ON ib – continuous function In supra topological space

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Abstract

In this paper, we introduce a new class of sets and functions between topological spaces called supra ib- open sets and supra ib- continuous functions , respectively. We introduce the concepts of supra ib-open functions and supra ib-closed functions and investigate several properties of them.

Key words and phrases: supra ib-open set, supra ib- continuous function, supra ib-open function, supra ib-closed function and supra topological space.

Introduction

In 1983, A.S mashhour [6] introduced the supra topological spaces. In 1996, D. Andrijevic, [2][2] introduced and studied aclass of generalized open sets in a topological space called b-open sets.This class of sets contained in the class of β-open sets [1] and contains all semi open sets [4] and all pre-open sets [5]. In 2010, O,R. sayed and Takashi Noiri [7] introduce the concepts of supra b-open sets and supra b-continuous maps.In 2011, s.w Askander [3] introduced the concept of i-open set, respectively. Now, we introduce the concepts of supra ib-open sets and study some basic properties of them, Also, we introduce the concepts of supra ib-continuous functions, supra ib-open functions and supra ib-closed functions and investigate several properties for these classes of functions.

Preliminaries

Throughout this paper (X,T), (Y,ϭ) and (Z,V) means topological spaces. For a subset A of X, the interior and closure of A are denoted by int (A) and cl (A) respectively. A sub collection $M\subset 2^x$ is called a supra topology [6] on X ifø, X∈ M and M is closed under arbitrary union. (X,M) is called a supra topological space. The elements of M are said to be supra open sets in (X,M) and the complement of a supra open set is called a supra closed set. The supra closure of a set A denoted by cl^m (A), is the intersection of supra closed sets including. A. The supra interior of a set A, denoted by Int^m (A), is the union of supra open sets included in A. The supra topology M on X is associated with the topology T if T⊂M. Now before we study the basic properties of supra ib-open sets we recall the following definitions.

Definition 2. 1 [3]: A subset A of a topological space (X,T) is called
i-open set if there exists open set ($o\neq \emptyset, X$) such that A ⊆ cl ($A \cap o$). The complement of an i-open set is called i-closed set.

Definition 2. 2[6]:Let (X,M) be a supra topological space. A set A is called a supra semi – open set if A ⊆ cl^m (int^m (A) .
The complement of supra semi- open set is called supra semi-closed set.

Definition 2. 3[7]: let (X,M) be a supra topological space. A set A is called a supra b-open sets if $A\subset cl^m (Int^m (A) \cup int^m (cl^m(A))$.
The complement of a supra b-open set is called a supra b- closed set.

1- Supra ib-open sets

In this section, we introduce a new class of generalized open sets called supra ib-open sets and study some of their properties.

Definition 3.1: let (X,M) be a supra topological space. A set A is called a supra ib-open set if there exists supra b-open set (o≠ø,X)such that A ⊆ cl (A∩o). The complement of supra ib-open set is called a supra ib- closed set.

The class of all ib- open set in (X,M) is denoted by supra ib O (X,M)

Definition 3.2: Let A be a subset of a supra topological space (X,M) then

1. The intersection of all supra ib-closed sets containing A is called supra ib-closure of A, denoted by $cl\Big|_{ib}^{m}$ (A).
2. The union of all supra ib-open sets of X containing in A is called supra ib-interior denoted by $int\Big|_{ib}^{m}$ (A).

Remark 3.3 : It is clear that

1. ∅ , X is a supra ib-open set.

