2}))] + \Phi^2 \\ &\leq \psi_1[d(\widetilde{\alpha_2}, \widetilde{\alpha_3}) + d(\widetilde{\alpha_1}, \widetilde{\alpha_2})] + \psi_2[d(\widetilde{\alpha_2}, \widetilde{\alpha_2}) + d(\widetilde{\alpha_1}, \widetilde{\alpha_3})] + \Phi^2 \\ &\leq \psi_1[d(\widetilde{\alpha_2}, \widetilde{\alpha_3}) + d(\widetilde{\alpha_1}, \widetilde{\alpha_2})] + \psi_2[d(\widetilde{\alpha_1}, \widetilde{\alpha_2}) + d(\widetilde{\alpha_2}, \widetilde{\alpha_3})] + \Phi^2 \end{aligned}$$

$$\begin{aligned} d(\widetilde{\alpha_2}, \widetilde{\alpha_3}) &\leq \frac{\psi_1 + \psi_2}{1-(\psi_1 + \psi_2)} d(\widetilde{\alpha_1}, \widetilde{\alpha_2}) + \Phi^2 \\ &\leq \Phi d(\widetilde{\alpha_1}, \widetilde{\alpha_2}) + \Phi^2 \\ &\leq \Phi\{\Phi d(\widetilde{\alpha_0}, \widetilde{\alpha_1}) + \Phi\} + \Phi^2 \end{aligned}$$

$$d(\widetilde{\alpha_2}, \widetilde{\alpha_3}) \leq \Phi^2 d(\widetilde{\alpha_0}, \widetilde{\alpha_1}) + 2\Phi^2$$

Similarly this process continue and we get a sequence $\{\widetilde{\alpha_n}\}$ such that $\widetilde{\alpha_{n+1}} \in S(\widetilde{\alpha_n})$ or $\widetilde{\alpha_{n+1}} \in T(\widetilde{\alpha_n})$

And

$$d(\widetilde{\alpha_{n+1}}, \widetilde{\alpha_n}) \leq \Phi^n d(\widetilde{\alpha_0}, \widetilde{\alpha_1}) + n\Phi^n.$$

Suppose $0 \ll u$ be given, choose that, a natural number N_1 such that $\Phi^n d(\widetilde{\alpha_0},\widetilde{\alpha_1}) + n\Phi^n \ll u \;\forall\; n \geq N_1$

$$\Rightarrow\; d(\widetilde{\alpha_{n+1}},\widetilde{\alpha_n}) \ll u.$$

$\therefore \{\widetilde{\alpha_n}\}$ is a Cauchy sequence in $(\tilde{X},\tilde{d},\tilde{E})$ is a $(\boldsymbol{SCMS})\exists\; p \in \tilde{X}$ such that $\widetilde{\alpha_n} \to p$. So choose a natural number N_2 such that

$d(\widetilde{\alpha_n},p) \ll \frac{u(1-(\psi_1+\psi_2))}{2v(1+(\psi_1+\psi_2))}$

and

$d(\widetilde{\alpha_{n-1}},p) \ll \frac{u(1-(\psi_1+\psi_2))}{2v(\psi_1+\psi_2)} \forall\; n \geq N_2$.

$$\begin{aligned}
d(T(p),p) &\leq d(p,\widetilde{\alpha_n}) + d(\widetilde{\alpha_n},T(p))\\
&\leq d(p,\widetilde{\alpha_n}) + H(S(\widetilde{\alpha_{n-1}}),T(p))\\
&\leq d(p,\widetilde{\alpha_n}) + \psi_1\left[d\left(\widetilde{\alpha_{n-1}},S(\widetilde{\alpha_{n-1}})\right) + d\left(p,T(p)\right)\right]\\
&\qquad +\psi_2[d(\widetilde{\alpha_{n-1}},T(p)) + d(p,S(\widetilde{\alpha_{n-1}}))]\\
&\leq d(p,\widetilde{\alpha_n}) + \psi_1\left[d(\widetilde{\alpha_{n-1}},\widetilde{\alpha_n}) + d\left(p,T(p)\right)\right]\\
&\qquad +\psi_2[d(\widetilde{\alpha_{n-1}},T(p)) + d(p,\widetilde{\alpha_n})]\\
&\leq d(p,\widetilde{\alpha_n}) + \psi_1\left[d(\widetilde{\alpha_{n-1}},p) + d(p,\widetilde{\alpha_n}) + d\left(p,T(p)\right)\right]\\
&\qquad +\psi_2[d((\widetilde{\alpha_{n-1}},p) + (p,T(p)) + d(p,\widetilde{\alpha_n})]
\end{aligned}$$

