Research Article

Mathematical modeling methods for estimation the thermophysical properties of heat-protective composite materials

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Abstract. The paper proposes a method of mathematical modeling for the identification of thermophysical properties using the developed software package based on a composite material model, presented as a combination of plates of alternating dissimilar components, of the material of fibers and air, oriented parallel and perpendicular to the heat flow. The influence of the angle of orientation of fibers and their volumetric content on the effective thermal conductivity is established.

Keywords: effective thermal conductivity, composite material, mathematical modeling, heat-shielding element

1. Introduction

At present, ever higher requirements are imposed on the accuracy of mathematical models of composite materials. With a tiled heat-shielding coating, the main structural element is a heat-shielding element, which consists of a fibrous heat-shielding tile, erosion-resistant and varnish coatings, a damping pad and an adhesive that connects the damping substrate with the tile and the heat-shielding element as a whole with the aircraft body (Fig. 1).

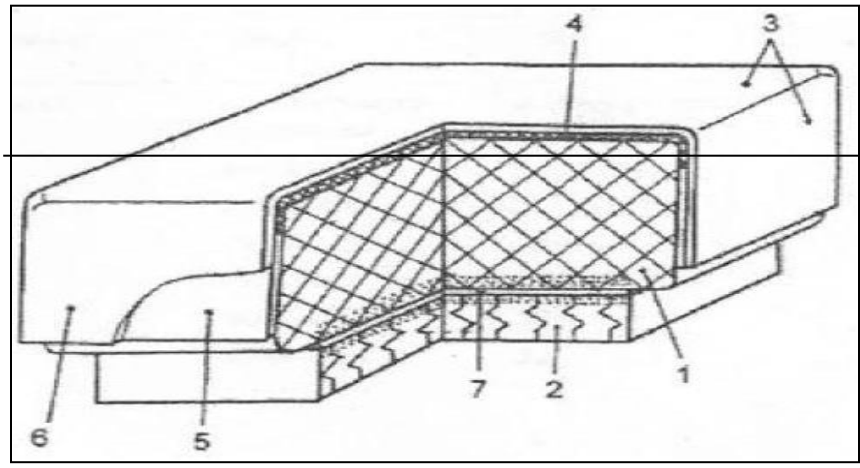

Fig. 1. The design of the heat-shielding element: 1- tiles made of fibrous heat-shielding composite material; 2- damping gasket; 3,6 - varnish moisture-proof coating; 4,5 - external and lateral glassy erosion-resistant coating; 7- adhesive layer.

In this design, each material fulfills its functional role, and in the absence of any of these materials, the design of the heat-shielding element as a whole, that is, the plate heat-shielding of the aircraft will not work. The main role is played by heat-shielding tiles. It is made of fibrous heat-shielding material and is a rigid spatial frame made of inorganic high-temperature fibers.

It is important to note that the properties of heat-shielding tiles (thermal conductivity, density, strength, coefficient of thermal expansion) are primarily determined by the fibers - their composition, structure, morphological features, etc. There is a sufficient number of works on the study of the thermophysical and mechanical properties of composite materials. As a rule, when modeling composite materials, two approaches are used: the theory of effective media [1-11] and the solution of inverse problems for the identification of thermophysical parameters of composite materials. To solve inverse coefficient problems, a modern methodology is proposed in the works of V.F. Formalev and S.A. Kolesnik [12-28]. In works [29-53] direct problems of determining the temperature fields in composite materials with significant anisotropy of properties, both analytical and numerical methods, are also solved, and experiments are described to determine the properties of composite materials. In this paper, a composite material is considered, in which the matrix is air, the inclusions are cylindrical inclusions-fibers. Modeling of thermophysical properties was carried out in a specially developed software package based on a composite material model, presented in the form of a combination of plates of alternating dissimilar components, in this case, from the material of fibers and air, oriented parallel and perpendicular to the heat flow.

