Analytical solution of the problem of thermoelasticity for a plate heated by a source with a constant heat supply on one surface

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Abstract. An analytical solution to the problem of thermoelasticity for a plate heated by a source with a constant heat supply on one surface is proposed. Numerous comparisons of the results obtained on the basis of the analytical solution with the numerical one are carried out. When comparing the analytical and numerical solutions, it is shown that the advantage of the analytical solution is that it allows you to determine the highest stress values in the entire heating time range, while using the numerical method, it is necessary to carry out a large number of calculations at fixed times. However, the finite element method makes it possible to determine the stresses in the areas of the slab far from its center, where the analytical solution turns out to be not valid.

Keywords: Analytical solutions, numerical solutions, thermoelasticity, surface

1. Introduction

Composite materials are currently widely used in rocket and space technology as heat-shielding materials due to their unique properties resulting from the technology of their manufacture. The matrix of fine-fiber fillers is impregnated with binders that degrade easily at moderate temperatures. As a result, various plastics are obtained: fiberglass, carbon fiber, asboplastics, etc. When using such materials as heat shielding materials at hypersonic flight speeds (Mach number greater than 5) of aircraft, it is important to study the effect of temperature on the mechanical characteristics of materials and the stress state on the temperature distribution in structural elements, such as thin heat shielding plates.

Currently, there is a sufficient number of works to solve the problems of thermoelasticity [1-17]. In works [18-33], analytical and numerical methods are proposed for solving the problems of heat conduction in super- and hypersonic flow around aircraft. And in works [33-41] various mechanical studies of heat-shielding composite materials are presented based on both analytical and numerical methods. Analytical solutions of thermoelasticity problems are important, including for the approbation of numerical solutions [42-56]. In this paper, an analytical solution to the problem of thermoelasticity for a plate heated by a source with a constant heat supply on one surface is proposed. A comparison of the results obtained on the basis of an analytical solution with a numerical.

2. Formulation of the problem

The problem is considered within the framework of linear thermoelasticity. Accordingly, the deformations are assumed to be small, as a result of which the following equations hold.

Equilibrium equations:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \rho X = 0, \tag{1}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + \rho Y = 0, \tag{2}$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial x} + \rho Z = 0, \tag{3}$$

where $\sigma_x, \sigma_y, \sigma_z$ – normal voltages, $\tau_{xy}, ..., \tau_{yz}$ - tangents; ρ - material density, X, Y, Z - body force intensity components.

Cauchy relations:

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \tag{4}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \tag{5}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}, \tag{6}$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z$ - longitudinal deformations, $\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ - shear deformation, u, v, w - displacement vector components.

Generalized Hooke's Law with Temperature Variation:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \upsilon\left(\sigma_y + \sigma_z\right)\right] + aT, \tag{7}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \upsilon\left(\sigma_x + \sigma_z\right)\right] + aT, \tag{8}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \upsilon\left(\sigma_x + \sigma_y\right)\right] + aT, \tag{9}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \gamma_{yz} = \frac{\tau_{yz}}{G}, \gamma_{xz} = \frac{\tau_{xz}}{G}, G = \frac{E}{2(1+\upsilon)}, \tag{10}$$

where a thermal expansion coefficient, T – temperature, υ – Poisson's ratio, G – shear modulus.

In subsequent calculations, we will determine the temperature stresses arising in the thermal protection plate under conditions of unsteady heating.

The problem of thermal conductivity for a plate heated from one side by a source that provides a constant supply of heat on one surface:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial x^2}, \tag{11}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0, \quad \left.\frac{\partial T}{\partial x}\right|_{x=L} = \frac{h}{\lambda}\left(T_{as} - \left.T\right|_{x=L}\right), \left.T\right|_{t=0} = 0. \tag{12}$$

3. Solution method

First, a thin rectangular plate of constant thickness is considered, in which the temperature change distribution is a function of Y (Fig. 1) and does not depend on X and Z.

