Research Article

Hybrid Fuzzy Bi-Ideals In Near-Rings

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ABSTRACT: In this paper, we introduce the concept of hybrid fuzzy bi-ideals in near rings and give some characterizations of hybrid fuzzy bi-ideals in near rings.

Key words: near-ring, hybrid fuzzy bi-ideal, hybrid structures.

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1. Introduction

In 1965, researcher L.A. Zadeh invented the innovative idea the fuzzy set [4]. M.Himaya Jaleela Begum and S. Jeya lakshmi [1] presented the concept of anti fuzzy bi-ideals in near-ring .The Hybrid structures and applications are introduced Young Baejun. Seok-Zunsong, G.Muhiuddin[5].Young Bae Jun, Madad Khan,Saima Anis [2] presented the concept of hybrid ideals in Semigroups. B.Elavarasam,K.Porselvi YoungBae Jin discussed hybrid generalized bi-ideals in Semi groups[3].In this research paper, we introduce the notion of hybrid fuzzy bi-ideals of Near –rings and illustrated with examples.

2.Preliminaries

Definition:2.1 [3] Let N be a near-ring with two binary operations as '+' and '·' which satisfy the following conditions:

(i) (N,+) be a group
(ii) (N, ·) be a semi-group.
(iii) (x+y) · z= x·z+y·z∀ x,y,z ∈N.
Precisely because it satisfies the right distributive law, it is a right near-ring. We would instead use the term "near-ring" of near ring right". We denote xy instead of x·y. Note that0 (x)=0 and(-x) y=-xy but in general x (0)≠0 for some x ∈N.

Definition:2.2 [3] Let N be a near-ring and let I be the non-empty subset of near- ring N that is called as an ideal of N which satisfies the following conditions:

(iv) (I,+) be a normal subgroup of (N,+),
(v) IN$\subseteq I$,
(vi) y(i+x)-yx $\in I \forall i \in I; x, y \in N$.

Definition:2.3 [1] Let N be a near-ring. A fuzzy set μ of N is called as an anti fuzzy bi-ideal of N if for all $x, y, z \in N$.

(i) $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$.
(ii) $\mu(xyz) \leq \max\{\mu(x), \mu(z)\}$.

Note: 2.4
Jun et al presented the basic representation of hybrid structure and associated outcome as result [4]. Let $\mathcal{P}(U)$ said to be the power set of an initial universal set U and let I be the unit interval.

Definition: 2.5[5] Let $\widetilde{f_\lambda}$ be a hybrid structure in N over U is defined as a mapping

$\widetilde{f_\lambda}:=(\tilde{f}, \lambda): N \rightarrow \mathcal{P}(U) \times I$ X $\mapsto (\tilde{f}(x), \lambda(x))$

Where $\tilde{f}: N \rightarrow \mathcal{P}(U)$ and $\lambda: N \rightarrow I$ are mapping.

The set of all hybrid structures in N over U is denoted by $\mathbb{H}(N)$.

Define a relation '≪' on as follows:

$\tilde{f} \ll \tilde{g} \Leftrightarrow \tilde{f} \mathrel{\tilde{\subseteq}} \tilde{g}$ $\lambda \succcurlyeq \mu \forall \tilde{f}, \tilde{g} \in \mathbb{H}(N)$ where $\tilde{f} \mathrel{\tilde{\subseteq}} \tilde{g}$ means that $\tilde{f}(x) \subseteq \tilde{g}(x)$ and $\lambda \succcurlyeq \mu$ means that $\lambda(x) \succcurlyeq \mu(x) \forall x \in$N.then $(\mathbb{H}(N), \ll)$ is a partially ordered set

