Comparison and Validation of Optimal Path Lengths in the Presence of Elliptical Forbidden Regions between CGA and V-Rep

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Abstract: This paper presents Comparison and Validation of Optimal path lengths in the presence of Elliptical Forbidden Regions between CGA and V-Rep simulation software. A systematic approach was made for developing results by applying a suitable CGA methodology that meets the solution requirements of routing problems that involve path planning procedures. CGA technique helps in reducing the evaluation time and also directly generates the shortest path for navigation to reach its destination point. The solutions thus obtained by CGA Technique absolutely agreed with the solutions by V-Rep and even produced optimal solutions when compared with V-Rep.

Keywords: Validation, Optimal path, Elliptical, Forbidden region, CGA, V-Rep.

1. Introduction

Locational situations normally have a bearing on various decision criteria like nearness to market, raw material, power, water etc., In due course of time, the very factors which were considered to be limitations might become influencing parameters, with change in operating environment. For example, places which are devoid of social necessities like health and education over a period of time, with change in social environment might become centers of excellence proving wrong earlier considerations drawn and thus influencing performance parameters. The above situation explains uncertain and unpredictable behavior of the factors influencing a location. Therefore many researchers emphasized more on distance as performance criteria, which seldom varies in spite of changing social environment.

In almost all works reported in locational aspects, researchers have developed methodologies that centered around determination of shortest unconstrained rectilinear / Euclidean distance depending on whether the problem involved is layout or locational. A significant number of works reported methods that yielded alternate feasible paths between a given pair of facilities under the influence of a single barrier / forbidden region/obstacle. Understandably, in spite of the potential strengths of the methods developed, they could not preclude the need for computation of path distances in more than one route before identifying the shortest path. This requires more computations and memory requirements resulting in the time taken being more.

Different researchers have contributed works towards generation of scientific methodologies to solve problems involving locational aspects and path planning. Several authors contributed works that reported methods to compute shortest un-constrained paths between interacting facilities in the presence of single/multi forbidden / barrier regions. Works reported, hither to, by various researchers have been reviewed here under.

Brady et al. [1] used interactive graphics approach to solve single and multiple facility location problems with min-max objective function in the presence of a forbidden region with an arbitrary shape. But there is a limitation in the developed methodology as paths are permitted through the forbidden region but the locations were not allowed with in it. Katz and Cooper [2] developed a mathematical method to solve single facility location problem with Euclidean distance norm in the presence of a single circular forbidden region. Ravindranath, K. et al. [3,4] developed a methodology to determine the shortest obstacle free path in the presence of a rectangular as well as circular forbidden region for both Euclidean and rectilinear norms of travel. The limitation of the method is regions which cannot be approximated to regular geometric shapes cited above shall make the method invalid.

Seshasayee, K.R., et al. [5] developed a methodology to solve a single facility location problem in the presence of convex polygonal forbidden region for both Euclidean and rectilinear norms using the direct search method due to Hook and Jeeves, but the method gives solution only for convex polygonal forbidden cases and exhibits severe limitations under the conditions of non-convex approximations. Raju, K.V.L., et al. [6] developed a methodology named sloping line search (SLS) technique to solve facility location problems in the presence of single convex / non-convex, regular / irregular polygonal forbidden regions. However SLS technique computes all alternate feasible paths before identifying the best and therefore involves more computations.Mohan, K.V.V., et al. [9] used SLS technique for path planning of robots in the presence of single convex / non-convex, regular / irregular polygonal forbidden regions and calculated all possible un-constrained paths to prepare a library of paths for networking analysis.

Ravindranath, P., [11], developed a technique called Coordinate Reference Frame (CRF) technique to compute various alternative shortest un-constrained paths involving single convex / non-convex polygonal forbidden regions for static source and target conditions. This method could overcome some of the limitations experienced in SLS technique, and therefore observed to be the best technique to date among all the heuristics available. However, he has applied it to problems involving collision free paths for robot applications. This also is a static application and does not involve optimization of a new location.

Reddy, V.V.R., et al. [10] has developed and used Coordinate Reference Frame technique in conjunction with CGA (CGACRF), SOD and RSA which provided solutions effectively. The in-transit data storage and process requirements are comparatively high when viewed against methods reported earlier like SLS heuristic. However, the limitations observed i.e., shadow zones in SLS technique are effectively overcome as CRF can provide pool proof solutions to both the cases of convex and non-convex polygonal forbidden region problems. Prakash M.A., et al. [12, 13] have applied CGA technique for the facility location in the presence of two elliptical forbidden regions and also mixed forbidden regions

A brief review of literature related to computations of shortest paths between a pair of facilities under the influence of elliptical forbidden regions has been made in this work. Comprehensive study has been made by considering numerous locational situations leading to identification and development of shortest path basing on a criterion developed by Reddy, V.V.R., et al. [10] named as Center of Gravity Approach (CGA).

Markedly review of the earlier works reveals the fact that the CGA technique has been applied for optimum facility location problems involving elliptical forbidden regions but the validation of the results has not been done. The author in the present work made an attempt to identify the shortest path in the presence of elliptical forbidden regions and validate for its accuracy by V-Rep simulation software.

2. Computation of Shortest Un-constrained Path

To determine the shortest path between any two planar points in the presence of a single Elliptical Forbidden region under constrained situations.

Understandably infinite number of unconstrained paths P_1, P_2, P_3.… P_n, not passing through the forbidden region, can be developed (Refer to Fig. 1).

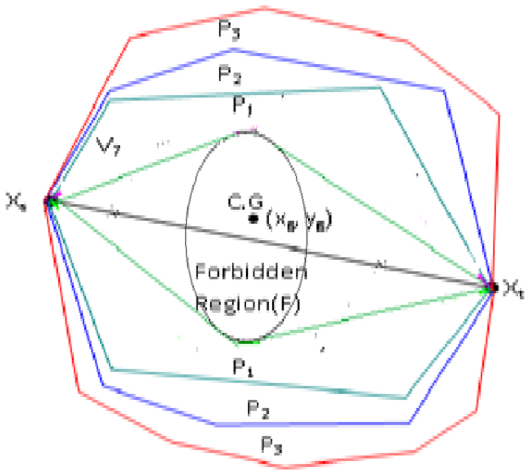


Fig. 1. Feasible alternate paths between two planar points.

