Research Article

Investigation of the mechanical properties of heat-protective highly porous composite materials using the effective medium model

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Abstract. The paper proposes a method for studying the mechanical properties of heat-shielding highly porous materials using an effective medium model. The mechanical characteristics of the composite material are calculated depending on the orientation angle and the volumetric content of the fibers. Stress-strain diagrams in tension and compression are plotted parallel and perpendicular to the plane.

Keywords: composite material, effective medium model, elastic modulus, mechanical characteristics, ultimate strength.

1. Introduction

Today, more and more requirements are imposed on the accuracy of mathematical models of composite materials. It should be noted that the properties of the composite material (thermal conductivity, density, strength, coefficient of thermal expansion) primarily determine the fibers - their composition. Currently, there are a large number of works that describe methods for studying the thermophysical and mechanical properties of composite materials [1-13]. One of the approaches to determining the characteristics of a composite material is to solve inverse problems for identifying the thermophysical and mechanical properties of composite materials. In works [14-27], a universal technique is proposed for solving direct and inverse coefficient problems based on both analytical and numerical solutions. Another approach is modeling composite materials using the effective media theory, or homogenization theory [28-41]. The task of such theories is to construct a procedure for the transition from structurally inhomogeneous media to homogeneous ones with averaging (effective) properties. It is assumed that an inhomogeneous medium containing inclusions or pores can be associated with a homogeneous medium with effective properties. The values of these properties are found by comparing the temperature drop, which is realized on the faces of the considered representative inhomogeneous and homogeneous fragments at a given heat flux. Recently, methods of numerical homogenization have been widely used, based on the variational approach and the method of asymptotic averaging [41-53]. In this paper, we propose a method for studying the mechanical properties of heat-shielding highly porous materials using an effective medium model.

2. Mathematical model

The final formula for determining the effective thermal conductivity of a medium with a spherical inclusion (or sometimes) has the form [3]:

$$R = R_M \left[1 + \frac{C}{\frac{1-C}{3} + \frac{R_M}{R_t - R_M}} \right] \quad (1)$$

Formula (1) corresponds to a polydisperse model of a medium with spherical inclusions.

$C = \left(\frac{a}{b} \right)^3$ – volume fraction of spherical inclusions;

R_M – effective thermal conductivity of the matrix;

R_t – effective coefficient of thermal conductivity of inclusions.

This expression is presented in work [3]; to obtain it, the self-consistent method of three Eshelby phases was used (Fig. 1).

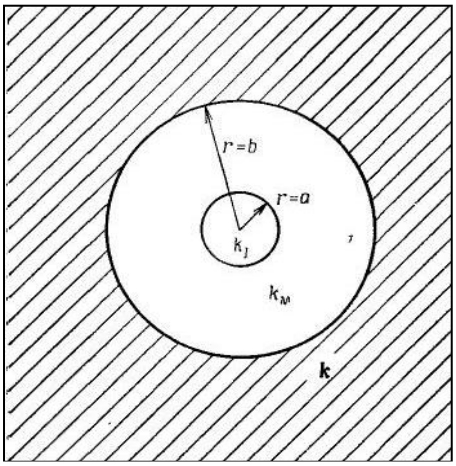

Fig. 1. Three-phase model of a medium with a spherical inclusion. Shaded medium with unknown effective properties

The exact solution for determining the effective elastic moduli of a medium with spherical inclusions is given in the work [7].

A model of a medium with a small volume fraction of spherical inclusions is presented in the work [3].

The shear modulus μ s determined by the following formula:

$$\frac{\mu}{\mu_M}=1-\frac{15\left(1-\upsilon_M\right)\left[1-\left(\frac{\mu_t}{\mu_M}\right)\right]C}{7-5\upsilon_M+2\left(4-5\upsilon_M\right)\left(\frac{\mu_t}{\mu_M}\right)}. \qquad (2)$$

$C=\left(\frac{a}{b}\right)^3$ – volume fraction of spherical inclusions;

υ_M – continuous phase Poisson's ratio (matrix);

μ_t – shear modulus of spherical inclusion.

Within the framework of the same model, the formula for the effective use of the volumetric module:

$$K=K_M+\frac{\left(K_t-K_M\right)C}{1+\frac{\left(K_t-K_M\right)}{K_M+\frac{4}{3}\mu_M}}. \qquad (3)$$

$K_M -$ volumetric matrix modulus;

$K_t -$ volumetric module of spherical inclusion;

μ_M – shear modulus of spherical matrix inclusion;

$C=\left(\frac{a}{b}\right)^3$ – volume fraction of spherical inclusions.

