Cycle Related Graphs on Square Difference Labeling

J. Rashmi Kumar [1] and K. Manimekalai[2]

[1,2] Department of Mathematics, Bharath Institute of Higher Education and Research, Chennai, India.
[1]Corresponding author: rashmilenny@gmail.com

Article History Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 28 April 2021

Abstract:
In this study, we prove that the graphs cycle C_n with parallel chords, $2-tuple$ of $Z-P_n$, Durer-graph, Moser-spindle, Herchel graph are Square difference graph (SDG).

Keywords: Square difference Labeling (SDL), Z-P_n, Durer-graph, Moser-spindle, Herchel graph, 2-tuple graph.

1.Introduction

All graphs in this paper are simple, undirected and finite. We refer J. A. Gallian for detailed study [1] and follow [2] for all terminology and notation. The Square difference labeling is introduced by Shiama [6]. A function of a graph G admits one to one and onto function $f: v(G) \to \{0,1,2, \ldots \ldots p-1\}$ such that the 1-1 function $f^*: E(G) \to N$ given by $f^*(uv) = |[f(u)^2 - f(v)^2|$, for all $uv \in E(G)$, distinct are said to be Square Difference graph[SDG] [1,4]. The concept of 2-tuple was introduced by P.L. Vihol [7]. P. Jagadeeswari investigated some various graphs for SDL. [4,5]. In this work, we prove Cycle related graphs on Square difference labeling.

We Commenced some preliminaries which are helpful for our work.

Definition 1.1[5]:
The graph $Z-Pn$ is acquired from the two paths $P_n{}'$ and $P_n{}''$. Let v_i and u_i, $i = 1,2 \ldots \ldots n-1$, are the vertices of path $P_n{}'$ and P_n'' respectively. To determine $Z-Pn$ attach i^{th} vertex of path $P_n{}'$ with $(i+1)^{th}$ vertex of path $P_n{}''$. for all $i = 1,2 \ldots \ldots n-1$.

Definition 1.2[5]:
Let $G = (V, E)$ be a simple graph and let $G' = (V', E')$ be another copy of G. Link each vertex V of G to the equivalent vertex V' of G' by an edge. The new graph thus gained is called 2-tuple of G. We signify 2-tuple graph of G by the notation $T^2(G)$.

Definition 1.3[5]:
The Durer graph is the graph termed by the vertices and edges of the durer solid. It is a cubic graph of girth 3 and diameter 4. The Durer graph is Hamiltonian. It has exactly 6 Hamiltonian cycles, each pair of which may be mapped into each other by a symmetry of the graph.

Definition 1.4[5]:
The Moser graph which is also called Moser spindle is an undirected graph with 7 vertices and 11 edges.

Definition 1.5[5]:
The Herschel graph is a bipartite undirected graph with 11 vertices and 18 edges, the smallest non-Hamiltonian polyhedral graph.

Main Results

Theorem 2.1:
The $2-tuple$ graph of $Z-P_n$ admits SDL.

Proof:
Let G be the graph with $4n$ vertices and $(8n-6)$ edges. Consider the vertex set $V = \{u_i, v_i \,/\, 1 \leqslant i \leqslant n\}$ and the edge set $E = \{u_iv_i, u_iu_{i+1}, v_iv_{i+1}, u_iu_{i+8}, v_iv_{i+8} \,/\, 1 \leqslant i \leqslant n-1\}$.
Determine the 1-1 and onto function as:

$f(u_i) = 2(i-1) \quad for\ 1 \leqslant i \leqslant n$
$f(v_i) = 2i - 1$
$f^*(u_iu_{i+1}) = 8i - 4 \quad for\ 1 \leqslant i \leqslant n-1$
$f^*(v_iv_{i+1}) = 8i$
$f^*(u_iv_i) = 4i - 3$
$f^*(u_iu_{i+8}) \equiv 0(mod\ 8)$
$f^*(v_iv_{i+8}) \equiv 0(mod\ 8)$

$Here\ f^*(u_iu_{i+8}) > f^*(v_iv_{i+8})$, therefore all the edge labeling are dissimilar. Hence the theorem is verified.

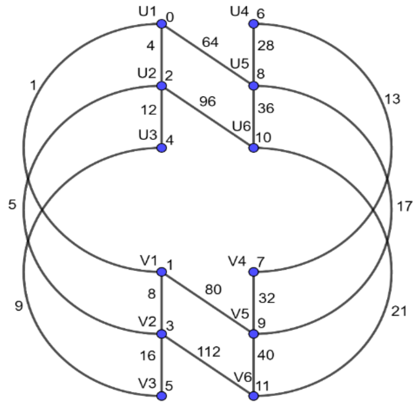

Figure 1. 2-tuple of Z-P_3

Theorem 2.2:

Every cycle C_n ($n \geq 6$) with parallel chords is Square difference graph.