$d(T(p),p) \leq \frac{\psi_1+\psi_2}{1-(\psi_1+\psi_2)} d(\widetilde{\alpha_{n-1}},p) + \frac{(1+(\psi_1+\psi_2)}{(1-(\psi_1+\psi_2)} d(\widetilde{\alpha_n},p) \forall\; n \geq N_2.$

$d(T(p),p) \ll \frac{u}{v}$ for all $v \geq 1$, we get $\frac{u}{v} - d(T(p),p) \in P$

And , as $n \to \infty$, we get $\frac{u}{v} \to 0$

And P is closed $-d(T(p),p) \in P$ but $d(T(p),p) \in P$.

Therefore $d(T(p),p) = 0$ and so $p \in T(p)$.

Similarly it can be established that $p \in S(p)$. Hence T and S has a common fixed point.

Theorem 4.2. Let $(\tilde{X},\tilde{d},\tilde{E})$ be a $(\boldsymbol{SCMS})$and $\tilde{d}$ be a metric on $\tilde{X}$ such that $(\tilde{X},\tilde{d})$ is complete soft metric set and let mappings $T,S\colon \tilde{X} \to \left(\widetilde{C(X)}\right)$

Satisfy the following conditions;

i. For each $\tilde{\alpha} \in \tilde{X}, T(\tilde{\alpha}), S(\tilde{\alpha}) \in$ closed set$(\tilde{X})$,

ii. $H\left(T(\tilde{\alpha}),S(\tilde{\beta})\right) \leq \psi_1\left[d\left(\tilde{\alpha},\tilde{\beta}\right) + d\left(\tilde{\beta},S\left(\tilde{\beta}\right)\right)\right] + \psi_2\left[d\left(\tilde{\beta},S(\tilde{\beta})\right) + d\left(\tilde{\alpha},T(\tilde{\alpha})\right)\right]$
$+\psi_3\left[d\left(\tilde{\beta},T(\tilde{\alpha})\right) + d\left(\tilde{\alpha},S(\tilde{\beta})\right)\right]$

where ψ_1,ψ_2,ψ_3 are non-negative real numbers and $\psi_1+\;\psi_2+\psi_3 < \frac{1}{3}$. Then T *and* S has a common fixed point.

Proof. Let $\widetilde{\alpha_0} \in \tilde{X}, T(\widetilde{\alpha_0})$ is a non-empty closed set of $\tilde{X}$. We can choose that $\widetilde{\alpha_1} \in T(\widetilde{\alpha_0})$, for this $\widetilde{\alpha_1}$ by the same reason mentioned above $S(\widetilde{\alpha_1})$ is non-empty closed subset of $\tilde{X}$.

Since $\widetilde{\alpha_1} \in T(\widetilde{\alpha_0})$ and $S(\widetilde{\alpha_1})$ are closed set of $\tilde{X}$, there exist $\widetilde{\alpha_2} \in S(\widetilde{\alpha_1})$ such that

$$d(\widetilde{\alpha_1},\widetilde{\alpha_2}) \leq H(T(\widetilde{\alpha_0}),S(\widetilde{\alpha_1})) + \Phi,$$

Where $\Phi = max\left\{\frac{\psi_1+\psi_2+\psi_3}{1-(\psi_1+\psi_2+\psi_3)},\frac{\psi_1+\psi_2+\psi_3}{1-(\psi_1+\psi_2+\psi_3)}\right\}$

