# Modified Modelling and Reliability Measure of Ammonia Synthesis Unit in a Fertilizer Plant 

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#### Abstract

The main focus of the paper is to discuss the reliability of the ammonia plant for transient state, consisting of six units in series. Ammonia plant is practically modelled to evaluate the reliability by victimisation of computer algebra system i.e. Mathematica. The reliability of each of its units is evaluated for the beneficial purpose of the plant.


Keywords: Transient state reliability, Practical Modelling, Markov process.

## 1. Introduction

Reliability of a system has become an integral part of research in the 20th century. It can be seen as one of the most effective decision-making tools so as to optimise the performance of a system over a period of time. Reliability analysis is performed based upon the repercussions of component failure rate on the failure rate of the system as whole. The sole purpose of this paper is to obtain the reliability of the modified system arising out of an ammonia synthesis unit in a fertilizer plant as described by Kumar and Tewari [1] and Garg et al. [9]. Kumar and Tewari [1] have discussed the performance evaluation and availability analysis of the system taken in steady state. Garg and Garg [9] have given the reliability analysis of the same system for transient state. However, this paper peeks into the reliability analysis of the modified system taken in time dependent transient state by utilising Markov birth-death process operated upon the mathematical model of the system and allied with the computer software "Mathematica" to solve the in-process system of complicated probabilistic equations. It also gives a variational study regarding reliability analysis of modified system.

Ammonia is one of the most extensively produced and used chemicals in the agriculture industry. Out of the total global energy, a unit percent is being used in the production of ammonia. Approximately $90 \%$ of the total produced ammonia is used in fertilizers. However, in recent times ammonia has also emerged as one of the most efficient refrigerants. Besides this, it also acts as a key component in the majority of household cleaning products as well, which are now an inseparable part of our lifestyles in these pandemic times.

The core of the whole production procedure of ammonia lies in the chemical process between two major inputs which are hydrogen and nitrogen. Apart from these, fuel gas mixture also contains noble gases like methane and argon. The input gases are exposed to thermal energy to raise the temperature and then streamed under pressure in the presence of a catalyst. After the removal of residual gases, chemical bonding between hydrogen and nitrogen results in synthesis of ammonia that is separated after cooling in a cold condenser.

### 2.1 The System Unit.

The system under consideration in this paper is a modified version of the ammonia synthesis unit described by Kumar and Tewari [1]. In this paper, the system is made up of six subunits placed in series [Fig.2].
"SU1: Subunit- 1 comprises 3 centrifugal compressors placed in series. The unit slips into a failed state when either of the compressors is failed.

SU2: Subunit-2 comprises 2 equipment's each, for hot heat exchanger and ammonia converter placed in parallel. The unit slips into a reduced capacity state when any one of them is in a failed state. However, the unit slips into a failed state only when both the equipment are failed.

SU3: Subunit-3 comprises a heat exchanger and its cold standby equipment. The unit works in full capacity until at least one of them is working. Hence the unit slips into a failed state when both the equipment are failed.

SU4: Subunit-4 is a cold condenser. Its failure slips the unit into a failed state.
SU5: Subunit-5 comprises ammonia separator and its cold standby equipment. The unit will work in full capacity until at least one of them is working. Hence the unit slips into a failed state when both the equipment are failed.

SU6: Subunit-6 comprises 3 heat exchangers placed in series. The unit slips into a failed state when either of the exchangers is failed."

### 2.2 Assumptions and Notations.

Assumptions used in the system are the same as given by Kumar and Tewari [1].
"Notations for depicting diversified states of its subunits are as under :
$>\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F}:$ Represents full operating states of all six subunits SU1 to SU6 respectively.
$>\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}:$ Represents the failed states of all six subunits SU1 to SU6 respectively.
$>\mathbf{B}_{1}$ : Represents the reduced state of subunit SU2.
$>$ Cs, Es: Represents standby states of subunit SU3 and SU5 respectively.
$>\boldsymbol{\alpha}_{1}, \boldsymbol{\alpha}_{2}, \boldsymbol{\alpha}_{3}, \boldsymbol{\alpha}_{4}, \boldsymbol{\alpha}_{5}, \boldsymbol{\alpha}_{6}:$ Mean failure rate in SU1, SU2, SU3, SU4, SU5, SU6.
$>\boldsymbol{\beta}_{\mathbf{1}}, \boldsymbol{\beta}_{\mathbf{2}}, \boldsymbol{\beta}_{\mathbf{3}}, \boldsymbol{\beta}_{\mathbf{4}}, \boldsymbol{\beta}_{\mathbf{5}}, \boldsymbol{\beta}_{\mathbf{6}}$ : Mean repair rate in SU1, SU2, SU3, SU4, SU5, SU6.
$>\mathbf{P}_{\mathrm{i}}(\mathrm{t})$ : Probability of the system unit working with full capacity at time ' t '; for $\mathrm{i}=0$.
$>$ Probability of the system unit working in cold standby state at time ' t '; for $\mathrm{i}=1,2,3$.
$>$ Probability of the system unit working in reduced capacity state at time ' t '; for $\mathrm{i}=4,5,6,7$.
$>$ Probability of the system unit working in failed state at time ' t '; for $\mathrm{i}=8-43$.
$>d / d t$ : Derivative w.r.t time."