Definition:2.6[5] Let $\widetilde{f_\lambda}$ be a hybrid structure in N over U. Then sets

$$\widetilde{f_\lambda}[\alpha, t] := \{x \in X \,|\, \tilde{f}(x) \supseteq \alpha, \lambda(x) \leq t\}$$

$$\tilde{f}_\lambda(\alpha,t] \coloneqq \{x \in X \mid \tilde{f}(x) \supsetneq \alpha, \lambda(x) \le t\}$$
$$\tilde{f}_\lambda[\alpha,t) \coloneqq \{x \in X \mid \tilde{f}(x) \supseteq \alpha, \lambda(x) < t\}$$
$$\tilde{f}_\lambda(\alpha,t) \coloneqq \{x \in X \mid \tilde{f}(x) \supsetneq \alpha, \lambda(x) < t\}$$

are called the $[\alpha,t]$- hybrid cut, $(\alpha,t]$ -hybrid cut $[\alpha,t)$ - hybrid cut (α,t)- the hybrid cut of $\tilde{f}_\lambda$respectively where $\alpha \in \wp(U)$ and t $\in I$

Obviously, $\tilde{f}_\lambda(\alpha,t) \subseteq \tilde{f}_\lambda(\alpha,t] \subseteq \tilde{f}_\lambda[\alpha,t]$ and $\tilde{f}_\lambda(\alpha,t) \subseteq \tilde{f}_\lambda[\alpha,t) \subseteq \tilde{f}_\lambda[\alpha,t]$

Definition: 2.7[3] Let $\tilde{f}_\lambda \in \mathbb{H}(N)$.For $\phi \neq A \subseteq N$,the characteristic hybrid structure in N over U is denoted by $\chi_A(\tilde{f}_\lambda)$ =($\chi_A(\tilde{f}), \chi_A(\lambda)$) and it is defined by

$$\chi_A(\tilde{f}): N \to \mathcal{P}(U) x \mapsto \begin{cases} U & x \in A \\ \phi & otherwise \end{cases}$$

and

$$\chi_A(\lambda): N \to I x \mapsto \begin{cases} 0 & x \in A \\ 1 & otherwise \end{cases}$$

Definition:2.8 [5] Let$\tilde{f}_\lambda$, $\tilde{g}_\mu \in \mathbb{H}(N)$, the hybrid intersection of $\tilde{f}_\lambda$and $\tilde{g}_\mu$ is denoted by $\tilde{f} \Cap \tilde{g}$ and is describes to be a hybrid structure

$\tilde{f}_\lambda \Cap \tilde{g}_\mu: N \to \mathcal{P}(U) \times I$ X $\mapsto ((\tilde{f} \,\tilde{\cap}\, \tilde{g})(x), (\lambda \vee \mu)(x))$ Where

$\tilde{f} \,\tilde{\cap}\, \tilde{g}: N \to \mathcal{P}(U)$ X $\mapsto \tilde{f}(x) \cap \tilde{g}(x)$

$\lambda \vee \mu : N \to I$ X $\mapsto \lambda(x) \vee \mu(x)$

2. Hybrid Fuzzy Bi-ideals

Definition: 3.1 Let $\tilde{f}_\lambda \in \mathbb{H}(N)$, $\tilde{f}_\lambda$ is called a hybrid fuzzy bi-ideal of N over U which satisfies the following conditions:

(HB1): $\tilde{f}(x+y) \supseteq \cap\{\tilde{f}(x), \tilde{f}(y)\}$

(HB2): $\lambda(x+y) \le \vee \{\lambda(x), \lambda(y)\}$

(HB3):$\tilde{f}(xyz) \supseteq \cap\{\tilde{f}(x), \tilde{f}(z)\} \forall x,y,z \in N$

(HB4): $\lambda(xyz) \le \vee \{\lambda(x), \lambda(z)\}$

Example:3.2 The universal set U is given by U=[0,1]

Let N={0,x,y,z} is a near-ring with the binary operation '+' and $'\cdot'$ defined by

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	x	0
z	z	y	0	x

·	0	x	y	z
0	0	0	0	0
x	0	0	0	0
y	0	0	0	x
z	0	0	0	x

Respectively.

Let $\tilde{f}_\lambda$be a hybrid structure in N over U which is given in the table

N	$\tilde{f}$
0	[0,0.6]
x	[0,0.2]
y	{0}
z	{0}

λ be any constant mapping from N to I . Then we say that $\tilde{f}_\lambda$ is a hybrid fuzzy bi-ideal of N over U.

Theorem:3.3 Let A be a non-empty subset of N and A is a hybrid fuzzy bi-ideal of N over U. Show that $\chi_A(\tilde{f}_\lambda)$ is a hybrid fuzzy bi-ideal of N.