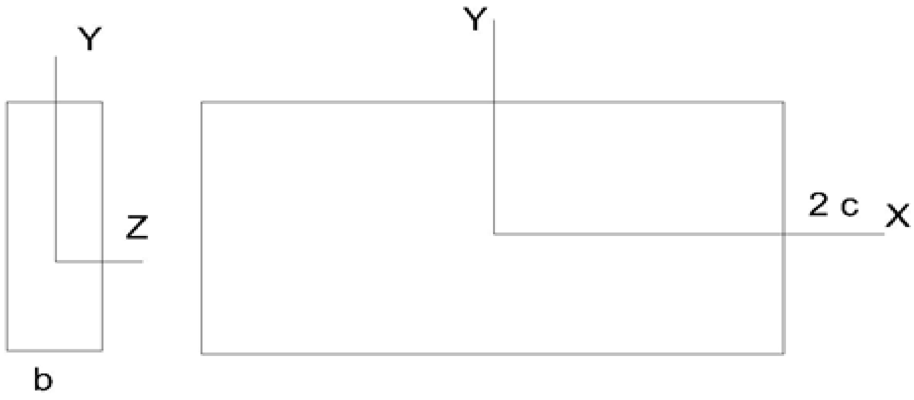

Fig. 1. Rectangular slab.

If the ends of the plate are fixed, then when heated, additional compressive stresses appear, which in the absence of buckling are expressed by the formula:

$$\sigma_x = -\alpha ET(y). \tag{13}$$

If the plate at the ends is free from external forces (not fixed), then to determine the resulting temperature stresses, it is necessary to add to the stresses calculated by formula (13), add the stresses caused in the plate by tensile forces of intensity $\alpha ET(y)$, distributed at the ends. These tensile stresses give the resulting force:

$$P_x = \int_{-c}^{c} \alpha ET(y) b dy, \tag{14}$$

and at a sufficient distance from the ends, uniformly distributed tensile stresses arise:

$$(\sigma_x)_{\text{strain}} = \frac{P_x}{F} = \frac{P_x}{2cb} = \frac{1}{2c}\int_{-c}^{c} \alpha ET(y) dy. \tag{15}$$

If the temperature distribution is asymmetric, then additional tensile stresses have a resultant moment:

$$M_Z = \int_{-c}^{c} \alpha ET(y) y b dy. \tag{16}$$

At a sufficient distance from the ends, the formula for normal bending stresses applies, so that this resultant moment will cause bending stresses:

$$(\sigma_x)_{flexion} = \frac{M_z y}{I_z} = \frac{3M_z y}{2c^3 b} = \frac{3y}{2c^3}\int_{-c}^{c} \alpha ET(y) y dy. \tag{17}$$

Thus, the total stress in a thin unfixed plate far from its ends is equal to (13), (15), (17):

$$\sigma_x = -aET(y) + \frac{1}{2c}\int_{-c}^{c} aET(y) dy + \frac{3y}{2c^3}\int_{-c}^{c} aET(y) y dy \tag{18}$$

Consider a thick plate with a temperature variable in thickness. We consider the value *b* significant and consider it as the width, a *2c* as plate thickness. Since the size along the *z* axis is significant and its elements expand in this direction in different ways due to the uneven distribution of *T(y)*, then in the plate will be created as stresses σ_y, and stress σ_x. When fully secured to the edges in the *X* and *Z* directions in the equation $\varepsilon_x = \varepsilon_z = 0$ and $\sigma_y = 0$, what gives:

$$\sigma_x = \sigma_z = -\frac{aET(y)}{1-\upsilon}. \tag{19}$$

Equation (18) in the case of a free thick plate for points far from the edges, taking into account (19), takes the form:

$$\sigma_x = \sigma_z = -\frac{aET(y)}{1-\upsilon} + \frac{1}{2c(1-\upsilon)}\int_{-c}^{c} aET(y) dy + \frac{3y}{2c^3(1-\upsilon)}\int_{-c}^{c} aET(y) y dy. \tag{20}$$

The outer surface of a thick padding or plate is heated by heat transfer by convection from the boundary layer, and heat is spread through the thickness of the plate by conduction. Accordingly, the equations of heat conduction and heat transfer through the interface between air and plate can be used.