: Full capacity state.

: Reduced capacity state.
: Failed state

## 3. Modified Modelling of the System:

With respect to all the transition states in Transition diagram given in the fig. 1 , following are the system of probabilistic differential equations:

$$
\begin{align*}
& {\left[\frac{d}{d t}+\sum_{\mathrm{i}=1}^{6} \alpha_{\mathrm{i}}\right] P_{0}(t)=\beta_{1} P_{8}(t)+\beta_{2} P_{5}(t)+\beta_{3} P_{3}(t)+\beta_{4} P_{9}(t)+\beta_{5} P_{1}(t)+\beta_{6} P_{10}(t) .}  \tag{1}\\
& {\left[\frac{d}{d t}+\sum_{\mathrm{i}=1}^{6} \alpha_{\mathrm{i}}+\beta_{5}\right] P_{1}(t)=\beta_{1} P_{11}(t)+\beta_{2} P_{6}(t)+\beta_{3} P_{2}(t)+\beta_{4} P_{12}(t)+\beta_{5} P_{13}(t)+\beta_{6} P_{14}(t)} \\
& +\alpha_{5} P_{0}(t) .  \tag{2}\\
& {\left[\frac{d}{d t}+\sum_{\mathrm{i}=1}^{6} \alpha_{\mathrm{i}}+\beta_{3}+\beta_{5}\right] P_{2}(t)=\beta_{1} P_{15}(t)+\beta_{2} P_{7}(t)+\beta_{3} P_{16}(t)+\beta_{4} P_{17}(t)+\beta_{5} P_{18}(t)+\beta_{6} P_{19}(t)} \\
& +\alpha_{3} P_{1}(t)+\alpha_{5} P_{3}(t) .  \tag{3}\\
& {\left[\frac{d}{d t}+\sum_{\mathrm{i}=1}^{6} \alpha_{\mathrm{i}}+\beta_{3}\right] P_{3}(t)=\beta_{1} P_{20}(t)+\beta_{2} P_{4}(t)+\beta_{3} P_{21}(t)+\beta_{4} P_{22}(t)+\beta_{5} P_{2}(t)+\beta_{6} P_{23}(t)} \\
& +\alpha_{3} P_{0}(t) .  \tag{4}\\
& {\left[\frac{d}{d t}+\sum_{i=1}^{6} \alpha_{i}+\beta_{2}+\beta_{3}\right] P_{4}(t)=\beta_{1} P_{24}(t)+\beta_{2} P_{25}(t)+\beta_{3} P_{26}(t)+\beta_{4} P_{27}(t)+\beta_{5} P_{7}(t)} \\
& +\beta_{6} P_{28}(t)+\alpha_{2} P_{3}(t)+\alpha_{3} P_{5}(t) .  \tag{5}\\
& {\left[\frac{d}{d t}+\sum_{i=1}^{6} \alpha_{i}+\beta_{2}\right] P_{5}(t)=\beta_{1} P_{29}(t)+\beta_{2} P_{30}(t)+\beta_{3} P_{4}(t)+\beta_{4} P_{31}(t)+\beta_{5} P_{6}(t)+\beta_{6} P_{32}(t)} \\
& +\alpha_{2} P_{0}(t) .  \tag{6}\\
& {\left[\frac{d}{d t}+\sum_{i=1}^{6} \alpha_{i}+\beta_{2}+\beta_{5}\right] P_{6}(t)=\beta_{1} P_{33}(t)+\beta_{2} P_{34}(t)+\beta_{3} P_{7}(t)+\beta_{4} P_{35}(t)+\beta_{5} P_{36}(t)} \\
& +\beta_{6} P_{37}(t)+\alpha_{2} P_{1}(t)+\alpha_{5} P_{5}(t)  \tag{7}\\
& {\left[\frac{d}{d t}+\sum_{\mathrm{i}=1}^{6} \alpha_{\mathrm{i}}+\beta_{2}+\beta_{5}\right] P_{7}(t)=\beta_{1} P_{38}(t)+\beta_{2} P_{39}(t)+\beta_{3} P_{40}(t)+\beta_{4} P_{41}(t)+\beta_{5} P_{42}(t)} \\
& +\beta_{6} P_{43}(t)+\alpha_{2} P_{2}(t)+\alpha_{3} P_{6}(t)+\alpha_{5} P_{4}(t) .  \tag{8}\\
& {\left[\frac{d}{d t}+\beta_{m}\right] P_{i}(t)=\alpha_{m} P_{j}(t) .}  \tag{9}\\
& m=1: i=8, j=0 ; i=11, j=1 ; i=15, j=2 ; i=20, j=3 ; i=24, j=4 ; i=29, j=5 \text {; } \\
& i=33, j=6 ; i=38, j=7 \text {. } \\
& m=2: i=25, j=4 ; i=30, j=5 ; i=34, j=6 ; i=39, j=7 \text {. } \\
& m=3: i=16, j=2 ; i=21, j=3 ; i=26, j=4 ; i=40, j=7 \text {. } \\
& m=4: i=9, j=0 ; i=12, j=1 ; i=17, j=2 ; i=22, j=3 ; i=27, j=4 ; i=31, j=5 \text {; } \\
& i=35, j=6 ; i=41, j=7 \text {. } \\
& m=5: i=13, j=1 ; i=18, j=2 ; i=36, j=6 ; i=42, j=7 \text {. } \\
& m=6: i=10, j=0 ; i=14, j=1 ; i=19, j=2 ; i=23, j=3 ; i=28, j=4 ; i=32, j=5 \text {; } \\
& i=37, j=6 ; i=43, j=7 \text {. }
\end{align*}
$$

with the initial conditions,

$$
P_{i}(t)=\left\{\begin{array}{l}
1 \text { for } i=0 \\
0 \text { for } i \neq 0
\end{array}\right.
$$