However it is technically infeasible and also undesirable to compute the sum of the distances along those infinite paths for identifying the shortest among them. Further, it can be observed that those infinite numbers of unconstrained paths are divided by the shortest constrained path into two halves. Further inferences can be drawn that there shall be only one path in each half of the plane that is shortest and also unconstrained. This analysis leads to one shortest unconstrained path in each half of the plane divided by the line $\overline{X_s - X_t}$.

i.e., $X_s - P_1 - P_3 - X_t$
$X_s - P_2 - P_4 - X_t$

Utilization of a method that can narrow down to the shortest of the above two paths forms the core criteria in formulating and obtaining the location of a new facility in relation to a set of existing facilities and in the presence of elliptical forbidden regions.

Works carried out, consisted of development of methodologies that could generate the unconstrained paths and also compute the path distances for determining the minimum of the paths generated. The author in this work intends to apply Centre of Gravity Approach (CGA) for identifying the shortest path among the paths without really evaluating all of them.

3. Methodology

Center of Gravity Approach (CGA)

The shortest unconstrained path connecting two planar point locations in *the presence of a single elliptical forbidden region, defined by major (a_1) and minor axis (b_1) always lies on the same side, with respect to Centre of Gravity, as that of the shortest constrained path joining X_s-X_t.*

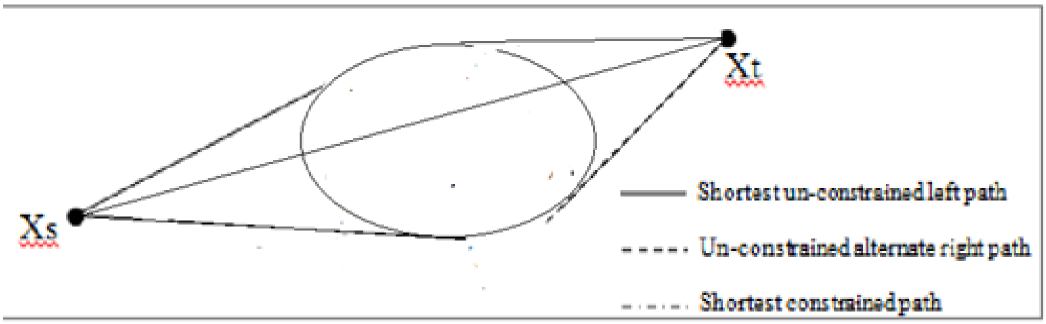
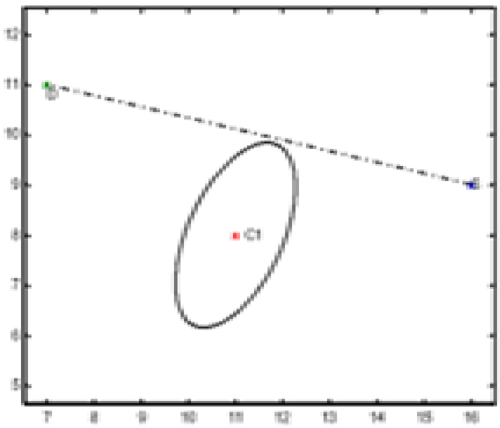


Fig. 2. Illustrate a case where the shortest un-constrained Euclidean path lies on the same side of the constrained X_s-X_t line with respect to C.G of the ellipse

Two conditions arise while connecting two planar locations in the presence of single elliptical forbidden region.

1. Constrained with region
2. Unconstrained with region



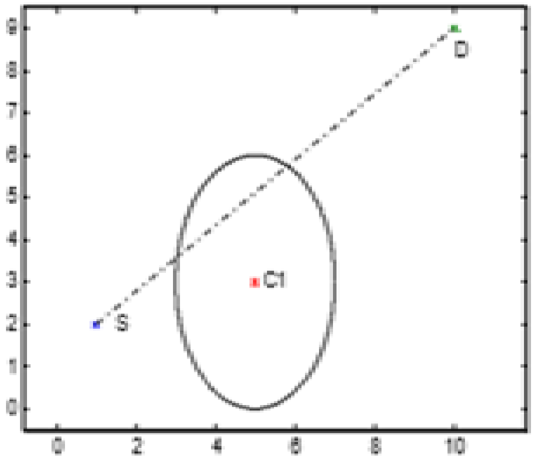


Fig. 3. Graphical representation of constrained and un-constrained paths in the presence of single elliptical forbidden region

Likewise CGA can be applied to two and three elliptical regions which has been illustrated below

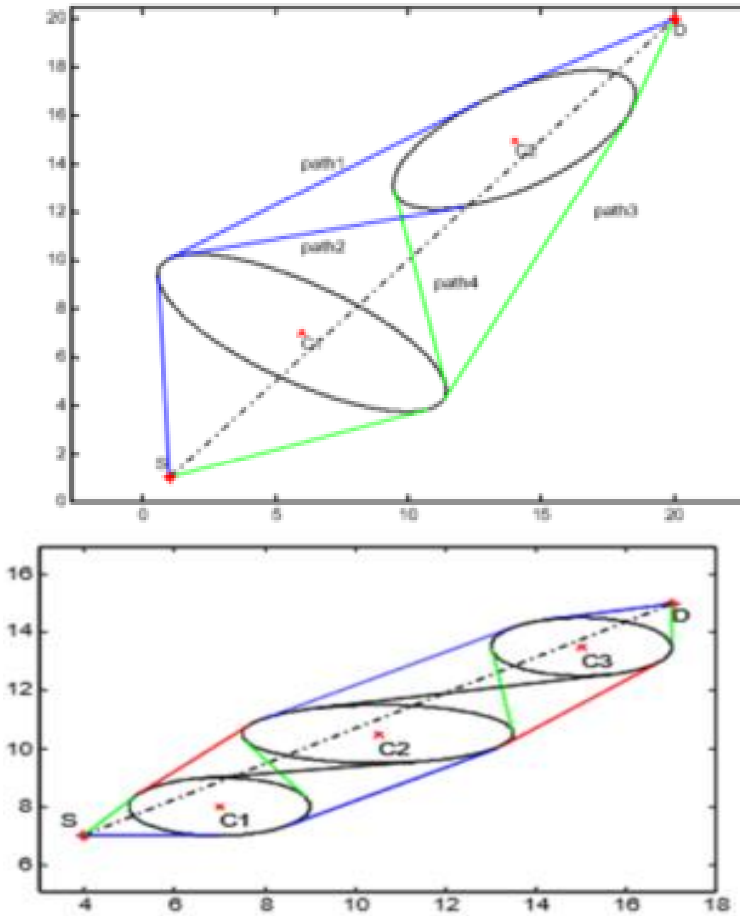


Fig. 4. CGA Technique illustration for two and three elliptical forbidden regions

