Research Article

On Interesting Integer Triple

R. Radha [1] and G. Janaki [2]

Assistant Professor [1], Associate Professor [2],
PG & Research Department of Mathematics,
Cauvery College for Women (Autonomous), Trichy-18.
Affiliated to Bharathidasan University
E-mail: [1] radha.maths@cauverycollege.ac.in & [2] janaki.maths@cauverycollege.ac.in
Corresponding Author- R. Radha, radha.maths@cauverycollege.ac.in

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Abstract: We search for non-zero distinct integer triples such that the sum of any two as well as twice the sum of these in each set is a nasty number.

Introduction:

The exciting part of Mathematics is the hypothesis of numbers where in Pythagorean triangles have been a subject important to various mathematicians and to the supporters of Mathematics, since it is a treasury wherein the quest for some, covered up association is an expedition. Number hypothesis is captivating because it has such an enormous number of open issues that appear to be open from an external perspective. Obviously, open issues in number hypothesis are open which is as it should be. Numbers, regardless of being straightforward, have a unimaginably rich design which we just comprehend partially. During the 20th century, Thus made a significant leap forward in the investigation of Diophantine conditions. His confirmation is one of the primary instances of the polynomial technique. His evidence affected a lot of later work in number hypothesis, including Diophantine conditions.

In [1-7], theory of numbers were discussed. Many mathematicians considered the problem of the existence of Diophantine triples with the property D(n) for any arbitrary integer n and also for any linear polynomials n [8-11]. In this paper, we exhibit the non-zero distinct integer triples such that the sum of any two as well as twice the sum of these in each set is a nasty number.

Definition:

A nasty number is a positive integer with at least four different factors such that the difference between the numbers in one pair of factors is equal to the sum of the numbers of another pair and the product of each pair is equal to the number.

Thus, a positive integer n is a nasty number, if n = ab = cd and a + b = c – d, where a, b, c, d are distinct positive integers.

Every integer n of the form $6(1^2+2^2+3^2+\ldots\ldots + k^2)$ is a nasty number.

Methodology:

Let a,b,c be three non-zero distinct integers such that

$$a+b=6t^2 \qquad (1)$$

$$a+c=6u^2 \qquad (2)$$

$$b+c=6v^2 \qquad (3)$$

Adding the equations (1),(2) and (3) we get,

$$2(a+b+c)=6w^2 \qquad (4)$$

Solving the equations (1) to (3) we get,

$$a=3t^2+3u^2-3v^2$$

$$b=3t^2-3u^2+3v^2$$

$$c=3u^2+3v^2-3t^2$$

Therefore (4) becomes,

$$2\left(3t^2+3u^2+3v^2\right)=6w^2$$

$$\Rightarrow t^2+u^2+v^2=w^2$$

Which is the well known Pythagorean equation. We present below two set of solutions.

SET 1

Three parametric solutions are given by

$$t = 2lm$$

$$u = 2ln$$

$$v = l^2 - m^2 - n^2$$

$$w = l^2 + m^2 + n^2$$

Substituting the values of t, u, v and w in a, b, c we have

$$a = 18l^2m^2 + 18l^2n^2 - 6m^2n^2 - 3(l^4 + m^4 + n^4)$$

$$b = 6l^2m^2 - 18l^2n^2 + 6m^2n^2 + 3(l^4 + m^4 + n^4)$$

$$c = 6l^2n^2 - 18l^2m^2 + 6m^2n^2 + 3(l^4 + m^4 + n^4)$$

The values represented by a, b and c satisfying the conditions are presented in the following table.

l	m	n	a	b	c	a+b	a+c	b+c	2(a+b+c)
2	1	3	372	-276	492	96	864	216	1176
1	1	1	21	3	3	24	24	6	54
2	2	2	336	48	48	384	384	96	864

SET 2

Four parametric solutions are given by

$$t = m^2 + n^2 - g^2 - h^2$$

$$u = 2(mg + nh)$$

$$v = 2(ng - mh)$$

$$w = m^2 + n^2 + g^2 + h^2$$

Few numerical example are given below.

m	n	g	h	a	b	c	a+b	a+c	b+c	2(a+b+c)
1	2	3	4	2604	-204	300	2400	2904	96	5400
1	1	1	2	123	-69	93	54	216	24	294
2	2	2	1	411	-357	453	54	864	96	1014

CONCLUSION

To Conclude, One may search for other patterns of integer triples under suitable constraints.

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