The given system is solved under real conditions.
The mathematical model so obtained is solved using Mathematica and the values of working states $P_{0}$, $\mathbf{P}_{1}, \mathbf{P}_{2}, \mathbf{P}_{3}, \mathbf{P}_{4}, \mathbf{P}_{5}, \mathbf{P}_{6}$ and $\mathbf{P}_{7}$ at a time $t$, are obtained as follows:
$\mathbf{P}_{\mathbf{0}}[\mathbf{t}]=7.13832 * 10^{-6} \mathrm{E}^{-3.13 \mathrm{t}}\left(1 . \mathrm{E}^{2.80465} \mathrm{t}+91.6426 \mathrm{E}^{2.85195}{ }^{\mathrm{t}}+1508.19 \mathrm{E}^{2.89127 \mathrm{t}}+0.408468 \mathrm{E}^{2.93749} \mathrm{t}+41.6024\right.$ $\mathrm{E}^{2.94231 \mathrm{t}}+1856.19 \mathrm{E}^{2.95507 \mathrm{t}}+43.3467 \mathrm{E}^{3.02005} \mathrm{t}+2801.67 \mathrm{E}^{3.06604} \mathrm{t}+3422.11 \mathrm{E}^{3.07885} \mathrm{t}+3.03195 * 10^{-21} \mathrm{E}^{3.08} \mathrm{t}+6.34258$ $\mathrm{E}^{3.0803} \mathrm{t}^{\mathrm{t}}+0.222969 \mathrm{E}^{3.08047} \mathrm{t}^{\mathrm{t}}+0.00850592 \mathrm{E}^{3.08058} \mathrm{t}+13.2801 \quad \mathrm{E}^{3.08227} \mathrm{t}+1159.79 \quad \mathrm{E}^{3.08527} \mathrm{t}^{\mathrm{t}}+2681.26 \mathrm{E}^{3.08908}$ ${ }^{\mathrm{t}}+0.00110192 \mathrm{E}^{3.09016 \mathrm{t}}+0.140827 \mathrm{E}^{3.09019} \mathrm{t}+4.99923 \mathrm{E}^{3.09025 \mathrm{t}}+0.926627 \mathrm{E}^{3.09074} \mathrm{t}+1206.04 \mathrm{E}^{3.0935}+10628.4 \mathrm{E}^{3.11906}$ ${ }^{t}+0.000152468 \mathrm{E}^{3.12003}{ }^{t}+0.0219708 \mathrm{E}^{3.12004} \mathrm{t}+0.863416 \mathrm{E}^{3.12005}{ }^{\mathrm{t}}+0.0631243 \mathrm{E}^{3.12011} \mathrm{t}+30.6989 \mathrm{E}^{3.12024 \mathrm{t}}+114590$. $\left.\mathrm{E}^{3.13 \mathrm{t}}\right)$
$\mathbf{P}_{1}[\mathbf{t}]=-7.30378 * 10^{-6} \mathrm{E}^{-3.13 \mathrm{t}}\left(1 . \mathrm{E}^{2.80465 \mathrm{t}}+44.9797 \mathrm{E}^{2.85195 \mathrm{t}}-28.7667 \mathrm{E}^{2.89127 \mathrm{t}}+0.409795 \mathrm{E}^{2.93749 \mathrm{t}}+20.4776\right.$ $\mathrm{E}^{2.94231 \mathrm{t}}-35.3127 \mathrm{E}^{2.95507 \mathrm{t}}+43.8893 \mathrm{E}^{3.02005 \mathrm{t}}+1467.17 \mathrm{E}^{3.06604 \mathrm{t}}-35.5958 \mathrm{E}^{3.07885 \mathrm{t}}+1.95785^{*} 10^{-21} \mathrm{E}^{3.08 \mathrm{t}}+0.0541949$ $\begin{array}{llllllllll}\mathrm{E}^{3.0803} & \mathrm{t} & -0.123465 & \mathrm{E}^{3.08047} & \mathrm{t}-0.00636122 & \mathrm{E}^{3.08058} & \mathrm{t}+7.70206 & \mathrm{E}^{3.08227} & \mathrm{t}+459.203 & \mathrm{E}^{3.08527} \\ \mathrm{t} & -57.4823 & \mathrm{E}^{3.08908}\end{array}$ ${ }^{\mathrm{t}}+0.000994248 \mathrm{E}^{3.09016 \mathrm{t}}+0.0620654 \mathrm{E}^{3.09019 \mathrm{t}}-0.105752 \mathrm{E}^{3.09025} \mathrm{t}+0.840802 \mathrm{E}^{3.09074 t}+545.693 \mathrm{E}^{3.0935 \mathrm{t}}-208.9 \mathrm{E}^{3.11906}$ ${ }^{\mathrm{t}}+0.000148286 \mathrm{E}^{3.12003 \mathrm{t}}+0.0104684 \mathrm{E}^{3.12004 \mathrm{t}}-0.0169597 \mathrm{E}^{3.12005} \mathrm{t}+0.061396 \mathrm{E}^{3.12011 \mathrm{t}}+14.629 \mathrm{E}^{3.12024 \mathrm{t}}-2239.88 \mathrm{E}^{3.13}$ $\left.{ }^{t}\right)$
$\mathbf{P}_{2}[\mathbf{t}]=7.76682 * 10^{-6} \mathrm{E}^{-3.13 \mathrm{t}}\left(1 . \mathrm{E}^{2.80465 \mathrm{t}}+45.9803 \mathrm{E}^{2.85195 \mathrm{t}}-31.1321 \mathrm{E}^{2.89127 \mathrm{t}}+0.152581 \mathrm{E}^{2.93749 \mathrm{t}}+12.1934 \mathrm{E}^{2.94231}\right.