Proof:

Let A be a hybrid fuzzy bi-ideal of N and Let x, y $\in N$

Case (i) If $x \notin A, y \notin A$ then $x+y \notin A$

$\therefore \chi_A(\tilde{f})(x+y) \supseteq \phi = \cap\{\chi_A(\tilde{f})(x), \chi_A(\tilde{f})(y)\}$ and

$\chi_A(\lambda)(x+y) \le 1 = \vee\{\chi_A(\lambda)(x), \chi_A(\lambda)(y)\}$

Let x, y, z $\in N$ If $x \notin A, y \notin A$ and $z \notin A$ then $xyz \notin A$

$\chi_A(\tilde{f})(xyz) \supseteq \phi = \cap\{\chi_A(\tilde{f})(x), \chi_A(\tilde{f})(z)\}$ and

$\chi_A(\lambda)(xyz) \le 1 = \vee\{\chi_A(\lambda)(x), \chi_A(\lambda)(z)\}$

Case (ii) Let x, y $\in N$ Case (ii) If $x \in A, y \in A$ then $x+y \in A$

$\therefore \chi_A(\tilde{f})(x+y) \supseteq U = \cap\{\chi_A(\tilde{f})(x), \chi_A(\tilde{f})(y)\}$ and

$\chi_A(\lambda)(x+y) \le 0 = \vee\{\chi_A(\lambda)(x), \chi_A(\lambda)(y)\}$

Let x, y, z $\in N$

If $x \in A, y \in A$ and $z \in A$ then $xyz \in A$

$\chi_A(\tilde{f})(xyz) \supseteq U = \cap\{\chi_A(\tilde{f})(x), \chi_A(\tilde{f})(z)\}$ and
$\chi_A(\lambda)(xyz) \leq 1 = \vee\{\chi_A(\lambda)(x), \chi_A(\lambda)(z)\}$
$\therefore \chi_A(\widetilde{f_\lambda})$ is a hybrid fuzzy bi-ideal of N.

Proposition:3.4 Let $\widetilde{f_\lambda}$ and $\tilde{g}_\mu$ are the two hybrid structures in N over U. For any $\beta, \delta \in \mathcal{P}(U)$ and $a, b \in I$, we have the following properties:

(i) If $\beta \subseteq \delta$ and $b \leq a$, then $\widetilde{f_\lambda}[\delta, b] \subseteq \widetilde{f_\lambda}[\beta, a]$
(ii) If $\widetilde{f_\lambda} \ll \tilde{g}_\mu$,then $\widetilde{f_\lambda}[\beta, b] \subseteq \tilde{g}_\mu[\beta, b]$
(iii) If $(\widetilde{f_\lambda} \Cap \tilde{g}_\mu)[\beta, b] = \widetilde{f_\lambda}[\beta, b] \cap \tilde{g}_\mu[\beta, b]$

Proof: (i) Let $\beta \subseteq \delta$ and $b \leq a$ and let $x \in \widetilde{f_\lambda}[\delta, b]$. Then $\tilde{f}(x) \supseteq \delta \supseteq \beta$ and $\lambda(x) \leq b \leq a$, which implies that $x \in \widetilde{f_\lambda}[\beta, a]$Thus $\widetilde{f_\lambda}\,[\delta, b] \subseteq \widetilde{f_\lambda}[\beta, a]$

(ii) Assume that $\widetilde{f_\lambda} \ll \tilde{g}_\mu$, and let $x \in \widetilde{f_\lambda}[\beta, b]$
Then $\tilde{g}\,(x) \supseteq \tilde{f}(x) \supseteq \beta$ and $\mu\,(x) \leq \lambda(x) \supseteq b$
Hence $x \in \tilde{g}_\mu[\beta, b]$ andso $\widetilde{f_\lambda}[\beta, b] \subseteq \tilde{g}_\mu\beta, b]$