$$\begin{aligned}
d(\widetilde{\alpha_1},\widetilde{\alpha_2}) &\leq H(T(\widetilde{\alpha_0}),S(\widetilde{\alpha_1})) + \Phi\\
&\leq \psi_1\left[d(\widetilde{\alpha_0},\widetilde{\alpha_1}) + d\left(\widetilde{\alpha_1},S(\widetilde{\alpha_1})\right)\right] + \psi_2\left[d(\widetilde{\alpha_1},S(\widetilde{\alpha_1})) + d\left(\widetilde{\alpha_0},T(\widetilde{\alpha_0})\right)\right]\\
&\qquad +\psi_3[d(\widetilde{\alpha_1},T(\widetilde{\alpha_0})) + d(\widetilde{\alpha_0},S(\widetilde{\alpha_1}))] + \Phi\\
&\leq \psi_1[d(\widetilde{\alpha_0},\widetilde{\alpha_1}) + d(\widetilde{\alpha_1},\widetilde{\alpha_2})] + \psi_2[d(\widetilde{\alpha_1},\widetilde{\alpha_2}) + d(\widetilde{\alpha_0},\widetilde{\alpha_1})]\\
&\qquad +\psi_3[d(\widetilde{\alpha_1},\widetilde{\alpha_1}) + d(\widetilde{\alpha_0},\widetilde{\alpha_2})] + \Phi\\
&\leq \psi_1[d(\widetilde{\alpha_0},\widetilde{\alpha_1}) + d(\widetilde{\alpha_1},\widetilde{\alpha_2})] + \psi_2[d(\widetilde{\alpha_1},\widetilde{\alpha_2}) + d(\widetilde{\alpha_0},\widetilde{\alpha_1})]\\
&\qquad +\psi_3[d(\widetilde{\alpha_1},\widetilde{\alpha_1}) + d(\widetilde{\alpha_0},\widetilde{\alpha_2}) + d(\widetilde{\alpha_2},\widetilde{\alpha_2})] + \Phi\\
&\leq \psi_1[d(\widetilde{\alpha_0},\widetilde{\alpha_1}) + d(\widetilde{\alpha_1},\widetilde{\alpha_2})] + \psi_2[d(\widetilde{\alpha_1},\widetilde{\alpha_2}) + d(\widetilde{\alpha_0},\widetilde{\alpha_1})]\\
&\qquad +\psi_3[d(\widetilde{\alpha_1},\widetilde{\alpha_1}) + d(\widetilde{\alpha_0},\widetilde{\alpha_2}) + d(\widetilde{\alpha_1},\widetilde{\alpha_2})] + \Phi
\end{aligned}$$

$d(\widetilde{\alpha_1},\widetilde{\alpha_2}) \leq \frac{\psi_1+\psi_2+\psi_3}{1-(\psi_1+\psi_2+\psi_3)} d(\alpha_o,\widetilde{\alpha_1}) + \Phi$

$d(\widetilde{\alpha_1},\widetilde{\alpha_2}) \leq \Phi d(\alpha_o,\widetilde{\alpha_1}) + \Phi$

Thus for this $\widetilde{\alpha_2},\; T(\widetilde{\alpha_2})$ is a non-empty closed set of $\tilde{X}$.

Since $\widetilde{\alpha_2} \in S(\widetilde{\alpha_1})$ and $S(\widetilde{\alpha_1})$ and $T(\widetilde{\alpha_2})$ are closed set of $\tilde{X}$, there exist $\widetilde{\alpha_3} \in T(\widetilde{\alpha_2})$ such that

$$\begin{aligned}
d(\widetilde{\alpha_2},\widetilde{\alpha_3}) &\leq H(T(\widetilde{\alpha_2}),S(\widetilde{\alpha_1})) + \Phi^2\\
&\leq \psi_1\left[d(\widetilde{\alpha_2},\widetilde{\alpha_1}) + d\left(\widetilde{\alpha_1},S(\widetilde{\alpha_1})\right)\right] + \psi_2\left[d(\widetilde{\alpha_1},S(\widetilde{\alpha_1})) + d\left(\widetilde{\alpha_2},T(\widetilde{\alpha_2})\right)\right]\\
&\qquad +\psi_3[d(\widetilde{\alpha_1},T(\widetilde{\alpha_2})) + d(\widetilde{\alpha_2},S(\widetilde{\alpha_1}))] + \Phi^2
\end{aligned}$$

$$\leq \psi_1[d(\widetilde{\alpha_2},\widetilde{\alpha_1}) + d(\widetilde{\alpha_1},\widetilde{\alpha_2})] + \psi_2[d(\widetilde{\alpha_1},\widetilde{\alpha_2}) + d(\widetilde{\alpha_2},\widetilde{\alpha_3})] + \psi_3[d(\widetilde{\alpha_1},\widetilde{\alpha_3}) + d(\widetilde{\alpha_2},\widetilde{\alpha_2})] + \Phi^2$$

$$\leq \psi_1[d(\widetilde{\alpha_2},\widetilde{\alpha_1}) + d(\widetilde{\alpha_1},\widetilde{\alpha_2})] + \psi_2[d(\widetilde{\alpha_1},\widetilde{\alpha_2}) + d(\widetilde{\alpha_2},\widetilde{\alpha_3})] + \psi_3[d(\widetilde{\alpha_1},\widetilde{\alpha_2}) + d(\widetilde{\alpha_2},\widetilde{\alpha_3}) + d(\widetilde{\alpha_2},\widetilde{\alpha_2})] + \Phi^2$$

$$d(\widetilde{\alpha_2},\widetilde{\alpha_3}) \leq \frac{\psi_1 + \psi_2 + \psi_3}{1-(\psi_1 + \psi_2 + \psi_3)} d(\widetilde{\alpha_1},\widetilde{\alpha_2}) + \Phi^2$$

$$\leq \Phi d(\widetilde{\alpha_1},\widetilde{\alpha_2}) + \Phi^2$$

$$\leq \Phi\{\Phi d(\widetilde{\alpha_0},\widetilde{\alpha_1}) + \Phi\} + \Phi^2$$