2- $\text{Int}\Big|_{ib}^{m}$ (A) is a supra ib-open set.
3- $\text{cl}\Big|_{ib}^{m}$ (A). is a supra ib-closed set.
4- A ⊆ $\text{cl}\Big|_{ib}^{m}$ (A); and A= $\text{cl}\Big|_{ib}^{m}$ (A) Iff A is a supra ib-closed set
5- $\text{Int}\Big|_{ib}^{m}$ (A) ⊆ A; and $\text{int}\Big|_{ib}^{m}$ (A) = A iff A is a supra ib-open set
6- X – $\text{Int}\Big|_{ib}^{m}$ (A) = $\text{cl}\Big|_{ib}^{m}$ (X-A)
7- X- $\text{cl}\Big|_{ib}^{m}$ (A) = $\text{Int}\Big|_{ib}^{m}$ (X-A)
8- $\text{Int}\Big|_{ib}^{m}$ (A) ∪ $\text{Int}\Big|_{ib}^{m}$ (B) ⊆ $\text{Int}\Big|_{ib}^{m}$ (A∪B)
9- $\text{cl}\Big|_{ib}^{m}$ (A∩B) ⊆ $\text{cl}\Big|_{ib}^{m}$ (A)∩ $\text{cl}\Big|_{ib}^{m}$ (B)

Theorem 3.4: Every supra – open set is supra ib- open set.

Proof : It is obvious.

Theorem 3.5: Every supra b- open set is supra ib- open set.

Proof: let (X,M) be a supra topological space, A ≠ X, ∅ be a supra b-open set in (X,M).

Since A ⊆ cl (A) , A∩A= A

Then A ⊆ cl (A∩A) when A ≠ ∅, X, (A a supra b-open set)

Then A is a supra ib-open set.

The following example show that the converse of theorem 3. 5 are not true in general.

Example 3.6: let (X,M) be a supra topological space

Where X= {1,2,3} and M= {∅, X, {1,2}, {2,3}} then {3} is a supra ib-open set, but is not supra b-open set.

Theorem 3.7: Every supra semi-open set is supra ib-open set.

Proof:

Let A be a supra semi- open set in (X,M) then A ⊆ cl^m (Int^m (A))

Hence A ⊆ cl^m (Int^m (A)) ∪ int^m (cl^m (A)) and A is supra b-open set

Then by (theorem 3.5) A is a supra ib-open set.

The following example show that the converse of theorem 3.7 are not true in general.

Example 3.8: let (X,M) be a supra topological space where X= {a,b,c} and M = {∅, X, {a} , {a,b} , {b,c}}, {a,c} is a supra ib- open set , but is not supra semi – open set.

4-Supra ib-continuous function

As an application of supra ib- open set, we introduce a new type of continuous function called a supra ib-continuous function and obtain some of their properties and characterizations.

Definition 4.1: let (X,T) and (y,σ) be two topological spaces and M be an associated supra topology with T . A function F: (X, T) → (y,σ) is called a supra ib-continuous function if the inverse image of each open set in y is a supra ib-open set in X.

Theorem 4.2: Every continuous function is supra ib-continuous function.

Proof: Let F: (X, T) →(y,σ) be continuous function and A is open set in y. then F^{-1} (A) is an open set in X. since M is associated with T , then T ⊆ M therefore F^{-1} (A) is a supra open set in X and it is a supra ib-open set in X (by theorem 3.4). Hence F is supra ib-continuous function.

The following example show that the converse of (theorem 4.2) are not true in general.

Example 4.3: let X= {1,2,3} and T = {∅,X,{1,2}} be a topology on X. The supra topology M is defined as follows M ={∅,X,{1}.{1,2}} let F: (X, T) → (X, T) be a function defined as follows: F (1) =1, F (2) =3 F (3) =2 the inverse image of the open set {1,2} is {1,3} which is not an open set but it is a supra ib-open set. Then F is supra ib-continuous function but is not continuous function.

Theorem 4.4: Every supra semi – continuous function is supra ib-continuous function.

Proof: Let F: (X, T) → (y,σ) be supra semi- continuous function and A is open set in y .Then F^{-1} (A) is supra semi-open set in X. since every supra semi- open set is supra ib- open set (by theorem 3.7) then F^{-1} (A) is supra ib-open set in X. Hence F is supra ib –continuous function.