To solve the partial differential equation (11), (12) in accordance with the method of separation of variables, we put:

$$T(x,t) = F_1(x) F_2(t), \tag{21}$$

whence follows:

$$\frac{1}{a^2}\frac{F_2'}{F_2} = \frac{F_1''}{F_2} = -\gamma^2,\ a^2 = \frac{\lambda}{c_p \rho}. \tag{22}$$

The solution to ordinary differential equations (22) has the form:

$$F_1\left(x\right)=C_1\sin\gamma x+C_2\cos\gamma x\,, \tag{23}$$

$$F_2\left(x\right)=C_3e^{-\gamma^2a^2t}\,, \tag{24}$$

and thus from (21), taking into account (23), (24):

$$T\left(x,t\right)=e^{-\gamma^2a^2t}\left(C_4\sin\gamma x+C_5\cos\gamma x\right). \tag{25}$$

In the case of a plate heated on one side by a source that provides a constant supply of heat on one surface in equations (12), we must put $T_{as}=T_E,\ h=\text{const},$ then from (25) it follows:

$$T\left(x,t\right)=Ae^{-\gamma^2a^2t}\cos\gamma x+B\,. \tag{26}$$

From the boundary conditions (12) we find:

$$B=T_E;\ \gamma\sin\gamma L=\frac{h}{\lambda}\cos\gamma L\,. \tag{27}$$

Thus, the equation for the eigenvalues has the form:

$$p_n=\frac{hL}{\lambda}\operatorname{ctg}p_n;\ \ p_n=\gamma_nL. \tag{28}$$

To fulfill the initial condition $T = 0$ at $t = 0$, we take:

$$T=T_E-\sum_{n=0}^{\infty}a_n\exp\left(-\frac{p_n^2a^2t}{t_1}\right)\cos\frac{p_nx}{L}\ ,$$

whence follows:

$$T_E=\sum_{n=0}^{\infty}a_n\cos\frac{p_nx}{L}\ ,\ a_n=\frac{T_E\int_0^L\cos\frac{p_nx}{L}dx}{\int_0^L\cos^2\frac{p_nx}{L}dx}=\frac{2T_E\sin p_n}{p_n+\sin p_n\cos p_n}\ .$$

Thus,

$$\frac{T}{T_E}=1-2\sum_{n=0}^{\infty}\frac{\sin p_n\exp\left(-\frac{p_n^2Wt}{t_1}\right)\cos\frac{p_nx}{L}}{p_n+\sin p_n\cos p_n}\ . \tag{29}$$

Here $T_E=T_{as}=const$, T_E – thermal equilibrium temperature, T_{as} – temperature of adiabatic deceleration in the boundary layer, t_1 – this is the time to reach the temperature of thermal equilibriumя.

Temperature stresses are determined using expression (20), which can be rewritten in the following form:

$$\frac{\left(1-\upsilon\right)\sigma_Y}{\alpha ET_E}=\frac{\left(1-\upsilon\right)\sigma_Z}{\alpha ET_E}=-\frac{T}{T_E}+H_C+H_B\left(\frac{x}{L}-\frac{1}{2}\right).$$

after expression substitution:

$$H_C=1-2\sum_{n=0}^{\infty}\frac{\sin^2p_n\exp\left(-\frac{p_n^2Wt}{t_1}\right)}{p_n\left(p_n+\sin p_n\cos p_n\right)}\ ;$$

$$H_B=12\sum_{n=0}^{\infty}\frac{\sin p_n\left[2\left(1-\cos p_n\right)-p_n\sin p_n\right]\exp\left(-\frac{p_n^2Wt}{t_1}\right)}{p_n^2\left(p_n+\sin p_n\cos p_n\right)};\ W=\frac{\lambda t_1}{\rho c_pL^2}\ .$$

For the case of fastening, excluding the possibility of bending, the term H_B is absent.

4. Comparison of numerical and analytical solutions for an isotropic plate

An analytical solution to the isotropic problem of a plate heated from one side by a source that provides constant heat supply to one surface was programmed in the Wolfram Mathematica symbolic computing environment. Let us compare this solution with the numerical solution obtained in Ansys.