The polydisperse model describes media with an arbitrary fraction of inclusions.

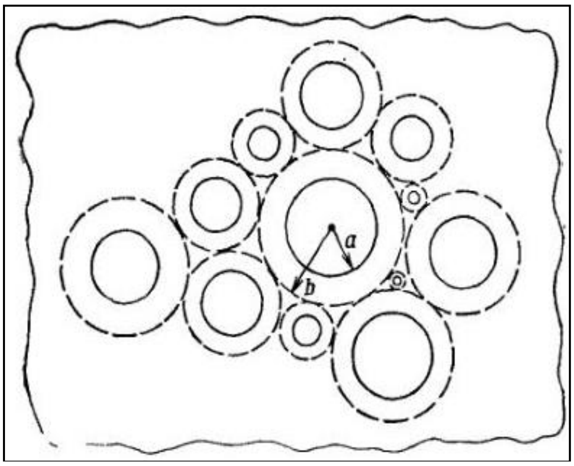

Fig. 2. Polydisperse model of a medium with spherical inclusions

Consider a continuous medium with spherical inclusions of various sizes. Dashed lines in Fig. 2 limited areas of the matrix associated with each individual inclusion. The ratio of the radii $\frac{a}{b}$ is assumed constant for each such particle, regardless of its absolute size. Therefore, the distribution of particle sizes should be such that the entire volume is filled with composite particles. Obviously, this distribution requires that the particle size be reduced to infinitesimal. Acceptable results cannot be expected from this model for systems containing large volume fractions of inclusions.

$$K = K_M + \frac{(K_t - K_M)C}{1 + (1-C)\frac{(K_t - K_M)}{K_M + \frac{4}{3}\mu_M}}; \qquad (4)$$

$$\frac{\mu}{\mu_t} = 1 - \frac{\left[1 - \frac{\mu_M}{\mu_t}\right]\left[7 - 5\upsilon_M + 2(4 - 5\upsilon_M)\frac{\mu_M}{\mu_t}\right]}{15(1-\upsilon_M)}(1-C). \qquad (5)$$

If the disparity between μ_t and μ_M is large, then it is necessary to use a three-phase model to determine the shear modulus.

Thermal expansion coefficients are determined by the following formulas:

$$a = \bar{a} + \frac{a_1 - a_2}{\frac{1}{k_1} - \frac{1}{k_2}}\left[\frac{1}{k} - \left(\frac{1}{\bar{k}}\right)\right], \qquad (6)$$

here $\bar{a} = C_1 a_1 + C_2 a_2$, $\frac{1}{\bar{k}} = \frac{C_1}{k_1} + \frac{C_2}{k_2}$,

k, k_1, k_2 – volumetric modules,

C_1, C_2 – volumetric phase content,

a_1, a_2 – thermal expansion coefficients of phases.

The specific heat capacities of two-phase composites are determined by the formula:

$$\frac{C_p - C_V}{T_0} = 9\left(\frac{a_2 - a_1}{\frac{1}{k_2} - \frac{1}{k_1}}\right)^2 \left[\frac{1}{k} - \frac{1}{\bar{k}}\right], \quad (7)$$

where $C_p - C_V = 3ka^2T_0$.

3. Determination of the mechanical characteristics of the composite material

The material under consideration is presented in the form of a composite, in which the matrix is cylindrical fibers, and the inclusions are air ellipsoidal pores. The mechanical properties of the fibers have been identified based on [15-18]. For this, the problem of modeling the typical structure of a composite material was considered. The problem of determining the effective moduli of elasticity and strength of the porous material was considered. In fact, the problem has been reduced to a model of an isolated elliptical inclusion-pore. The pore size (aspect ratio of the ellipse) was chosen based on the data on the structural structure of the heat-shielding material. Fiber orientation 45 deg. corresponded to the inclusion in the form of a sphere. If the fibers in the material are not oriented isotropically, but are inclined to the normal to the surface of the slab at a certain angle, then it was assumed that the pores in the material can be represented in the form of ellipsoids, the dimensions of which are determined by the angle of inclination of the fibers. As a result of calculations, it is possible to obtain a prediction of the values of the elastic moduli of an anisotropic material containing elliptical pores and ultimate strength in different directions and under different loading conditions. To determine the ultimate strength, it was required to set the strength values of the "carcass" formed by the fibers. By comparing the calculation results and experimental data (elastic moduli and ultimate strength in the plane and in the normal direction), the characteristics of the inherent strength and rigidity of the "frame" formed by the fibers were found. It was taken into account that the structure of the material is not isotropic. It is known that fibers in a composite material have an average inclination angle of 60 degrees with respect to the normal [7]. Further, all effective characteristics of the board were found for various orientation angles and fiber volumetric contents. For this purpose, the found characteristics of the rigidity and strength of the fiber “frame” were fixed in the model, and the parameters that determine the structure of the material were changed. Fixed aspect ratio (AR) was calculated using the formula $\mathrm{ctg}(\alpha) = AR$ and is shown in the table 1.