The following example show that the converse of theorem 4.4 are not true in general.

Example 4.5: Let X = {1,2,3,4} and T={∅, X, {1,3} , {2,4}} be a topology on X, the supra topology M is defined as follows M = {∅, X, {1,3},{2,4},{1,3,4}}, Y = {x,y,z} and σ= {∅,Y,{z}} be a topology on Y. let F: (X, T) →(Y,σ) be a function defined as follows F (1)= y, F (2)= F (3) = z, F (4) = x,

The inverse image of the open set {z} is {2,3} which is a supra ib-open set but is not supra semi-open set ,then F is supra ib- continuous function but is not supra semi – continuous function.

Theorem 4.6: Every supra b-continuous function is supra ib- continuous function.

Proof: Let F: (X, T) → (y,σ) be supra b- continuous function and A is open set in y. then F^{-1} (A) is a supra b-open set in X, since every supra b –open set is a supra ib-open set (by theorem 3.5) then F^{-1} (A) is supra ib- open set in X. Hence F is supra ib- continuous function.

The following example show that the converse of theorem 4.6 are not true in general.

Example 4.7: Let X = {1,2,3} and T ={∅, X, {1} , {1,2}} be a topology on X the supra topology M is defined as follows M = { ∅ , X, {1}, {1,2} , {2,3}} Y= {x,y,z} and σ={∅, Y,{x}} be a topology on Y, let F: (X, T) → (Y,σ) be a function defined as follows F (1) = F (2) =z , F (3)= x. The inverse image of the open set {x} is {3} which is a supra ib- open set but is not a supra b-open set. Then F is supra ib- continuous function but is not supra b-continuous function.

Theorem 4.8: let (X,T) and (Y,σ) be two topological spaces and M be an associated supra topology with T. Let F: (X, T) → (Y,σ) then F is a supra ib- continuous function if and only if the inverse image of a closed set in Y is a supra ib-closed set in X.

Proof: Let F: (X, T) → (Y,σ) be a supra ib- continuous function ↔let A be a closed set in Y↔ then A^c is an open set in Y↔ then F^{-1} (A^c) is a supra ib- open set↔ It follows that F^{-1} (A) is a supra ib- closed set in X.

Theorem 4.9: let (X,T) and (Y,σ) be two topological spaces , M and V be the associated supra topologies with T and σ, respectively. Then F:(X, T) → (Y,σ) is a supra ib- continuous function, if one of the following holds:

1. F^{-1} (int$\Big|_{ib}^{v}$ (B)) ⊆ in t (F^{-1} (B)) for every set B in Y.
2. Cl(F^{-1}(B)) ⊆ F^{-1} (cl$\Big|_{ib}^{v}$ (B)) for every set B in Y.
3. F (cl (A)) ⊆ cl$\Big|_{ib}^{m}$ (F(A)) for every set A in X.

Proof : let B be any open set of Y. if condition (1) is satisfied, then F^{-1} (int$\Big|_{ib}^{v}$ (B)) ⊆ int (F^{-1} (B)), we get F^{-1}(B) ⊆ int (F^{-1} (B)). Therefore F^{-1} (B) is an open set. Every open set is supra ib- open set. Hence F is a supra ib- continuous function.

If condition (2) is satisfied, then by theorem (4.8) we can easily prove that F is a supra ib - continuous function.

Let condition (3) be satisfied and B be any open set of Y. Then F^{-1} (B) is a set in X and F (cl (F^{-1}(B)) ⊆ cl$\Big|_{ib}^{m}$ (F(F^{-1}(B)). This implies F (cl F^{-1} (B)) ⊆ cl$\Big|_{ib}^{m}$ (B). This is nothing but condition (2). Hence F is a supra ib- continuous function.

5-Supra ib- open functions and supra ib-closed functions

Definition 5.1: A function F: (X, T) → (Z,V) is called a supra ib- open (resp., supra ib closed) if the image of each open (resp. closed) set in X is supra ib- open (resp., supra ib- closed) set in (Z,V).