2. Mathematical model

To assess the thermophysical properties of the materials under consideration, we used a model of the structure of a chaotic fibrous system, represented as a combination of plates of alternating dissimilar components, in this case, of the material of fibers and air, oriented parallel and perpendicular to the heat flow. The thermal conductivity of such a model can be represented as a function of the thermal conductivity of two models-packets of flat plates, some of which are oriented parallel, and some are perpendicular to the direction of the heat flow.

$$L = \alpha_1 \left[L_1 \left(1 - m_2\right) + L_2 m_2 \right] + \frac{\alpha_2}{\frac{1-m_2}{L_1} - \frac{m_2}{L_2}} \quad . \qquad (1)$$

$L -$ thermal conductivity of the system under consideration;

$L_1 -$ thermal conductivity of fiber material;

$L_2 -$ thermal conductivity of gas;

$m_2 -$ porosity of fibrous material;

$\alpha_1 -$ numerical coefficient characterizing the volume concentration of a package of plates located *parallel* to the direction of heat flow;

$\alpha_2 -$ numerical coefficient characterizing the volumetric concentration of a package of plates located *perpendicular* to the direction of heat flow.

Determination of thermal conductivity in the direction parallel and perpendicular to the plane of the slab depending on the orientation angle and the volumetric content of fibers

We will consider a composite material in which the matrix is air, inclusions are cylindrical inclusions-fibers.

Modeling of thermophysical properties was carried out in a specially developed software package using two different methods for specifying the angle of orientation of fibers. To choose the most suitable method, we will conduct a test study for mullite fibers, after which we apply the results obtained to determine the thermal conductivity of a plate of heat-shielding material.

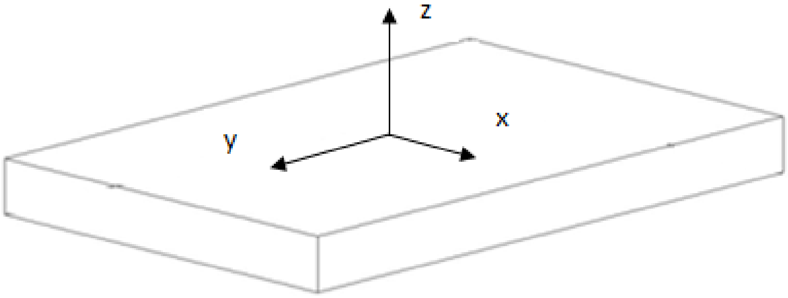

Fig. 2. Panel of heat-shielding composite material

The initial data were set as follows: specific heat capacity of air 1000 $\mathrm{J/(kg \cdot K)}$, thermal conductivity of air at 20 C^0 0.025 $\mathrm{W/(m \cdot K)}$, density has no significant effect on the thermal conductivity. Fiber properties: specific heat of mullite 1000 $\mathrm{J/(kg \cdot K)}$, thermal conductivity of mullite at 20 C^0 0.35 $\mathrm{W/(m \cdot K)}$. The ratio of the longitudinal and transverse dimensions of the fiber was 100. This parameter is used to define an elongated ellipsoidal shape. When this parameter is equal to one, the fiber acquires a spherical shape.

Fiber orientation models can be specified using two methods:

1. fixed - with fixed fiber orientation
2. tensor - using the fiber orientation tensor, which determines the probability of fiber orientation in different directions

The components of the main diagonal of the fiber orientation tensor were put in accordance with the solution of the system of equations:

$$\begin{cases} A \cdot \tan\alpha = B\sqrt{2} \\ A + 2B = 1 \end{cases} \quad (2)$$

the solution of this system has the form in which the values of the components of the main diagonal of the orientation tensor are assigned to a specific mean angle of fiber orientation.

When constructing a solution for isotropic cases with different volumetric fiber content, a model was used that corresponds to an isotropic material, while the fibers are arranged in a chaotic manner and are oriented in all directions with equal probability.

Below in Fig. 3-4 shows a comparative analysis of the results obtained using different models of fiber orientation. Here λ, - coefficient of thermal conductivity, [W /(m·K)], φ - fiber orientation angle.