In the above section the conditions under which a given path between source and target can be constrained or un-constrained have been enumerated through Fig.3 and also the objective criterion has been spelt-out under enumeration. Understandably, there are two possible shortest, yet dissimilar, paths that connect the source and target circumventing a single elliptical forbidden region, four for two ellipse and eight paths for three elliptical forbidden regions which are shown in fig4. To compute the path distances and determine the routes, method of drawing a pair of tangents from a point to an ellipse of known co-ordinate locations and major and minor axis of ellipse made use-of. This yields points of intersection of tangency from both source and target. Subsequently, using the concept of location of a point with respect to a line i.e. to the left or right, the points of tangency are grouped as either left or right. The angle subtended by the radial lines connecting the center of the ellipse with that of left pair and right pair of tangency points is then computed. Further, some of the distances of lengths of tangents falling on the same side of the path added to the arc length on the same side of the path give the total distance function to the left as well as right of the constrained path SD.

4. Algorithm

Step 1: Begin

Step 2: Input the coordinates of the center of the elliptical regions and their major and minor axes and orientation of ellipses.

Step 3: Input the source point coordinates S(Sx, Sy).

Step 4: Input the target point coordinates D(Dx,Dy).

Step 5: Check the type of constraint:

a) If not constrained with any region then go to step 6

b) If constrained with single region then go to step 7

c) If constrained with two regions then go to step 8

d) If constrained with three regions then go to step 9

Step 6: Find the un-constrained direct Euclidean path distance between the source point S (Sx , Sy) and destination point D(Dx, Dy) as:

$D = \sqrt{(S_x - D_x)^2 + (S_y - D_y)^2}$
go to step 10.

Step 7: Identify the region with which the direct path from source point to target point is constrained.

7.1 Apply the technique with respect to the constrained region which yields two obstacle-free paths with respect to the current obstacle.
7.2 Check if any of the above two paths is constrained with other obstacles. If so, go to Step 7.4, else go to Step 7.3
7.3 Calculate the path lengths and store the path points coordinates and length of each path in the results file. Go to step 7.9
7.4 Check if the path is constrained with single obstacle or two obstacles
7.5 Apply the technique accordingly and obtain obstacle-free sub paths
7.6 Join these sub paths with the main path
7.7 Calculate the paths lengths and store the path points and length of each path in the results file
7.8 Check if the second main path is also constrained with other obstacles. If so, repeat the steps 7.4 to 7.7, else go to Step 7.9
7.9 Read the paths data from the results file and arrange the paths in the increasing value of the path lengths
7.10 The first path in the revised results file now gives the shortest obstacle-free path between the source point and target point. The other paths in the file provide alternate feasible obstacle-free paths with lengths in increasing order which are more than the optimum value. Go to step 10.

Step 8: Number the two constrained regions with respect to their distance from the source point. The nearest one is the first region and the farthest one is the second region. Go to 8.1

8.1 Apply the technique with respect to the two constrained regions.
8.2 Check if any of the above paths is constrained with the third region. If so, go to Step 8.4, else go to Step 8.3
8.3 Calculate the path lengths and store the path points, coordinates and length of each path in the results file. Go to Step 8.8
8.4 Apply the technique and obtain obstacle-free sub paths
8.5 Join these sub paths with the main path
8.6 Calculate the path lengths and store the path points and length of each path in the results file.
8.7 Check if any of the remaining paths is also constrained with the third region. If so, repeat the Steps 8.4 to 8.6 else go to step 8.8
8.8 Read the paths data from the results file and arrange the paths in the increasing value of the path lengths.
8.9 The first path in the revised results file now gives the shortest obstacle-free path between the source point and target point. The other paths in the file provide alternate feasible obstacle-free paths with lengths in increasing order, which are more than the optimum value. Go to step 10.

Step 9: Number the three constrained regions with respect to their distance from the source point. The nearest one will be the first region, the farthest one will be the third region and the remaining one will be the second region. Go to 9.1.

9.1 Apply the technique with respect to the three constrained regions.
9.2 Calculate the path lengths and store the path points, coordinates and length of each path in the results file.
9.3 Read the paths data from the results file and arrange the paths in the increasing value of the path lengths.
9.4 The first path in the revised results file now gives the shortest obstacle-free path between the source point and target point. The other paths in the file provide alternate feasible obstacle-free paths with lengths in increasing order, which are more than the optimum value.