Proof:

Contemplate the graph G with $V = \{v_i \ / \ 0 \ \leqslant \ i \ \leqslant \ n-1\}$ and $E \ = \ \{v_i v_{i+1}, v_i v_{n-i} / 0 \ \leqslant \ i \ \leqslant \ n-1\}$ .The labeling f for the vertices and the labeling f^* for the edges are given respectively in the following two cases depending on n being even and n being odd .

Also,

$|V(G)| \ = \ n$ and

$$| \, E(G)| \ = \begin{cases} \frac{(3n-3)}{2}, \ n \ is \ odd \\ \frac{(3n-2)}{2}, n \ is \ even \end{cases}$$

Case (i) : *n* is even

Define the bijective function g and the edge labeling g^* as:

$g(v_i) \ = \ i, 0 \ \leqslant \ i \ \leqslant \ n-1$

$g^*(v_i v_{i+1}) = 2i \ + \ 1$

$g^*(v_i v_{n-i}) \equiv 0 (mod \ n)$

$g^*(v_0 v_{n-1}) = (n-1)^2$

$g^*(v_0 v_1) = 1$

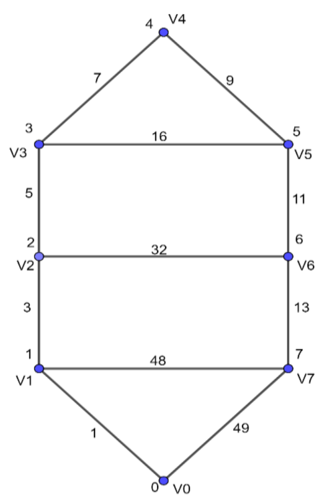

Figure 2. Cycle C_8 with parallel chords

Case(ii): *n* is odd

$g\,(v_i) = i,\ 0 \leqslant i \leqslant n-1$

$g^*(v_i v_{i+1}) = 2i + 1$

$g^*(v_0 v_1) = 1$

$g^*(v_i v_{n-1}) \equiv 0(mod\ n)$

$g^*(v_0 v_{n-1}) = (n-1)^2$

For the above labeling pattern, the induced edge labeling function $g^*: E\,(G) \to N$ defined by $g^*(uv) = |\,[g(u)^2 - g(v)^2|$, for every $uv \in E(G)$ are all diverse. such that $g^*(e_i) \neq g^*(e_j)$ for every $e_i \neq e_j$. Hence the graph *G* admits *SDL*.

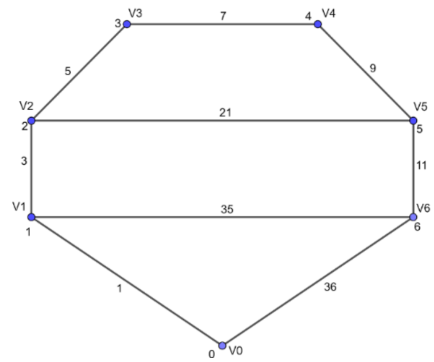

Figure 3. Cycle C_5 with parallel chords

Theorem 2.3:

The Durer graph acknowledges SDL.

Proof:

Contemplate the graph G with 12 vertices and 18 edges . Let v_i be the vertex set for $0 \leqslant j \leqslant 11$. Now define the function $f : V \to \{0,1,\ldots,11\}$ as :

$f(v_j) = j$ for $0 \leqslant j \leqslant n-1$

and the induced function *f* * satisfies the condition of square difference labeling and it yields the edge labels as

$f^*(v_0 v_1) = 1$

$f^*(v_0 v_5) = 25$

$f^*(v_i v_{i+1}) = 2i + 1\,,\ \ i = 1\ to\ 5$

$f^*(v_i v_{i+7}) \equiv 0(mod\ 7)\ \ i = 1\ to\ 4$

$f^*(v_i v_{i+2}) \equiv 0(mod\ 4)\ \ i = 6\ to\ 9$

$f^*(v_i v_{i+4}) \equiv 0(mod\ 8)\ \ i = 6, 7$

Thus the entire 11 edges acquire the discrete edge labels. Hence the theorem is verified.

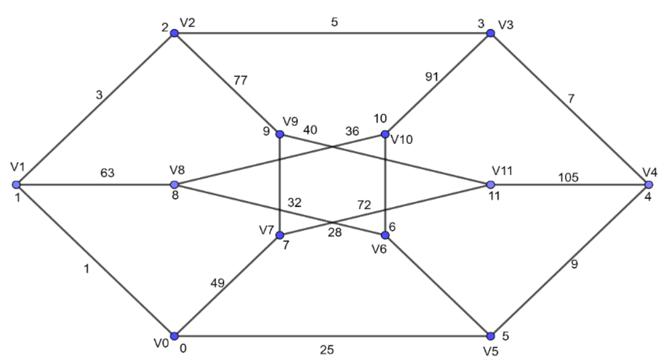

Figure 4. Durer graph

Theorem 2.4:

The Moser - Spindle graph is Square Difference graph.