$ ${ }^{\mathrm{t}}-27.5828 \mathrm{E}^{2.95507 \mathrm{t}}-1.06347 \mathrm{E}^{3.02005 \mathrm{t}}-34.8926 \mathrm{E}^{3.06604} \mathrm{t}+0.843914 \mathrm{E}^{3.07885 \mathrm{t}}-2.1206 * 10^{-22} \mathrm{E}^{3.08 \mathrm{t}}+0.050553 \mathrm{E}^{3.0803 \mathrm{t}}-$ $0.115172 \mathrm{E}^{3.08047 \mathrm{t}}-0.00593409 \mathrm{E}^{3.08058 \mathrm{t}}-0.182467 \mathrm{E}^{3.08227 \mathrm{t}}-10.872 \mathrm{E}^{3.08527 \mathrm{t}}+1.35992 \mathrm{E}^{3.08908 \mathrm{t}}+0.000929301 \mathrm{E}^{3.09016}$ ${ }^{\mathrm{t}}+0.0580115 \mathrm{E}^{3.09019} \mathrm{t}-0.0988452 \mathrm{E}^{3.09025 \mathrm{t}}-0.0198853 \mathrm{E}^{3.09074 \mathrm{t}}-12.8992 \mathrm{E}^{3.0935} \mathrm{t}+4.91808 \mathrm{E}^{3.11906 \mathrm{t}}+0.000139267$ $\left.\mathrm{E}^{3.12003 \mathrm{t}}+0.00983171 \mathrm{E}^{3.12004 \mathrm{t}}-0.0159283 \mathrm{E}^{3.12005 \mathrm{t}}-0.00144523 \mathrm{E}^{3.12011 \mathrm{t}}-0.344351 \mathrm{E}^{3.12024} \mathrm{t}+52.6585 \mathrm{E}^{3.13 \mathrm{t}}\right)$
$\mathbf{P}_{3}[\mathbf{t}]=-7.59088^{*} 10^{-6} \mathrm{E}^{-3.13 \mathrm{t}}\left(1 . \mathrm{E}^{2.80465} \mathrm{t}+93.6814 \mathrm{E}^{2.85195} \mathrm{t}+1632.21 \mathrm{E}^{2.89127 \mathrm{t}}+0.152087 \mathrm{E}^{2.93749} \mathrm{t}+24.7722\right.$ $\mathrm{E}^{2.94231} \mathrm{t}+1449.87 \mathrm{E}^{2.95507 \mathrm{t}}-1.05032 \mathrm{E}^{3.02005 \mathrm{t}}-66.6298 \mathrm{E}^{3.06604 \mathrm{t}}-81.1321 \mathrm{E}^{3.07885 \mathrm{t}}+4.66097 * 10^{-22} \mathrm{E}^{3.08} \mathrm{t}+5.91636$ $\mathrm{E}^{3.0803} \mathrm{t}^{\mathrm{t}}+0.207993 \mathrm{E}^{3.08047} \mathrm{t}+0.00793479 \mathrm{E}^{3.08058} \mathrm{t}-0.314614 \mathrm{E}^{3.08227} \mathrm{t}-27.4591 \quad \mathrm{E}^{3.08527} \mathrm{t}-63.4332 \quad \mathrm{E}^{3.08908}$ ${ }^{\mathrm{t}}+0.00102994 \mathrm{E}^{3.09016 t}+0.131628 \mathrm{E}^{3.09019} \mathrm{t}+4.67274 \mathrm{E}^{3.09025 \mathrm{t}}-0.0219151 \mathrm{E}^{3.09074 \mathrm{t}}-28.5086 \mathrm{E}^{3.0935 \mathrm{t}}-250.222 \mathrm{E}^{3.11906}$ ${ }^{\mathrm{t}}+0.000143194 \mathrm{E}^{3.12003 \mathrm{t}}+0.0206345 \mathrm{E}^{3.12004 \mathrm{t}}+0.810904 \mathrm{E}^{3.12005 \mathrm{t}}-0.00148591 \mathrm{E}^{3.12011 \mathrm{t}}-0.722622 \mathrm{E}^{3.12024 \mathrm{t}}-2693.95$ $\left.\mathrm{E}^{3.13 \mathrm{t}}\right)$
$\mathbf{P}_{4}[\mathbf{t}]=7.76682 * 10^{-6} \mathrm{E}^{-3.13 \mathrm{t}}\left(1 . \mathrm{E}^{2.80465 \mathrm{t}}+45.9803 \mathrm{E}^{2.85195 \mathrm{t}}-31.1321 \mathrm{E}^{2.89127 \mathrm{t}}+0.152581 \mathrm{E}^{2.93749 \mathrm{t}}+12.1934 \mathrm{E}^{2.94231}\right.$ ${ }^{\mathrm{t}}-27.5828 \mathrm{E}^{2.95507 \mathrm{t}}-1.06347 \mathrm{E}^{3.02005 \mathrm{t}}-34.8926 \mathrm{E}^{3.06604 \mathrm{t}}+0.843914 \mathrm{E}^{3.07885 \mathrm{t}}+2.33225 * 10^{-23} \mathrm{E}^{3.08 \mathrm{t}}+0.050553 \mathrm{E}^{3.0803 \mathrm{t}}-$ $0.115172 \mathrm{E}^{3.08047 \mathrm{t}}-0.00593409 \mathrm{E}^{3.08058 \mathrm{t}}-0.182467 \mathrm{E}^{3.08227 \mathrm{t}}-10.872 \mathrm{E}^{3.08527 \mathrm{t}}+1.35992 \mathrm{E}^{3.08908 \mathrm{t}}+0.000929301 \mathrm{E}^{3.09016}$ ${ }^{\mathrm{t}}+0.0580115 \mathrm{E}^{3.09019} \mathrm{t}-0.0988452 \mathrm{E}^{3.09025} \mathrm{t}-0.0198853 \mathrm{E}^{3.09074 \mathrm{t}}-12.8992 \mathrm{E}^{3.0935} \mathrm{t}^{2}+4.91808 \mathrm{E}^{3.11906 \mathrm{t}}+0.000139267$ $\left.\mathrm{E}^{3.12003 \mathrm{t}}+0.00983171 \mathrm{E}^{3.12004 \mathrm{t}}-0.0159283 \mathrm{E}^{3.12005 \mathrm{t}}-0.00144523 \mathrm{E}^{3.12011 \mathrm{t}}-0.344351 \mathrm{E}^{3.12024 \mathrm{t}}+52.6585 \mathrm{E}^{3.13 \mathrm{t}}\right)$