$d(\widetilde{\alpha_2},\widetilde{\alpha_3}) \leq \Phi^2 d(\widetilde{\alpha_0},\widetilde{\alpha_1}) + 2\Phi^2$

Similarly this process continue and we get a sequence $\{\widetilde{\alpha_n}\}$ such that $\widetilde{\alpha_{n+1}} \in S(\widetilde{\alpha_n})$ or $\widetilde{\alpha_{n+1}} \in T(\widetilde{\alpha_n})$ and $d(\widetilde{\alpha_{n+1}},\widetilde{\alpha_n}) \leq \Phi^n d(\widetilde{\alpha_0},\widetilde{\alpha_1}) + n\Phi^n$.

Suppose $0 \ll u$ be given, choose that, a natural number N_1 such that $\Phi^n d(\widetilde{\alpha_0},\widetilde{\alpha_1}) + n\Phi^n \ll u \; \forall \, n \geq N_1$

$$\Rightarrow d(\widetilde{\alpha_{n+1}},\widetilde{\alpha_n}) \ll u.$$

$\therefore \{\widetilde{\alpha_n}\}$ is a Cauchy sequence in $(\tilde{X},\tilde{d},\tilde{E})$ be a $(\boldsymbol{SCMS})$, $\exists\, p \in \tilde{X}$ such that $\widetilde{\alpha_n} \to p$. So choose a natural number N_2 such that

$d(\widetilde{\alpha_n},p) \ll \frac{u(1-(\psi_1+\psi_2+\psi_3))}{2v(1+(\psi_1+\psi_2+\psi_3))}$

and

$d(\widetilde{\alpha_{n-1}},p) \ll \frac{u(1-(\psi_1+\psi_2+\psi_3))}{2v(\psi_1+\psi_2+\psi_3)} \forall\, n \geq N_2$.

$$d(T(p),p) \leq d(p,\widetilde{\alpha_n}) + d(\widetilde{\alpha_n},T(p))$$

$$\leq d(p,\widetilde{\alpha_n}) + H(S(\widetilde{\alpha_{n-1}}),T(p))$$

$$\leq d(p,\widetilde{\alpha_n}) + \psi_1\left[d(\widetilde{\alpha_{n-1}},p) + d\big(p,T(p)\big)\right] + \psi_2\left[d(p,T(p)) + d\big(\widetilde{\alpha_{n-1}},S(\widetilde{\alpha_{n-1}})\big)\right] + \psi_3[d(p,S(\widetilde{\alpha_{n-1}})) + d(\widetilde{\alpha_{n-1}},T(p))]$$

$$\leq d(p,\widetilde{\alpha_n}) + \psi_1\left[d(\widetilde{\alpha_{n-1}},p) + d\big(p,T(p)\big)\right] + \psi_2[d(p,T(p)) + d(\widetilde{\alpha_{n-1}},\widetilde{\alpha_n})] + \psi_3[d(p,\widetilde{\alpha_n}) + d(\widetilde{\alpha_{n-1}},T(p))]$$

$$\leq d(p,\widetilde{\alpha_n}) + \psi_1\left[d(\widetilde{\alpha_{n-1}},p) + d\big(p,T(p)\big)\right] + \psi_2[d(p,T(p)) + d(\widetilde{\alpha_{n-1}},p) + d(p,\widetilde{\alpha_n})] + \psi_3[d(p,\widetilde{\alpha_n}) + d((\widetilde{\alpha_{n-1}},p) + (p,T(p))]$$

$d(T(p),p) \leq \frac{\psi_1+\psi_2+\psi_3}{1-(\psi_1+\psi_2+\psi_3)} d(\widetilde{\alpha_{n-1}},p) + \frac{1+(\psi_2+\psi_3)}{1-(\psi_1+\psi_2+\psi_3)} d(\widetilde{\alpha_n},p) \forall\, n \geq N_2$.