Tab. 1 Solution of the system of equations (2), which determines the value of the diagonal components of the fiber orientation angle tensor

Angle α	A	B
0	1	0
10	0,800407242	0,099796379
20	0,660182948	0,169908526
30	0,550510257	0,224744871
40	0,457317196	0,271341402
50	0,372384821	0,313807589
60	0,289897949	0,355051026
70	0,204686509	0,397656745
80	0,110859784	0,444570108
90	0	0,5

$\alpha[1,1] = A;\ \alpha[2,2] = \alpha[3,3] = B$.

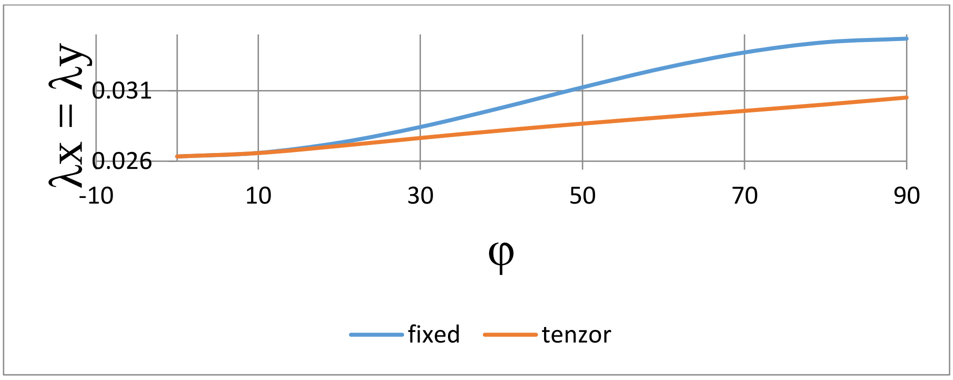

Fig. 3. Thermal conductivity in the direction of the *X* and *Y* axes (volumetric content of mullite m1=0,03%)

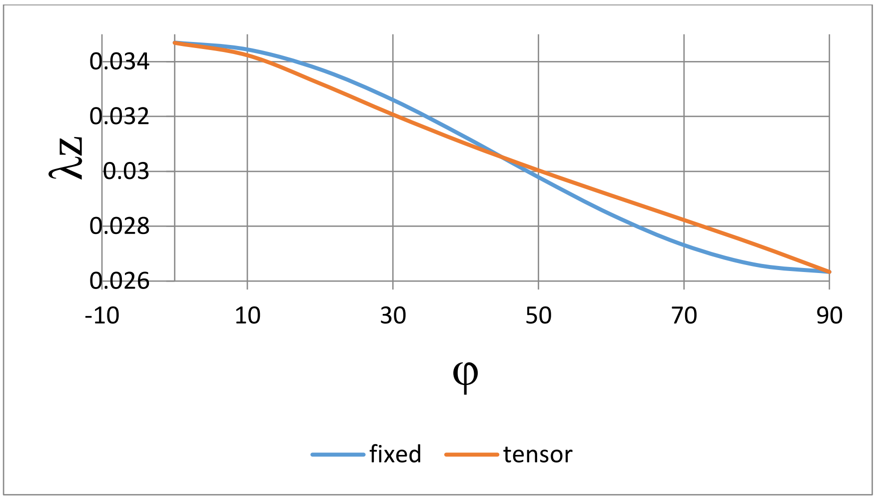

Fig. 4. Thermal conductivity in the direction of the Z-axis (volumetric content of mullite m1=0,03%)

The second tensor model is more consistent with a structure with a chaotic arrangement of fibers. This model assumes that the fibers have a fixed average angle of inclination to the normal to the surface of the slab, while they can be oriented arbitrarily in the plane of the slab (that is, they can "rotate" around the normal and each location will be equally probable). That is why the model with the specification of the fiber orientation tensor demonstrates lower values of the thermal conductivity coefficient - there is no selected direction in the fiber plane, which is fixed when choosing the method for specifying the fiber orientation with a fixed angle.

Based on the chosen model, we will assess the thermal conductivity of a heat-shielding element with different fiber orientation angles. In Figures 5-6, dots on the graphs indicate the values of thermal conductivities in isotropic cases.