$\mathbf{P}_{5}[\mathbf{t}]=-7.30378^{*} 10^{-6} \mathrm{E}^{-3.13 \mathrm{t}}\left(1 . \mathrm{E}^{2.80465 \mathrm{t}}+44.9797 \mathrm{E}^{2.85195 \mathrm{t}}-28.7667 \mathrm{E}^{2.89127 \mathrm{t}}+0.409795 \mathrm{E}^{2.93749 \mathrm{t}}+20.4776 \mathrm{E}^{2.94231}\right.$ ${ }^{\mathrm{t}}-35.3127 \mathrm{E}^{2.95507 \mathrm{t}}+43.8893 \mathrm{E}^{3.02005 \mathrm{t}}+1467.17 \mathrm{E}^{3.06604 t}-35.5958 \mathrm{E}^{3.07885 \mathrm{t}}-9.9606 * 10^{-22} \mathrm{E}^{3.08}+0.0541949 \mathrm{E}^{3.0803 \mathrm{t}}-$ $0.123465 \mathrm{E}^{3.08047 \mathrm{t}}-0.00636122 \mathrm{E}^{3.08058 \mathrm{t}}+7.70206 \mathrm{E}^{3.08227 \mathrm{t}}+459.203 \mathrm{E}^{3.08527 \mathrm{t}}-57.4823 \mathrm{E}^{3.08908 \mathrm{t}}+0.000994248 \mathrm{E}^{3.09016}$ ${ }^{t}+0.0620654 \mathrm{E}^{3.09019} \mathrm{t}-0.105752 \mathrm{E}^{3.09025} \mathrm{t}+0.840802 \mathrm{E}^{3.09074 t}+545.693 \mathrm{E}^{3.0935 \mathrm{t}}-208.9 \mathrm{E}^{3.11906 \mathrm{t}}+0.000148286 \mathrm{E}^{3.12003}$ $\left.{ }^{\mathrm{t}}+0.0104684 \mathrm{E}^{3.12004 \mathrm{t}}-0.0169597 \mathrm{E}^{3.12005 \mathrm{t}}+0.061396 \mathrm{E}^{3.12011 \mathrm{t}}+14.629 \mathrm{E}^{3.12024 \mathrm{t}}-2239.88 \mathrm{E}^{3.13 \mathrm{t}}\right)$
$\mathbf{P}_{6}[\mathrm{t}]=7.47307 * 10^{-6} \mathrm{E}^{-3.13 \mathrm{t}}\left(1 . \mathrm{E}^{2.80465} \mathrm{t}-1.75075 \mathrm{E}^{2.85195} \mathrm{t}+0.548685 \mathrm{E}^{2.89127} \mathrm{t}^{2}+0.411125 \mathrm{E}^{2.93749 \mathrm{t}}-0.794778\right.$ $\mathrm{E}^{2.94231}{ }^{\mathrm{t}}+0.671801 \mathrm{E}^{2.95507}{ }^{\mathrm{t}}+44.4387 \mathrm{E}^{3.02005}{ }^{\mathrm{t}}-53.5234 \mathrm{E}^{3.06604}{ }^{\mathrm{t}}+0.370258 \mathrm{E}^{3.07885}{ }^{\mathrm{t}}+9.30994 * 10^{-23} \quad \mathrm{E}^{3.08}$ ${ }^{\mathrm{t}}+0.000463074 \mathrm{E}^{3.0803 \mathrm{t}}-0.00425962 \mathrm{E}^{3.08047 \mathrm{t}}+0.00475728 \mathrm{E}^{3.08058 \mathrm{t}}+4.46696 \mathrm{E}^{3.08227 \mathrm{t}}-22.1568 \mathrm{E}^{3.08527 \mathrm{t}}+1.23234$ $\mathrm{E}^{3.08908} \mathrm{t}+0.000897095 \mathrm{E}^{3.09016 \mathrm{t}}-0.00269037 \mathrm{E}^{3.09019} \mathrm{t}+0.00223702 \mathrm{E}^{3.09025} \mathrm{t}+0.762926 \mathrm{E}^{3.09074} \mathrm{t}-23.0403 \mathrm{E}^{3.0935}$ $\mathrm{t}+4.1059 \mathrm{E}^{3.11906 \mathrm{t}}+0.00014422 \mathrm{E}^{3.12003 \mathrm{t}}-0.000419733 \mathrm{E}^{3.12004 \mathrm{t}}+0.000333133 \mathrm{E}^{3.12005 \mathrm{t}}+0.059715 \mathrm{E}^{3.12011 \mathrm{t}}-0.586476$ $\left.\mathrm{E}^{3.12024 \mathrm{t}}+43.7827 \mathrm{E}^{3.13 \mathrm{t}}\right)$
$\mathbf{P}_{7}[\mathrm{t}]=-7.94685 * 10^{-6} \mathrm{E}^{-3.13 \mathrm{t}}\left(1 . \mathrm{E}^{2.80465 \mathrm{t}}-1.7897 \mathrm{E}^{2.85195 \mathrm{t}}+0.593802 \mathrm{E}^{2.89127 \mathrm{t}}+0.153076 \mathrm{E}^{2.93749 \mathrm{t}}-0.47325 \mathrm{E}^{2.94231}\right.$ ${ }^{\mathrm{t}}+0.524744 \mathrm{E}^{2.95507 \mathrm{t}}-1.07678 \mathrm{E}^{3.02005} \mathrm{t}+1.2729 \mathrm{E}^{3.06604 \mathrm{t}}-0.00877816 \mathrm{E}^{3.07885 \mathrm{t}}-1.70513 * 10^{-23} \mathrm{E}^{3.08 \mathrm{t}}+0.000431956$ $\mathrm{E}^{3.0803 \mathrm{t}}-0.00397352 \mathrm{E}^{3.08047} \mathrm{t}+0.00443785 \mathrm{E}^{3.08058} \mathrm{t}-0.105825 \mathrm{E}^{3.08227} \mathrm{t}+0.524582 \mathrm{E}^{3.08527 \mathrm{t}}-0.0291546 \mathrm{E}^{3.08908}$ ${ }^{t}+0.000838494 E^{3.09016}{ }^{\mathrm{t}}-0.00251465 \mathrm{E}^{3.09019}{ }^{\mathrm{t}}+0.00209093 \mathrm{E}^{3.09025} \mathrm{t}_{-} 0.0180435 \mathrm{E}^{3.09074} \mathrm{t}^{+}+0.544632 \mathrm{E}^{3.0935} \mathrm{t}_{-}$ $0.0966642 \mathrm{E}^{3.11906} \mathrm{t}+0.000135448 \mathrm{E}^{3.12003} \mathrm{t}-0.000394205 \mathrm{E}^{3.12004} \mathrm{t}^{t}+0.000312872 \mathrm{E}^{3.12005} \mathrm{t}-0.00140566 \mathrm{E}^{3.12011}$ $\left.\left.{ }^{\mathrm{t}}+0.0138051 \mathrm{E}^{3.12024 \mathrm{t}}-1.02931 \mathrm{E}^{3.13 \mathrm{t}}\right)\right]$