Proof:

Consider the Moser Spindle graph with 7 vertices and 11 edges. Let u_j be the vertex set for $0 \leqslant j \leqslant 6$. Then the vertex function f and edge function f^* is defined as

$f(u_j) = j, \quad 0 \leqslant j \leqslant n-1$

$f^*(u_j u_{j+1}) = 2j+1, \ j = 0 \ to \ 3$

$f^*(u_0 u_j) = j^2, \ j = 1,4,5,6$

$f^*(u_j u_{j+3}) \equiv 0(mod\ 7), \ j = 2,3$

$f^*(u_1 u_5) \equiv 0(mod\ 8)$

$f^*(u_1 u_4) \equiv 0(mod\ 4)$

Hence the theorem.

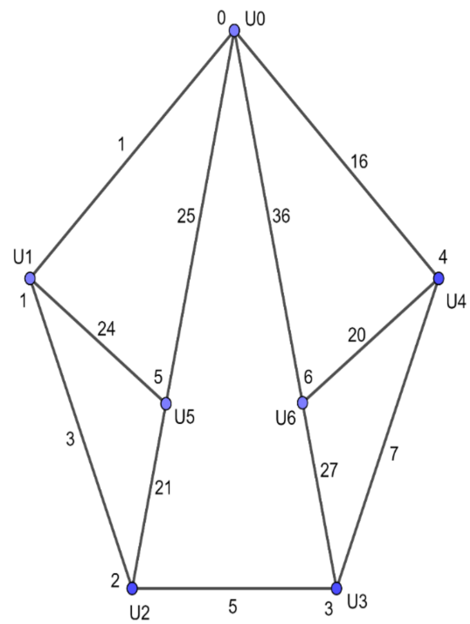

Figure 5. Moser – Spindle graph

Theorem 2.5:

The Herschel graph is SDG.

Proof:

Consider the Herschel graph with 11 vertices and 18 edges . The vertex set $V = \{v, u\}$. Then the vertex valued function f and edge function f^* is defined as

$f(u_i) = 2i+1, \ 0 \leqslant i \leqslant 4$

$f(v_j) = 2j, \ 0 \leqslant j \leqslant 5$

$f^*(v_1 u_1) = 5$

$f^*(v_0 u_2) = 25$

$f^*(v_1 u_3) = 45$

$f^*(v_4 u_1) = 55$

$f^*(v_3 u_0) = 35$

$f^*(v_2 u_4) = 65$

$f^*(v_5 u_2) = 75$

Hence $f^*(e_i) \neq f^*(e_j) \ \forall e_i, e_j \in E(G)$. i.e., all the edge labeling are diverse. Therefore, the theorem is verified.

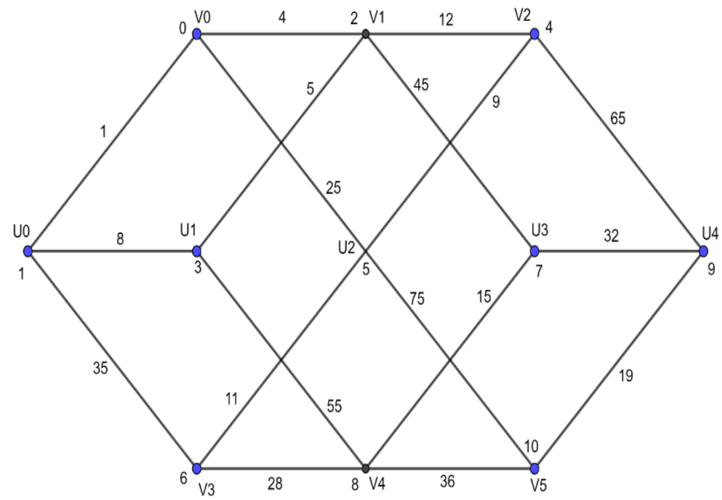

Figure 6. Herschel graph

REFERENCES:

1. J. A. Gallian, "A Dynamic survey of graph labeling", The electronics journal of Combinatories, 17(2010), # DS6.
2. F. Harary, Graph Theory, Narosa publication House Reading, New Delhi, 1998.
3. P.Jagadeeswai, "Considering Square difference labeling for validating Theta graphs of dynamic machinaries" , International journal of Innovative Technology and Exploring Engineering, ISSN : 2278-3075, vol-9, Issue-2S4 dec2019.
4. P.Jagadeeswai, K.Manimekalai, K.Bhuvaneswari , "Square difference labeling of cycle , path and tree related graphs", Advancement in Engineering, science & Technology J.Mech.Cont.&Math.sci.,Special issue, No.-2, August (2019) pp 627-631.
5. Mydeenbibi & M.Malathi , " Equality labeling on special graphs" , International journal of modern trends in Engineering & Research, vol5 , issue 04 (Apr2018) ISSN(online) 2349-9745 ISSN(print) 2393-8161.
6. J.Shiama , " Square difference labeling for certain graph " International journal of computer applications (0975-08887), 44(4) (2012).
7. P. L. Vihol et. al., "Difference Cordial Labeling of 2-tuple Graphs of Some Graphs", International Journal of Mathematics and its Applications, Volume 4, Issue 2–A (2016), 111–119.