Initial data took the following values: platinum height 80 mm; plate length and width 500 mm; coefficient of thermal conductivity $\lambda = 0,061905 \frac{\mathrm{W}}{\mathrm{m} \cdot \mathrm{K}}$; heat transfer coefficient *h*=100; density $\rho = 144\,\mathrm{kg}/\mathrm{m}^3$; Poisson's ratio $\upsilon = 0,33$; specific heat $c_p = 1000 \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$; thermal expansion coefficient 5,5E-7; end warm-up time 300 s; Young's modulus *E*=140 MPa; thermal equilibrium temperature $T_E = 1250^0 C$.

An advantage of the analytical approach is the ability to immediately find the extreme stress values. Having determined the extrema of the stress function, you can immediately find the time and area in the slab in which the maximum and minimum stresses are realized. This was done in Mathematica system. It was found that the maximum tensile stresses arise in the plate at 14 seconds near the heated plate surface.

Stress distribution over thickness (at the 14th second) - the duration of the maximum stresses in the plate.

Tab. 1 Comparison of numerical and analytical solutions.

	Analytical solution	Numerical solution
Maximum tensile stresses, MPa	0,026	0,022
Maximum compressive stress, MPa	0,108	0,081

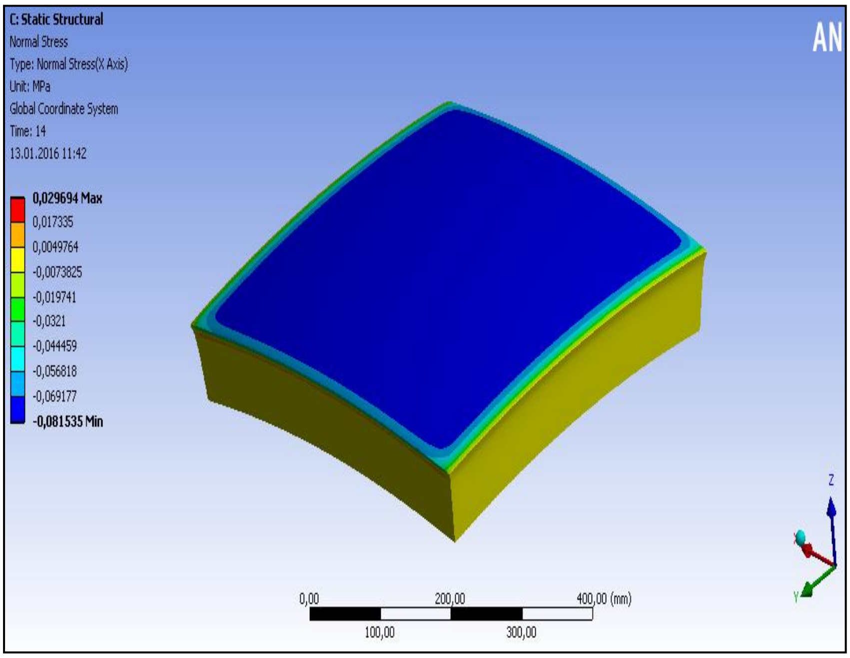

Fig. 2 a) Distribution of normal stresses $\sigma_x = \sigma_y$ n the plate

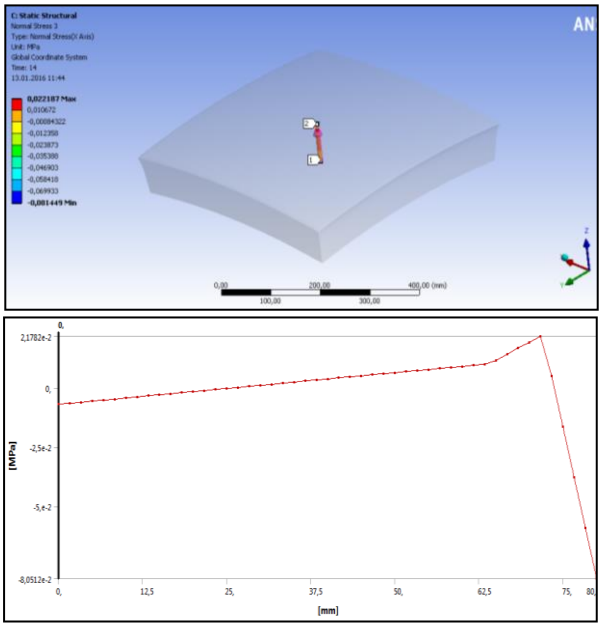

Fig. 2 b) Distribution of normal stresses $\sigma_x = \sigma_y$ by the thickness of the slab away from its edges