Step10: Exit

FLOWCHART

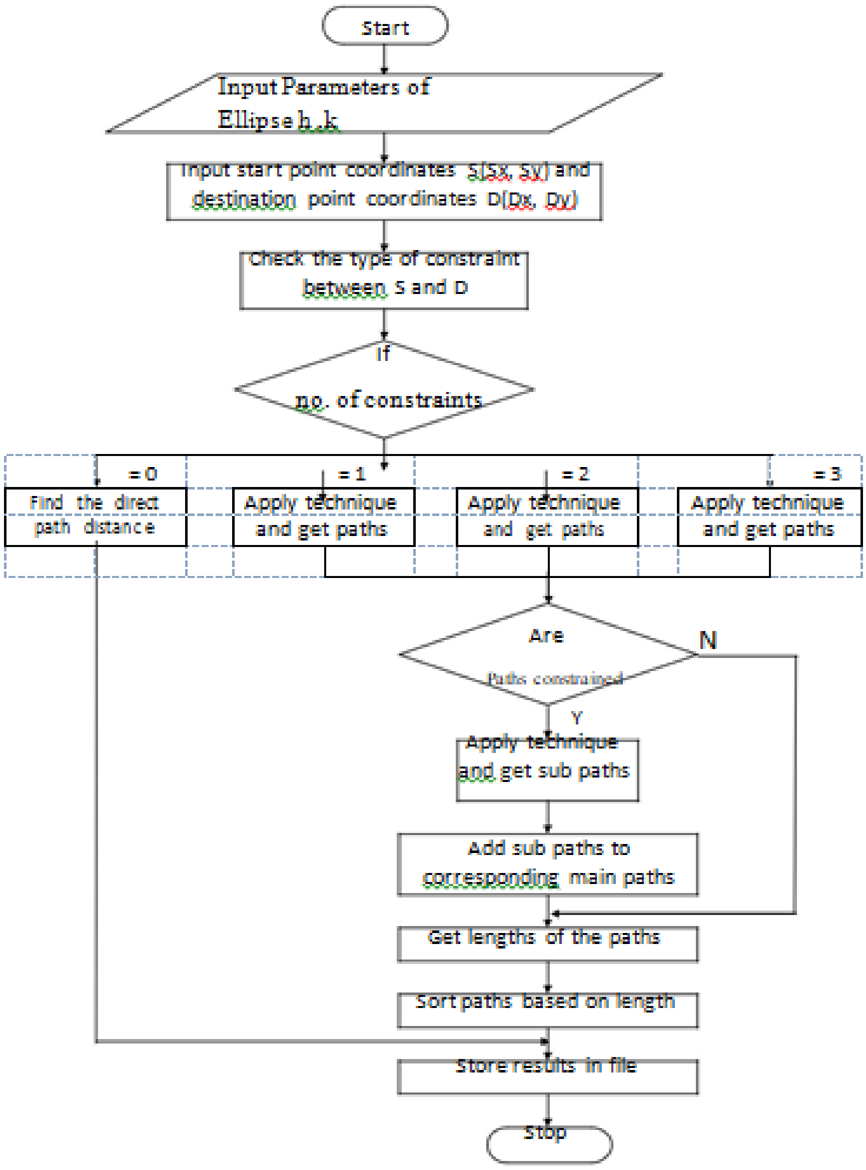


Fig. 5. Flowchart showing the procedure for finding alternate obstacle-free paths in the presence of one, two and three elliptical obstacles

The methodology applied is validated by considering cases for the problems involving single two and three elliptical forbidden regions through MAT LAB simulation software.

The problem data and solution for the first case study is presented in Table 1. The problem data table lists the Cartesian coordinates (X, Y) of the source point (i.e. start point), destination point (i.e. target) and the center, major and minor axis of the ellipse located in the solution space. The solution data table provides the results of the technique and the path followed.

The individual obstacle-free paths are also listed in the same table with the information about the path number, number of points in the path, points along the path and length of the path. The above referred numerical data and solution is also shown through graphical display for easy visualization. Fig. 7 shows the graphical display of individual obstacle-free paths from source to target. Likewise problem data and solution for other two cases are presented in Table 2 and 3 and solutions are shown through graphical display for easy visualization. Fig. 9 and 11 shows the graphical display of individual obstacle-free paths The efficacy of the methodology and solutions mentioned in the above section has been test validated for elliptical forbidden regions by solving locational problems through V-Rep robotic simulation software and are shown in fig. 8, 10 and 12 applications as no works have been reported earlier.

V-Rep

V-Rep is a versatile and ideal for multi-robot. V-REP is used for path planning, fast script development, factory automation simulations, fast verification, robotics related education, remote monitoring, etc. A simulation process in V-REP can be started paused and stopped. The process can be done by using the bullet dynamics engine, the process of simulation in this method to be created to determine the collision free path between the obstacles in the process of the avoidance of the static obstacles for the process of the optimal path creation. Fig 6 shows the simulation of robot on the path which was created between two points by avoiding the static obstacles.

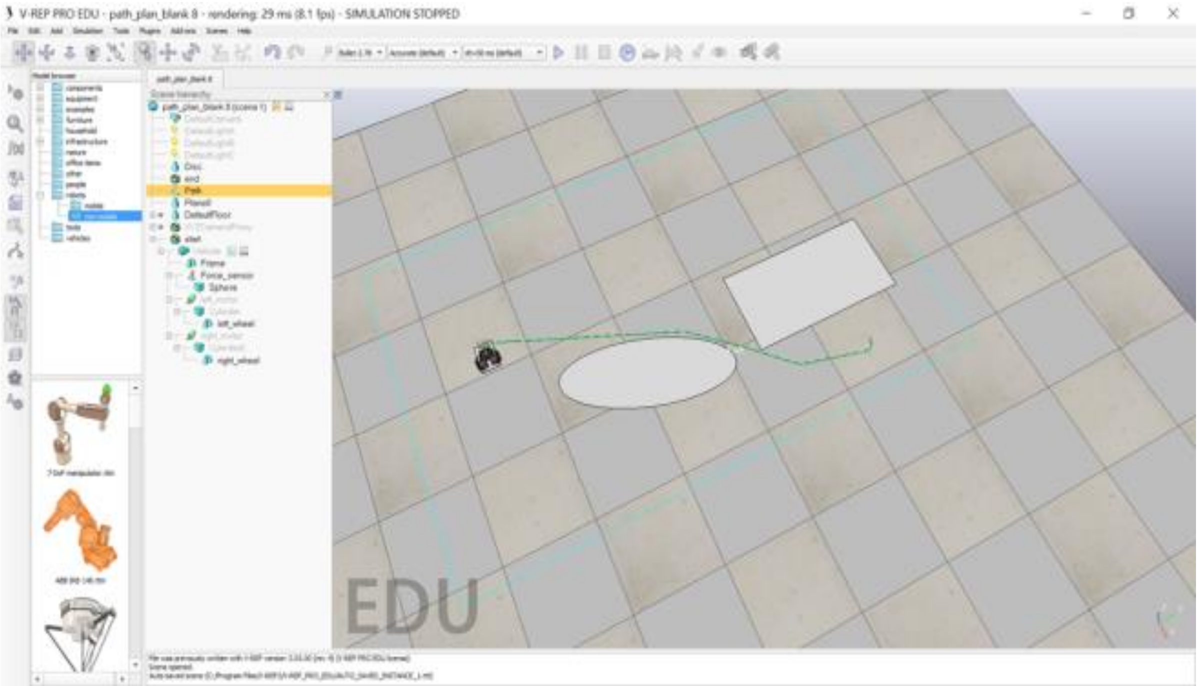


Fig. 6. Simulation of Robot Path between two points by avoiding the static obstacles