Since the system is in working state when it is in either of the states $\mathrm{P}_{0}, \mathrm{P}_{1}, \mathrm{P}_{2}, \mathrm{P}_{3}, \mathrm{P}_{4}, \mathrm{P}_{5}, \mathrm{P}_{6}$ and $\mathrm{P}_{7}$ The reliability of the system is calculated as sum of probabilities of the working states $\mathrm{P}_{0}, \mathrm{P}_{1}, \mathrm{P}_{2}, \mathrm{P}_{3}, \mathrm{P}_{4}, \mathrm{P}_{5}, \mathrm{P}_{6}$ and $\mathrm{P}_{7}$ i.e.

$$
\begin{gather*}
\mathbf{R}[t]=\mathbf{P}_{0}[t]+\mathbf{P}_{1}[t]+\mathbf{P}_{2}[t]+\mathbf{P}_{3}[t]+\mathbf{P}_{4}[t]+\mathbf{P}_{5}[t]+\mathbf{P}_{6}[t]+\mathbf{P}_{7}[t]  \tag{11}\\
\text { Fig. } 1 \text { Transition Diagram. }
\end{gather*}
$$




Fig. 2

## 4. Performance analysis of the system:

We analyse the reliability with fluctuation in values of failure and repair rates.
a). Variational study:

We analyse the reliability of the system for various values of failure rates as: $a_{1}=0.001,0.006 \& 0.011$ and other values of failure and repair rates as: $a_{2}=0.001, a_{3}=0.005, a_{4}=0.001, a_{5}=0.001$, $\mathrm{a}_{6}=0.001, \mathrm{~b}_{1}=0.04, \mathrm{~b}_{2}=0.05, \mathrm{~b}_{3}=0.2, \mathrm{~b}_{4}=0.05, \mathrm{~b}_{5}=0.05, \mathrm{~b}_{6}=0.01$ are kept constant.

Similarly, we analyse the reliability of the system for various values of repair rates as: $b_{1}=0.004,0.008 \&$ 0.012 and other values of failure and repair rates as: $\mathrm{a}_{1}=0.001, \mathrm{a}_{2}=0.001, \mathrm{a}_{4}=0.001, \mathrm{a}_{5}=0.001, \mathrm{a}_{6}=0.001$, $\mathrm{b}_{2}=0.05, \mathrm{~b}_{3}=0.2, \mathrm{~b}_{4}=0.05, \mathrm{~b}_{5}=0.05, \mathrm{~b}_{6}=0.01$ are kept constant.

Table 1: Variation of SU1 with respect to failure rate with passage of time

| $\mathbf{T}$ | $\mathbf{a}_{1}=\mathbf{0 . 0 0 1}$ | $\mathbf{a}_{\mathbf{1}=} \mathbf{0 . 0 0 6}$ | $\mathbf{a}_{\mathbf{1}}=\mathbf{0 . 0 1 1}$ | $\mathbf{b}_{\mathbf{1}}=\mathbf{0 . 0 0 4}$ | $\mathbf{b}_{\mathbf{1}=\mathbf{0 . 0 0 8}}$ | $\mathbf{b}_{\mathbf{1}=\mathbf{0 . 0 1 2}}$ |
| :---: | :---: | :---: | :---: | :--- | :--- | :--- | :--- |
| $\mathbf{6}$ | 0.983567 | 0.957740 | 0.932645 | 0.983567 | 0.98353 | 0.927521 |
| $\mathbf{1 2}$ | 0.970134 | 0.925355 | 0.882974 | 0.970134 | 0.970724 | 0.867044 |
| $\mathbf{1 8}$ | 0.959139 | 0.900502 | 0.846301 | 0.95914 | 0.960856 | 0.816433 |
| $\mathbf{2 4}$ | 0.950028 | 0.881301 | 0.819095 | 0.950028 | 0.953006 | 0.773364 |
| $\mathbf{3 0}$ | 0.942376 | 0.866348 | 0.798792 | 0.942376 | 0.946542 | 0.736253 |
| $\mathbf{3 6}$ | 0.935868 | 0.854600 | 0.783534 | 0.935868 | 0.941066 | 0.704025 |
| $\mathbf{4 2}$ | 0.930269 | 0.845281 | 0.771974 | 0.930269 | 0.936324 | 0.675903 |
| $\mathbf{4 8}$ | 0.925400 | 0.837812 | 0.763132 | 0.9254 | 0.932145 | 0.651288 |
| $\mathbf{5 4}$ | 0.921124 | 0.831758 | 0.756296 | 0.921124 | 0.928413 | 0.629697 |
| $\mathbf{6 0}$ | 0.917334 | 0.826793 | 0.750945 | 0.917334 | 0.925044 | 0.610728 |

Similarly, variation of failure and repair rate of other states showing fluctuations as:
We analyse the reliability of the system for various values of failure rates as: $a_{3}=0.005,0.010 \& 0.015$ and other values of failure and repair rates as: $a_{1}=0.001, a_{2}=0.001, a_{4}=0.001, a_{5}=0.001, a_{6}=0.001, b_{1}=0.04, b_{2}=0.05, b_{3}=0.2, b_{4}=0.05, b_{5}=0.05, b_{6}=0.01$ are kept constant.

Similarly, we analyse the reliability of the system for various values of repair rates as: $b_{3}=0.2,0.4 \& 0.6$ and other values of failure and repair rates as: $a_{1}=0.001, a_{2}=0.001, a_{3}=0.005, a_{4}=0.001, a_{5}=0.001, a_{6}=0.001, b_{1}=0.04$, $\mathrm{b}_{2}=0.05, \mathrm{~b}_{4}=0.05, \mathrm{~b}_{5}=0.05, \mathrm{~b}_{6}=0.01$ are kept constant.