$d(T(p),p) \ll \frac{u}{v}$ for all $v \geq 1$, we get $\frac{u}{v} - d(T(p),p) \in P$

And

as $n \to \infty$, we get $\frac{u}{v} \to 0$ and P is closed $-d(T(p),p) \in P$ but $d(T(p),p) \in P$.

Therefore

$$d(T(p),p) = 0 \text{ and so } p \in T(p).$$

Hence T and S has a fixed point.

Corollary 4. 3. Let $(\tilde{X},\tilde{d},\tilde{E})$ be $(\boldsymbol{SCMS})$ and $\tilde{d}$ be a metric on $\tilde{X}$ such that $(\tilde{X},\tilde{d},\tilde{E})$ be a complete $(\boldsymbol{SCMS})$and let mappings $T,S: \tilde{X} \to C(\tilde{X})$

i. For each $\tilde{\alpha} \in X, T_1(\tilde{\alpha}), T_2(\tilde{\alpha}) \in$ closed set(X),

ii. $H\big(T_1(\tilde{\alpha}),T_2(\tilde{\beta})\big) \leq \psi a\left[d(\tilde{\alpha},\tilde{\beta}) + d\left(\tilde{\beta},T_2(\tilde{\beta})\right)\right] + \psi c\left[d(\tilde{\beta},T_1(\tilde{\alpha})) + d(\tilde{\alpha},T_2(\tilde{\beta}))\right]$

Where a and c are non-negative real numbers and $a + c < \frac{1}{2}$. Then T has a fixed point.

References

1. Bhardwaj, R.K., Rajput S.S., Choudhry S, Yadava, R.N. (2008),"Some fixed point theorems in compact metric spaces". Int. Journal of Math. Analysis, Vol. 2.2. No. 11, 543-550.
2. Chen, D., Tsang, E. C. C., Yeung, D. S. and Wang, X. (2005). The parameterization reduction of soft sets and its applications, *Compute. Math. Appl.* ***49(5);*** (2005) 757–763.
3. Chiney, M. and Samanta, S. K. (2015). Vector soft topology, *Ann. Fuzzy Math. Inform.* ***10(1);*** 45–64.
4. Das, S. and Samanta, S. K. (2012). Soft real sets, soft real numbers and their properties, *J. Fuzzy Math*. 20(3); 551–576.
5. Das, S. and Samanta, S. K. (2013). On soft complex sets and soft complex numbers, *J. Fuzzy Math.* 21(1***);*** 195–216.
6. Das, S. and Samanta, S. K. (2013). On soft metric spaces, *J. Fuzzy Math*. ***21(3);*** 707–734.

7. Das, S. and Samanta, S. K. (2013). Soft metric, *Ann. Fuzzy Math. Inform.* ***6 (1);*** 77–94.
8. Fisher, B. (1977). On three fixed point mappings for compact metric space, *Indian J. Pure and Appl. Math.* 8(4); 479-481.
9. Jalil, Sayyed., Reddy, surendranath, B. (2017). On soft compact, sequentially compact and locally compactness, Advances in Fuzzy mathematics. 12(4); 835-844.
10. Kabir, Q. A., Jamal, R., Mishra, J., Mohammad, M. and Bhardwaj, R. (2018)**.** New Common Fixed Point Results in Hyperconvex Ultrametric Spaces, International journal of pure and applied mathematics, Vol. 119, No. 10, pp. 1261-1271.
11. Maji, P. K., Biswas, R. and Roy, A. R. (2003). Soft set theory, *Compute. Math. Appl*. ***45(4);*** 555–562.
12. Maji, P. K., Roy, A. R. and Biswas, R. (2002). An application of soft sets in a decision making problem, *Compute. Math. Appl*. ***44(8);*** 1077–1083.
13. Mohammad, M., Jamal, R. and Kabir, Q. A. (2017). Soft cone metric spaces and common fixed point theorems, *International Journal of mathematical Archive,* **8(9);** 1-6.
14. Molodtsov, D. (1999). Soft set theory-first results, *Comput. Math. Appl.* ***37(4);*** 19-31
15. Ryszard E, (1989) “General Topology”, Heldermann Verlag Berlin.
16. Shabir, M. and Naz, M. (2011). On soft topological spaces, *Compute. Math. Appl.* ***61(7);*** 1786 – 1799.
17. Zorlutuna. I, Akdag, M., Min. W.K. and Atmaca, S. (2012) “Remarks on soft topological spaces”, Annals of Fuzzy mathematics and informatics Vol.3, pp. 171-185.