Generation of Various Obstacle-Free Paths between Source and Target and Validation

Case 1

Table 1. Problem Data and Solution in the Presence of Single Elliptical Obstacle

Source	**Ellipse 1**				**Destination**	
	Center (h, k)	**Major axis a**	**Minor axis b**	**Angle θ**	**X**	**Y**
	5,3	3	2	0	1 0	9

Path No.	**Path**	**Length of the path**
1	Left Path	**11.6635**
2	Right Path	15.0422

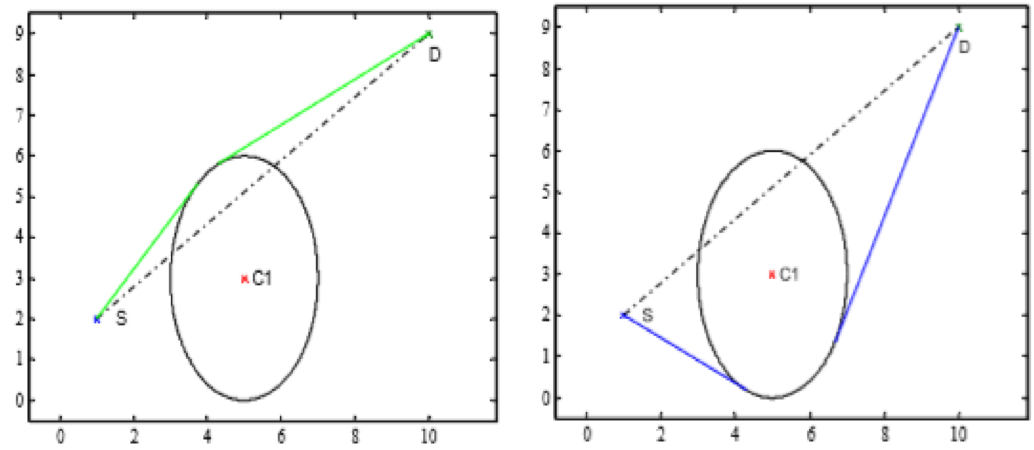


Fig. 7 Graphical representation of individual obstacle-free paths from source to target for the problem data shown in Table 1

Validation of optimal path length for case 1 with V-Rep

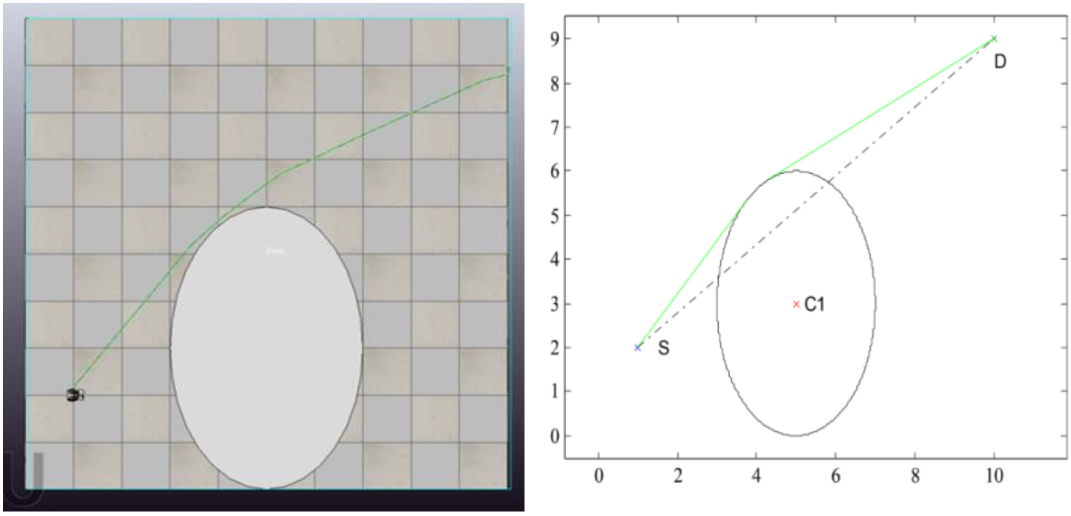


Fig. 8. Optimal path length in V-REP - 11.78 **Optimal path length in CGA - 11.66**

Case 2

Table 2: Problem Data and Solution in the Presence of Two Elliptical Obstacle

Source	**Ellipse 1**				**Ellipse 2**				**Destination**	
	Center (h₁, k₁)	**Major axis a1**	**Minor axis b1**	**Angle Θ_1**	**Centre (h, k)**	**Major axis a1**	**Minor axis b1**	**Angle Θ_2**	**X**	**Y**
	8,6	5	3	0	15, 18	5	3	70	20	20

Path No.	**Paths**	**Length of the path**
1	Left-Left Path	32.3770
2	Left-Right Path	**29.3649**