Theorem 5.2: A function F: (X, T) → (Z,V) is supra ib-open function if and only if F(in t (A)) ⊆ int$\Big|_{ib}^{v}$(F(A)) for each set A in X.

Proof: suppose that F is a supra ib-open function. Since int (A) ⊆ A then F (int (A)) ⊆ F (A).By hypothesis, F (int (A)) is a supra ib- open set and int $\Big|_{ib}^{v}$ (F(A)) is the largest supra ib-open set contained in F(A). Hence F(int (A)) ⊆ int$\Big|_{ib}^{v}$(F(A)).

Conversely, suppose A is an open set in X then F(int(A)) ⊆ int$\Big|_{ib}^{v}$(F(A)).Since int (A) = A, then F(A) ⊆ int$\Big|_{ib}^{v}$ (F(A)).Therefore F(A) is a supra ib- open set in (Z,V) and F is a supra ib- open function.

Theorem 5.3: A function F: (X,T) → (Z,V) is supra ib- closed if and only if cl$\Big|_{ib}^{v}$ (F(A)) ⊆ F (cl (A)) for each set A in X.

Proof: suppose F is a supra ib-closed function. Since for each set A in X, cl (A) is closed set in x, then F (cl(A)) is a supra ib-closed set in Z. Also, since F (A) ⊆ F(cl(A)), then cl$\Big|_{ib}^{v}$(F(A)) ⊆ F (cl(A)). conversely, let A be a closed set in x. Since C$l\Big|_{ib}^{v}$ (F(A)) is the smallest supra ib-closed set contining F (A), then F(A) ⊆ cl$\Big|_{ib}^{v}$ (F(A)) ⊆ F(cl(A) = F(A). thus, F(A)= cl$\Big|_{ib}^{v}$ F(A) Hence, F(A) is a supra ib-closed set in Z. Therefore, f is a supra ib-closed function.

Theorem 5.4: let (X,T), (Y,ϭ) and (Z,V) be three topological space and F:(X,T) →(Y,ϭ) and g: (Y,ϭ) →(Z,V) be two functions, then.

1- If goF is supra ib-open and F is continuous surjective, then g is supra ib- open function.
2- If goF is open and g is supra ib- continuous injective, then F is supra ib-open function.

Proof:

1- Let A be an open set in Y. then F^{-1} (A) is an open set in X. since goF is a supra ib-open function, then (gof) (F^{-1}(A)) = g (F(F^{-1}(A)) = g (A) (because f is surjective) is a supra ib- open set in Z. therefore, g is supra ib-open function.
2- Let A be an open set in X, then g (F(A)) is an open set in Z, therefore, g^{-1} (g F(A)) = F(A) (because g is injective) is a supra ib-open set in Y. Hence, F is a supra ib-open function.

Theorem 5.5: let (X,T) and (y,ϭ) be two topological spaces and F: (X,T) → (Y,ϭ) be a bijective function, then the following are equivalent:

1- F is a supra ib-open function.
2- F is a supra ib-closed function.
3- F^{-1} is a supra ib-continuous function.

Proof:

(1) →(2): let B is a closed set in X. Then X-B is an open set in X and by (1) F (X-B) is a supra ib-open set in Y. since F is bijective, then F (X-B) = Y-F(B). Hence, F (B) is a supra ib- closed set in Y. Therefore, F is a supra ib- closed function.

(2) → (3): let F is a supra ib- closed function and B a closed set in x. since F is bijective then $(F^{-1})^{-1}$ (B) = F (B) which is a supra ib-closed set in Y. therefore, by theorem (4.8), F is a supra ib- continuous function.

(3) →(1): let A be an open set in X. since F^{-1} is a supra ib-continuous function, then $(F^{-1})^{-1}$ (A) = F (A) is a supra ib-open set in Y. Hence, F is a supra ib-open function.

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