Table 2 : Variation of SU3 with respect to failure rate with passage of time

| $\mathbf{T}$ | $\mathbf{a}=\mathbf{0 . 0 0 5}$ | $\mathbf{a}=\mathbf{0 . 0 1 0}$ | $\mathbf{a} 3=\mathbf{0 . 0 1 5}$ | $\mathbf{\mathbf { b } _ { 3 } = \mathbf { 0 . 2 }}$ | $\mathbf{\mathbf { b } _ { 3 } = \mathbf { 0 . 4 }}$ | $\mathbf{\mathbf { b } _ { 3 } = \mathbf { 0 . 6 }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{6}$ | 0.983567 | 0.982968 | 0.981992 | 0.983567 | 0.983357 | 0.927706 |
| $\mathbf{1 2}$ | 0.970134 | 0.968953 | 0.967049 | 0.970134 | 0.969986 | 0.866773 |
| $\mathbf{1 8}$ | 0.959139 | 0.957686 | 0.955357 | 0.95914 | 0.959076 | 0.814871 |
| $\mathbf{2 4}$ | 0.950028 | 0.948479 | 0.945998 | 0.950028 | 0.950007 | 0.770415 |
| $\mathbf{3 0}$ | 0.942376 | 0.940806 | 0.938291 | 0.942376 | 0.942372 | 0.732177 |
| $\mathbf{3 6}$ | 0.935868 | 0.934305 | 0.931800 | 0.935868 | 0.935870 | 0.699166 |
| $\mathbf{4 2}$ | 0.930269 | 0.928721 | 0.926239 | 0.930269 | 0.930272 | 0.670571 |
| $\mathbf{4 8}$ | 0.925400 | 0.923868 | 0.921413 | 0.9254 | 0.925403 | 0.645729 |
| $\mathbf{5 4}$ | 0.921124 | 0.919608 | 0.917178 | 0.921124 | 0.921127 | 0.624091 |
| $\mathbf{6 0}$ | 0.917334 | 0.915833 | 0.913425 | 0.917334 | 0.917337 | 0.605200 |

Now we analyse the reliability of the system for various values of failure rates as: $\mathrm{a}_{6}=0.001,0.006 \& 0.011$ and other values of failure and repair rates as: $a_{1}=0.001, a_{2}=0.001, a_{3}=0.005, a_{4}=0.001, a_{5}=0.001, b_{1}=0.04, b_{2}=0.05$, $\mathrm{b}_{3}=0.2, \mathrm{~b}_{4}=0.05, \mathrm{~b}_{5}=0.05, \mathrm{~b}_{6}=0.01$ are kept constant.

Similarly, we analyse the reliability of the system for various values of repair rates as: $b_{6}=0.01,0.06 \& 0.11$ and other values of failure and repair rates as: $a_{1}=0.001, a_{2}=0.001, a_{3}=0.005$, $a_{4}=0.001, a_{5}=0.001, a_{6}=0.001, b_{1}=0.04, b_{2}=0.05, b_{3}=0.2, b_{4}=0.05, b_{5}=0.05$ are kept constant.

Table 3 : Variation of SU6 with respect to failure rate with passage of time

| $\mathbf{t}$ | $\mathbf{a}_{\mathbf{6}}=\mathbf{0 . 0 0 1}$ | $\mathbf{a}_{\mathbf{6}}=\mathbf{0 . 0 0 6}$ | $\mathbf{a}_{\mathbf{6}}=\mathbf{0 . 0 1 1}$ | $\mathbf{b}_{\mathbf{6}}=\mathbf{0 . 0 1}$ | $\mathbf{b}_{\mathbf{6}}=\mathbf{0 . 0 6}$ | $\mathbf{b}_{\mathbf{6}}=\mathbf{0 . 1 1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| $\mathbf{6}$ | 0.983567 | 0.960942 | 0.938848 | 0.983567 | 0.983746 | 0.941476 |
| $\mathbf{1 2}$ | 0.970134 | 0.927380 | 0.886588 | 0.970134 | 0.971655 | 0.910435 |
| $\mathbf{1 8}$ | 0.959139 | 0.898413 | 0.841770 | 0.95914 | 0.963000 | 0.893947 |


| $\mathbf{2 4}$ | 0.950028 | 0.8732071 | 0.803112 | 0.950028 | 0.956777 | 0.884874 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{3 0}$ | 0.942376 | 0.851105 | 0.769588 | 0.942376 | 0.952260 | 0.879623 |
| $\mathbf{3 6}$ | 0.935868 | 0.831600 | 0.740385 | 0.935868 | 0.948954 | 0.876410 |
| $\mathbf{4 2}$ | 0.930269 | 0.814288 | 0.714845 | 0.930269 | 0.946519 | 0.874335 |
| $\mathbf{4 8}$ | 0.925400 | 0.798852 | 0.692435 | 0.9254 | 0.944718 | 0.872931 |
| $\mathbf{5 4}$ | 0.921124 | 0.785031 | 0.672713 | 0.921124 | 0.943381 | 0.871945 |
| $\mathbf{6 0}$ | 0.917334 | 0.772614 | 0.655313 | 0.917334 | 0.942386 | 0.871233 |

b). Graphical Analysis


## 5. Conclusion

Study of the intended model infers that reliability of the system is updated with noticeable increment by decomposing the cold condenser and ammonia separator as individual units and adjoining a cold standby subunit along with ammonia separator. Deployment of an additional standby subunit also pushes the engineers and management towards manufacturing more robust units with increased life-longevity. Also, its reliability increases with increase in repair rate and decrease with increase of failure rate. Optimum reliability achieved is nearly about $80 \%$ to $90 \%$ which is beneficial for plant owners. Further, the variational study discussed above gives us a meaningful technical tool to troubleshoot the failure mechanisms and get rid of them in an effective way.

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