3	Right-Right Path	29.6791
4	Right-Left Path	37.6933

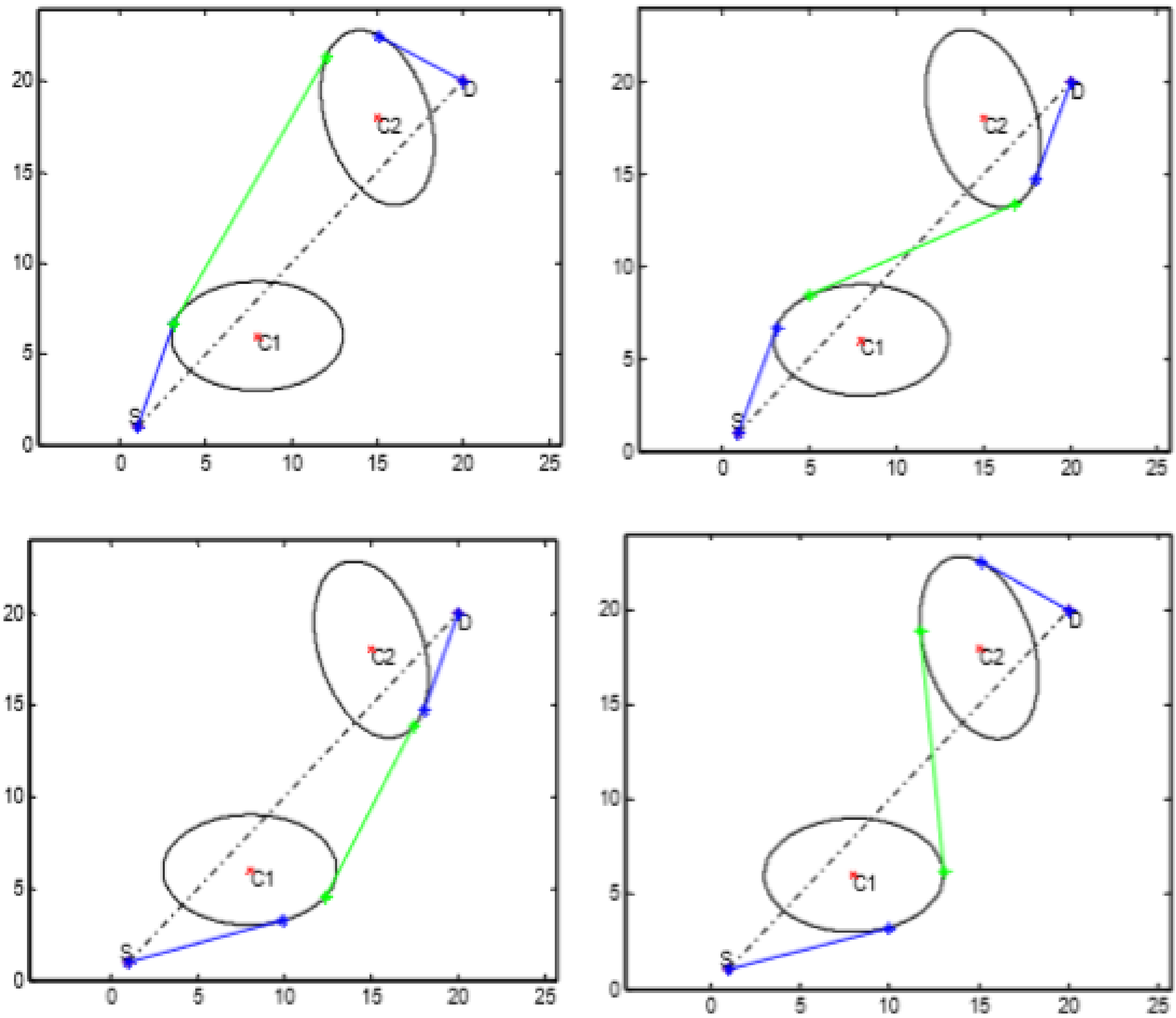
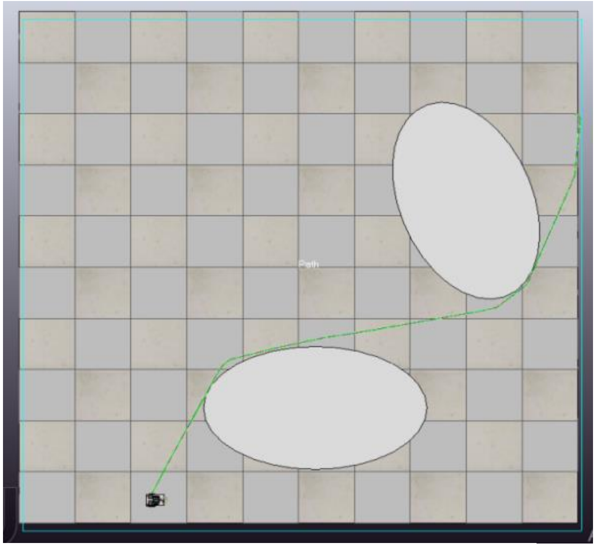


Fig. 9. Graphical representation of individual obstacle-free paths from source to target for the problem data shown in Table 2

Validation of Optimal Path Length for Case 2 with V-Rep



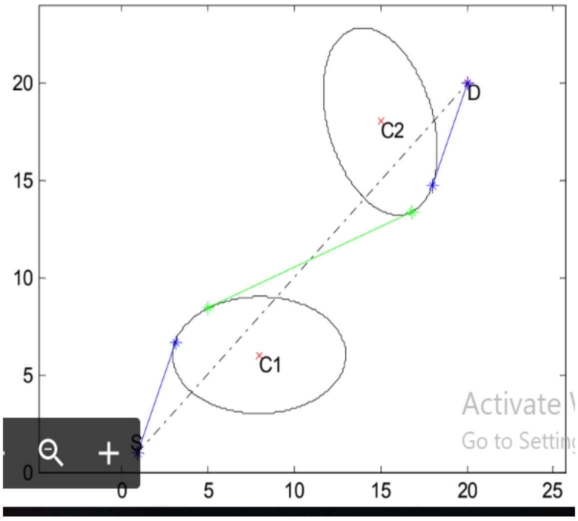


Fig. 10. Optimal path length in V-REP - 29.45 **Optimal path length in CGA - 29.36**