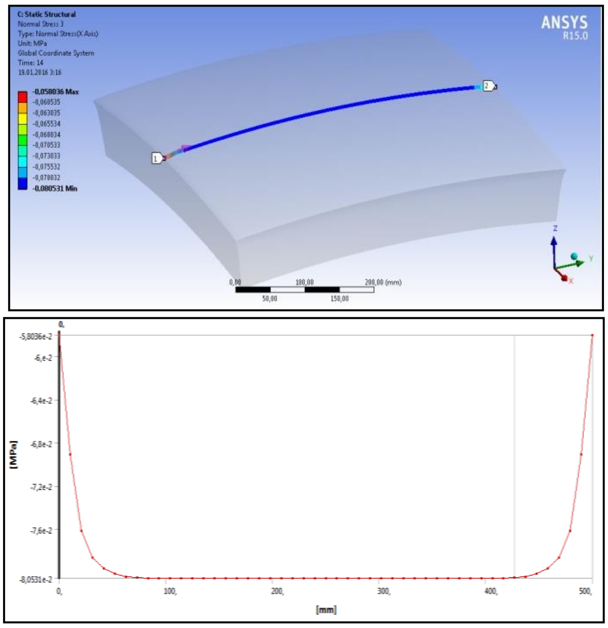

Fig. 3 Distribution of normal stresses $\sigma_x = \sigma_y$ on the heated surface of the plate.

As can be seen from Figure 3, normal stresses $\sigma_x = \sigma_y$ on the heated surface, the plates are not equally distributed in its central area and along the edges. From the figure it is possible to determine the zone of the edge effect by normal stresses, it is 0,911 of the slab thickness.

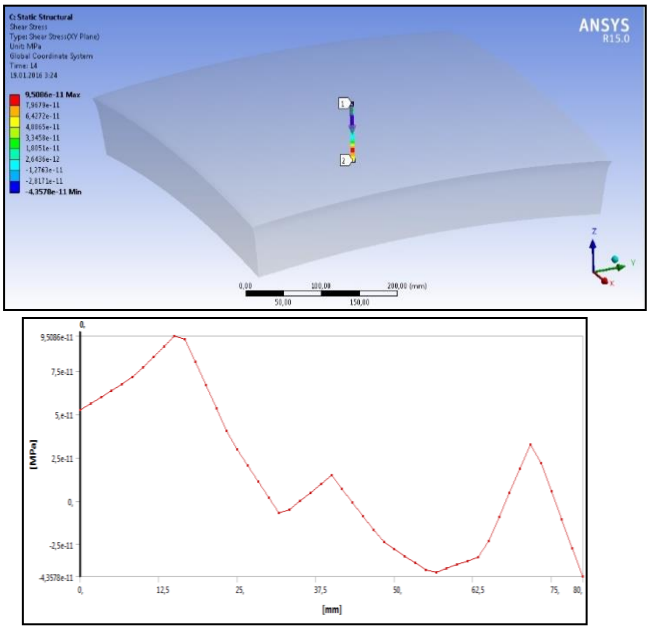

Fig. 4. Distribution of shear stresses τ_{xy} over the thickness of the slab away from its edges.

From the analytical solution it follows that in the problem of a plate heated from one side by a source that provides a constant supply of heat on one surface, only normal stresses arise $\sigma_x = \sigma_y$. This is confirmed by the numerical solution presented in the figure 4. Shear stress order τ_{xy} in 10^{-11} degree shows that, in fact, these stresses are absent.