Table 1 Correspondence of the ellipsoidal pore sizes used in the model and the average angles of fiber orientation in the structure

α , average fiber orientation angle	AR
0	85
10	5,67
20	2,75
30	1,73
40	1,19
50	0,839
60	0,577
70	0,36
80	0,176
90	0,001

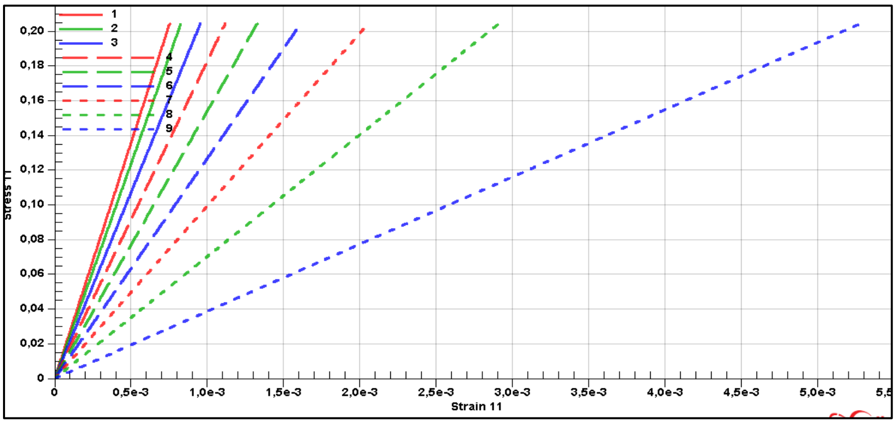

Fig. 3. Tensile stress-strain diagrams (perpendicular to the plane) for pore volumetric content 93,2%

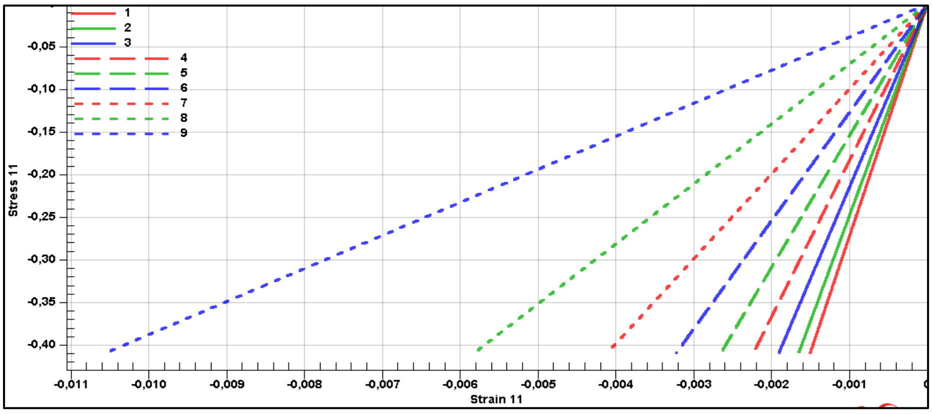

Figure: 4. Diagrams of stress-strain in compression (perpendicular to the plane) for the volumetric pore content 93,2%

In fig. 5 and 6 show diagrams of stress - strain in tension and compression, respectively (parallel to the plane). Volumetric pore content 93,2%. The pore orientation angle took the following values: $1 - 0^0$, $2 - 10^0$, $3 - 20^0$, $4 - 30^0$, $5 - 40^0$, $6 - 50^0$, $7 - 60^0$, $8 - 70^0$, $9 - 80^0$.