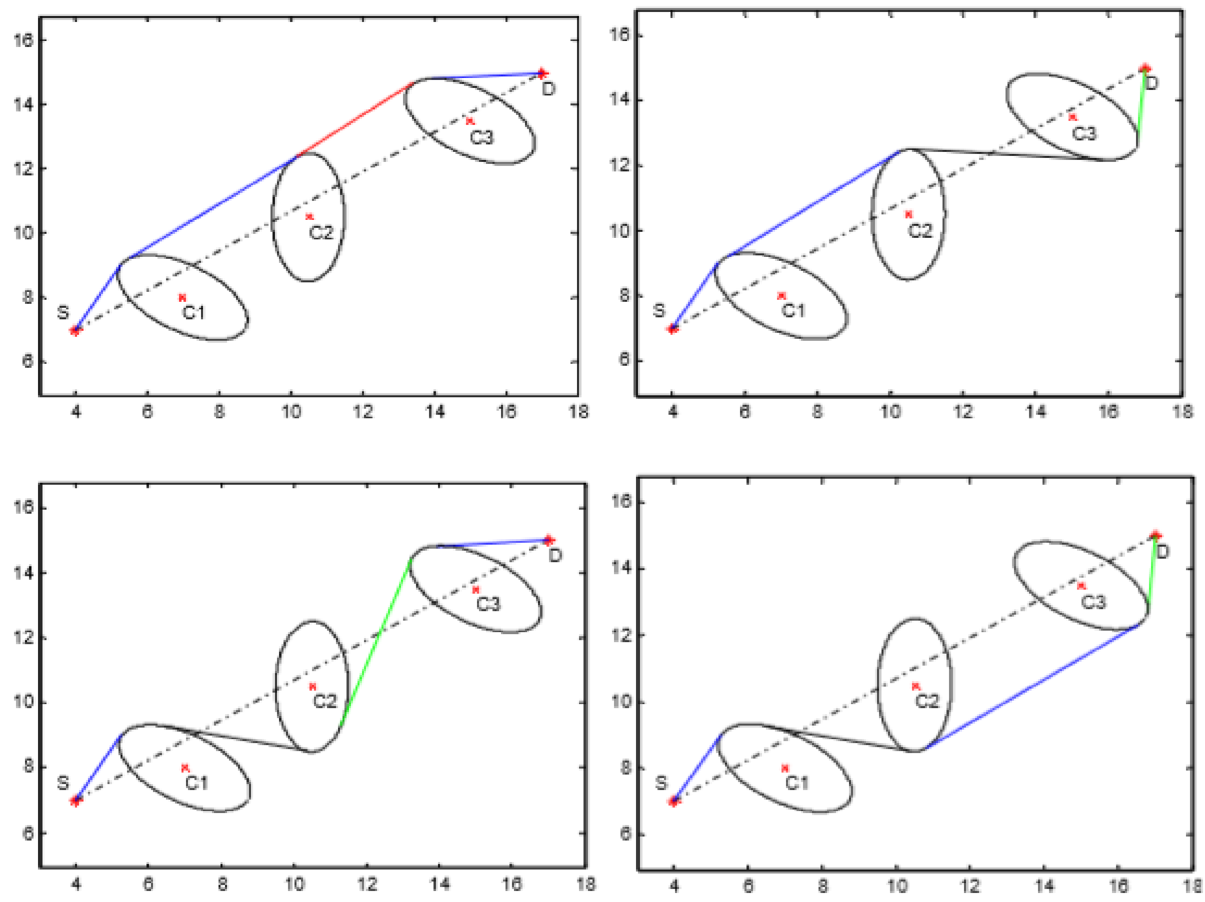
CASE 3

Table 3: Problem Data and Solution in the Presence of Three Elliptical Obstacles

Source	**Ellipse 1**				**Ellipse 2**				**Ellipse 3**				**Destination**	
	Center	**Major axis**	**Minor axis**	**Angle**	**Center**	**Major axis**	**Minor axis**	**Angle**	**Center**	**Major axis**	**Minor axis**	**Angle**	**X**	**Y**
	(h_1, k_1)	**a_1**	**b_1**	**Θ_1**	**(h_2,k_2)**	**a_2**	**b_2**	**Θ_2**	**(h_3,k_3)**	**a_3**	**b_3**	**Θ_3**		
	7, 8	**2**	**1**	**30**	**10.5, 10.5**	**2**	**1**	**90**	**15, 13.5**	**2**	**1**	**30**	**17**	**15**

Path No.	**Paths**	**Length of the path**

1	Left- Left-Left path	**16.1846**
2	Left-Left-Right path	17.6082
3	Left-Right-Left path	18.5193
4	Left-Right-Right path	17.8663
5	Right-Right-Right path	17.1813
6	Right-Right-Left path	19.4319
7	Right-Left-Left path	19.7782
8	Right-Left-Right path	18.2614