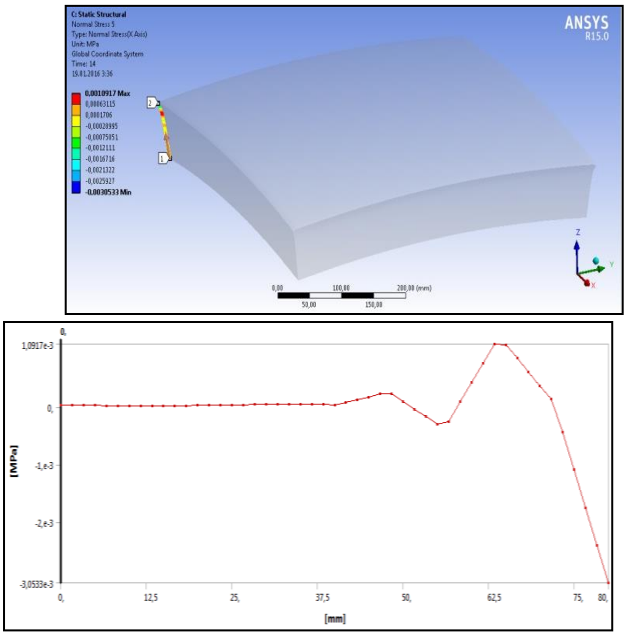

Fig. 5. Distribution of normal stresses $\sigma_x = \sigma_y$ by the thickness of the slab at the maximum distance from its central part

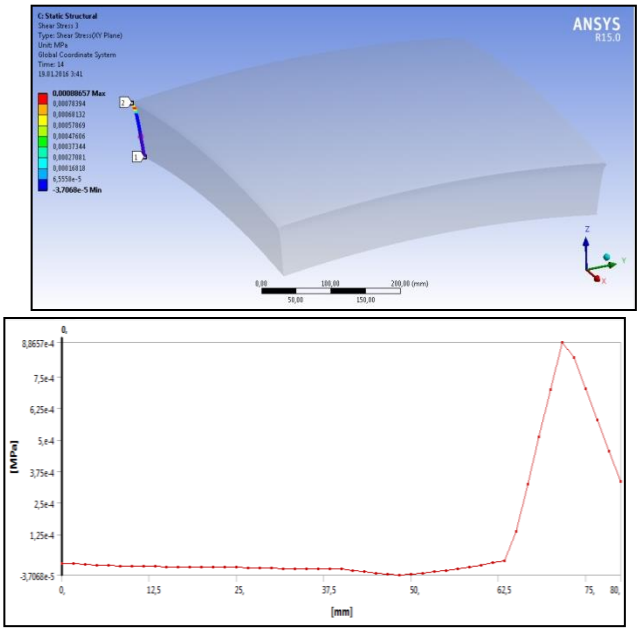

Fig. 6. Distribution of shear stresses τ_{xy} by the thickness of the slab at the maximum distance from its central part

Comparing Figures 2 b) and 5 for normal stresses $\sigma_x = \sigma_y$ and Figures 4 and 6 tangentially τ_{xy} , their difference becomes obvious in the center of the slab and at its edges.

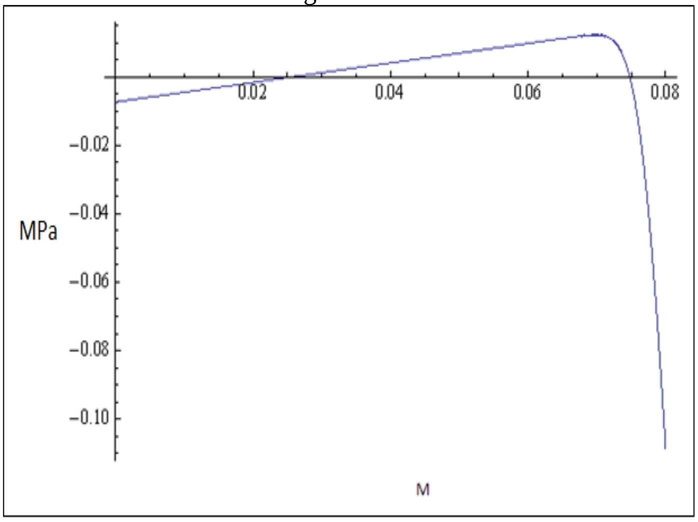

Fig. 7 a) Distribution of normal stresses $\sigma_x = \sigma_y$ along the thickness of the slab away from its edges (based on the analytical solution)