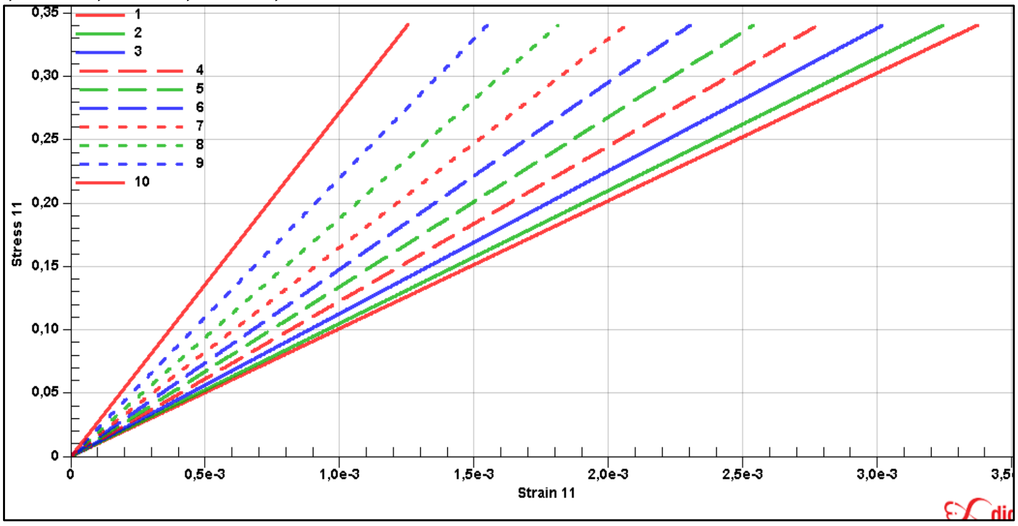

Fig. 5. Tensile stress-strain diagrams (parallel to plane) for pore volumetric content 93,2%

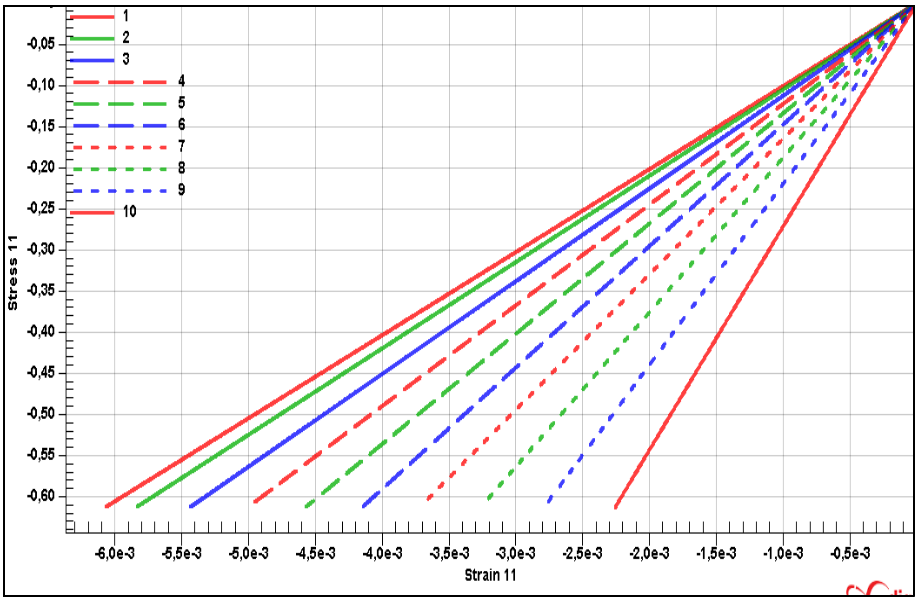

Figure: 6. Compression stress-strain diagrams (parallel to plane) for pore volumetric content 93,2%

Figures 3 - 5 show a significant effect of the dependence of stresses on deformations on the angle of fiber orientation.

4. Conclusion

1. A method for studying the mechanical properties of heat-shielding highly porous materials using an effective medium model is proposed.
2. Calculated mechanical characteristics of the composite material depending on the angle of orientation and the volumetric content of fibers.
3. Stress-strain diagrams were plotted in tension and compression parallel and perpendicular to the plane.
4. A significant effect of the dependence of stresses on deformations on the angle of fiber orientation is shown.

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