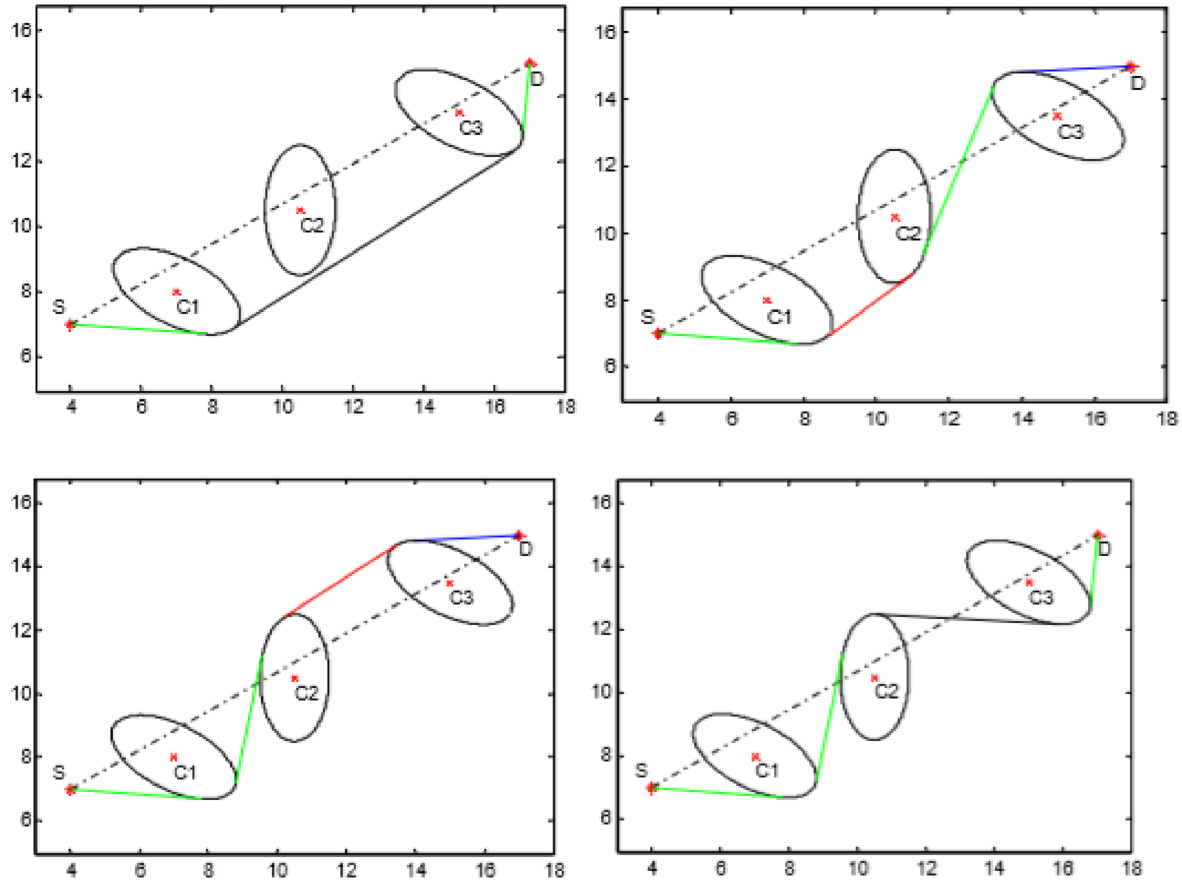


Fig. 11. Graphical representation of individual obstacle-free paths from source to target for the problem data shown in Table 3

Validation of Optimal Path Length for Case 3 with V-Rep

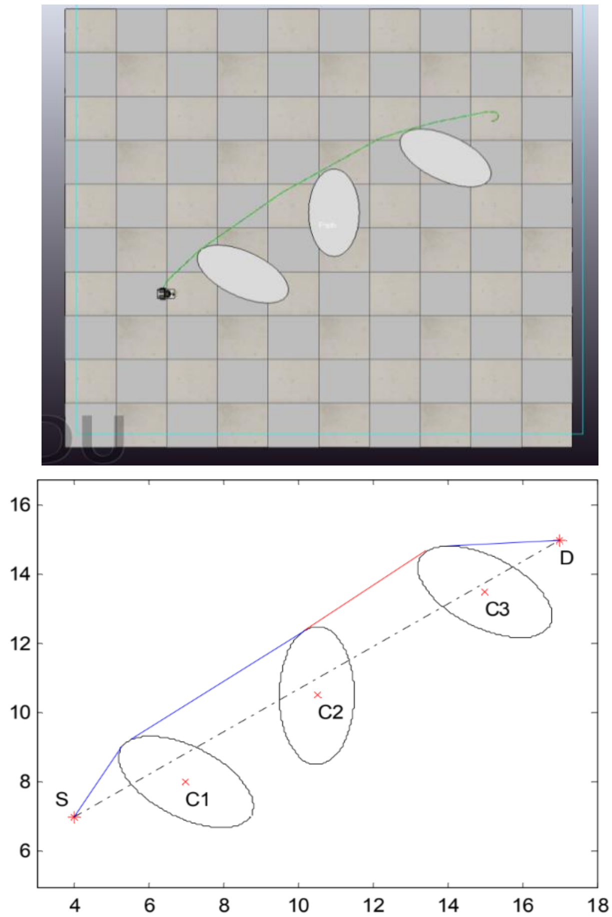


Fig. 12. Optimal path length in V-Rep - 16.3 Optimal path length in CGA - 16.1

The solutions to the above test cases amply demonstrate the usefulness of the above statement. Understandably this concept would foresee the solutions obtained basing on the criteria of CGA method has revealed certain limitations leading to about two to three percent of the test cases failing in the identification of the shortest unconstrained path. A careful comparison of the results obtained need for computation of distances on alternate paths. However, an exhaustive analysis show that the deviation in the function value i.e., the difference of distances between left and right paths is less than two percent of the function value. Though the deviation from the function value is insignificant, the same when considered in a dynamic environment involving multiple facilities the cumulative deviation might become substantial leading to imposing specific limitations on optimality of the function value. The results obtained by CGA were test validated by V-Rep and the results are encouraging.

5. Conclusions

In the present work a systematic approach was made for developing results by applying a suitable methodology that meets the solution requirements of routing problems that involve path planning procedures. Combinations of feasible paths are attained from the start point to the target point. Among these paths the shortest unconstrained path is required in order to minimize the distance of travel. During the process of attaining these paths it is also necessary to optimize the time of programming in order to acquire the desired results. This can be done by incorporating a CGA technique to the developed equation in the programming. This technique helps in reducing the evaluation time and also directly generates the shortest path for the navigation of a robot to reach its destination point. To begin with a path planning problem in the presence of a single elliptical barrier region problem was considered to develop alternate feasible and dissimilar un-constrained paths between source and target.

The same has been extended to include situations that involve two and three elliptical barrier conditions with possible four and eight un-constrained feasible paths respectively.

The methodology adopted generates a library of feasible un-constrained paths under the conditions of above situations that can be used in an order of priority to obviate the limitations normally experienced in operating conditions. The methodology is simulated for its efficacy by considering problems with suitable situations. The solutions obtained conform to the theoretical expressions. The solutions thus obtained by CGA Technique absolutely agreed with the solutions by V-Rep and even produced optimal solutions when compared with V-Rep thus proving that the algorithm developed for the elliptical forbidden regions is efficient than V-Rep simulation software and can be applicable to elliptical forbidden regions.

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