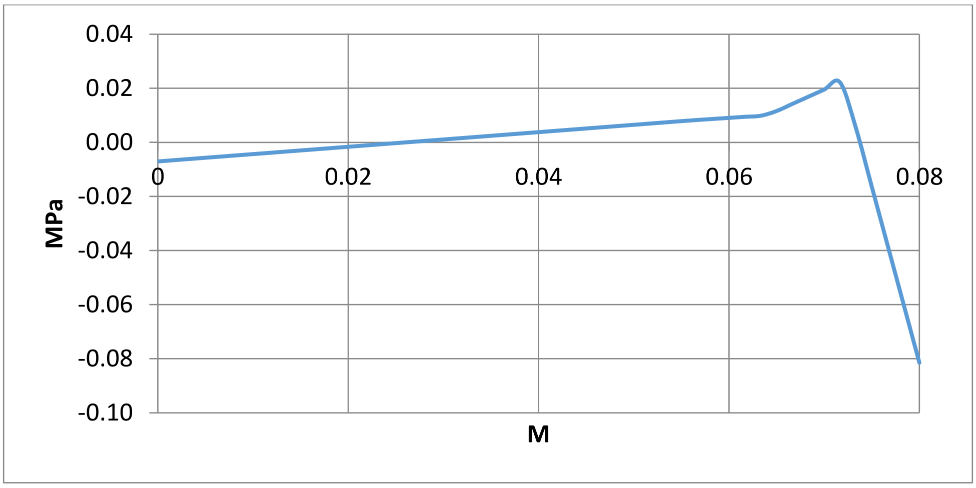

Fig. 7 b) Distribution of normal stresses $\sigma_x = \sigma_y$ along the thickness of the TPM slab far from its edges (based on a numerical solution)

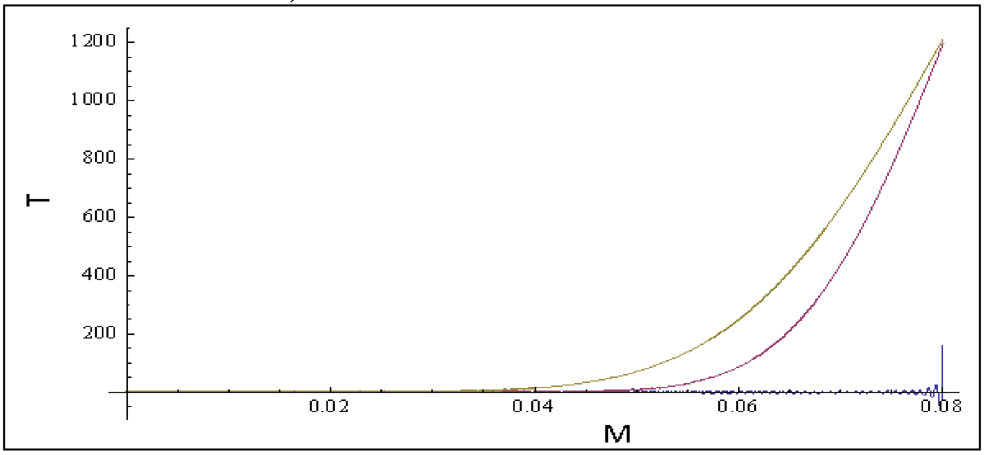

Fig. 8. Graph of temperature distribution over the thickness of the slab at different moments t=0; t = 0,5t1; t = t1, where t1 =300 seconds warm-up time. Volumetric pore content 93,2%

Figure 8 shows the temperature distribution over the thickness of the slab at different points in time plotted using the analytical solution. At the initial moment of time, some instability is visible due to the limited number of terms in the series in the analytical solution.

5. Conclusion

An analytical solution to the problem of thermoelasticity for a plate heated by a source with a constant heat supply on one surface is obtained. The obtained analytical solution is compared with the numerical. When comparing the analytical and numerical solutions, it can be noted that the advantage of the analytical solution is that it allows one to determine the highest stress values in the entire heating time range, while using the numerical method, it is necessary to carry out a large number of calculations at fixed time values. However, the finite element method makes it possible to determine the stresses in the areas of the slab that are remote from its center, where the analytical solution is not valid. Also, the numerical method clearly demonstrates the stress-strain state of the slab, which, as it turned out, bends during heating, and on its heated and "cold" surfaces, stresses of the same sign appear.

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