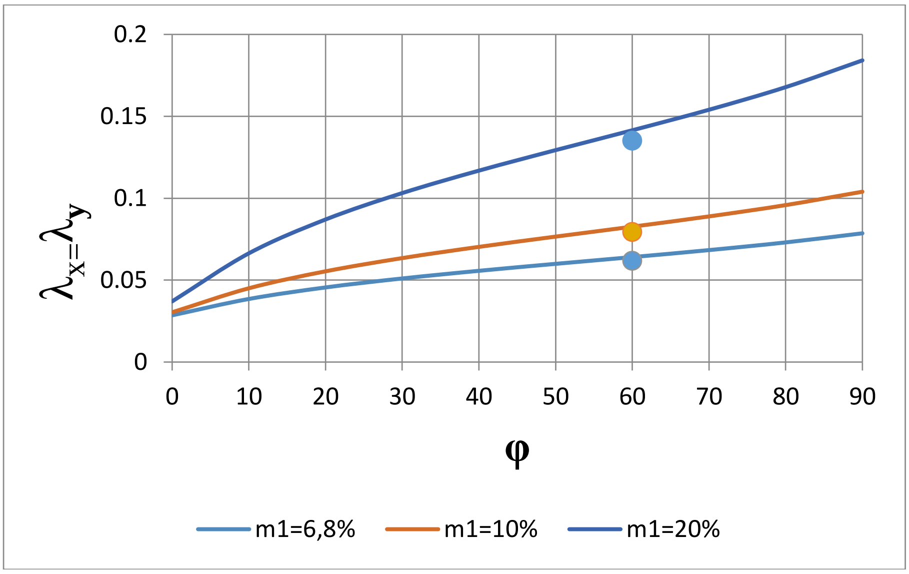

Fig. 5. Thermal conductivity in the plane of the plate of the heat-shielding composite material depending on the angle of orientation of the fibers and their volumetric content

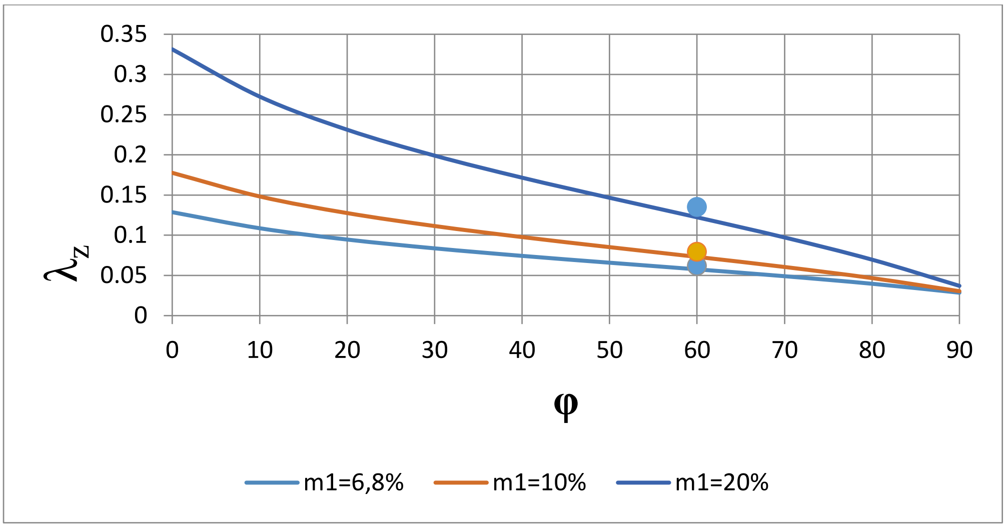

Fig. 6. Thermal conductivity in the selected direction of the plate of the heat-shielding composite material depending on the angle of orientation of the fibers and their volumetric content

3. Conclusion

In this work, the microstructure of a plate of a heat-shielding composite material was investigated. The modeling of thermophysical properties was carried out using the developed software package based on a composite material model, presented in the form of a combination of plates of alternating dissimilar components, in this case, from the material of fibers and air, oriented parallel and perpendicular to the heat flow. The influence of the angle of orientation of fibers and their volumetric content on the effective thermal conductivity is established.

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