(iii) Let $x \in N$ we have $x \in \widetilde{f_\lambda} \Cap \tilde{g}_\mu)[\beta, b] \Leftrightarrow (\tilde{f} \tilde{\cap} \tilde{g})(x) \supseteq \beta, (\lambda \vee \mu)(x) \leq b$
$\Leftrightarrow \tilde{f}(x) \cap \tilde{g}(x)) \supseteq \beta\,, \vee\{\lambda(x), \mu(x)\} \leq b$
$\Leftrightarrow \tilde{f}(x) \supseteq \beta, \tilde{g}(x)) \supseteq \beta, \lambda(x) \leq b, \mu(x) \leq b$
$\Leftrightarrow x \in \widetilde{f_\lambda}[\beta, b], x \in \tilde{g}_\mu[\beta, b]$
$\Leftrightarrow x \in \widetilde{f_\lambda}[\beta, b] \cap \tilde{g}_\mu[\beta, b]$.

Theorem:3.5 If $(A_1, A_2, A_3, \ldots A_n)$ are hybrid fuzzy bi-ideal of N ,then $A = \bigcap_{i=1}^{n} A_i$ is also a hybrid fuzzy bi-ideal of N.

Proof: Let $A_i = \{(\widetilde{f}_{A_i}, \lambda_{A_i}) : i \in I\}$ be a non-empty family of hybrid fuzzy bi-ideal of N

Let x, y, z $\in N$ we have

(i) $\left(\bigcap_{i\in I} \widetilde{f}_{A_i}\right)(x+y) = \bigcap_{i\in I}(\widetilde{f}_{A_i}(x+y)) \supseteq \bigcap_{i\in I}(\cap\{\widetilde{f}_{A_i}(x), \widetilde{f}_{A_i}(y)\})$

$= \cap\{\bigcap_{i\in I}(\widetilde{f}_{A_i}(x), \bigcap_{i\in I}(\widetilde{f}_{A_i}(y)\}$

$Therefore \left(\bigcap_{i\in I}(\widetilde{f}_{A_i}\right)(x+y) \supseteq \cap\left\{\bigcap_{i\in I}(\widetilde{f}_{A_i}(x), \bigcap_{i\in I}(\widetilde{f}_{A_i}(y)\right\}$

(ii) $\bigvee_{i\in I} \lambda_{A_i}(x+y) = \bigvee_{i\in I}\left(\lambda_{A_i}(x+y)\right) \leq \bigvee_{i\in I}\left(\vee\left\{\lambda_{A_i}(x), \lambda_{A_i}(y)\right\}\right)$
$\leq \vee\{\bigvee_{i\in I} \lambda_{A_i}(x), \bigvee_{i\in I} \lambda_{A_i}(y)\}$

Therefore

$\bigvee_{i\in I} \lambda_{A_i}(x+y) \leq \vee\{\bigvee_{i\in I} \lambda_{A_i}(x), \bigvee_{i\in I} \lambda_{A_i}(y)\}$

(iii) $\left(\bigcap_{i\in I} \widetilde{f}_{A_i}\right)(xyz) = \bigcap_{i\in I}(\widetilde{f}_{A_i}(xyz)) \supseteq \bigcap_{i\in I}(\cap\{\widetilde{f}_{A_i}(x), \widetilde{f}_{A_i}(z)\})$

$= \cap\{\bigcap_{i\in I}(\widetilde{f}_{A_i}(x), \bigcap_{i\in I}(\widetilde{f}_{A_i}(z)\}$

$Therefore \left(\bigcap_{i\in I}(\widetilde{f}_{A_i}\right)(xyz) \supseteq \cap\left\{\bigcap_{i\in I}(\widetilde{f}_{A_i}(x), \bigcap_{i\in I}(\widetilde{f}_{A_i}(z)\right\}$

(iv) $\bigvee_{i\in I} \lambda_{A_i}(xyz) = \bigvee_{i\in I}(\lambda_{A_i}(xyz)) \leq \bigvee_{i\in I}(\vee\left\{\lambda_{A_i}(x), \lambda_{A_i}(z)\right\}) \leq \vee\{\bigvee_{i\in I} \lambda_{A_i}(x), \bigvee_{i\in I} \lambda_{A_i}(z)\}$

Therefore

$\bigvee_{i\in I} \lambda_{A_i}(xyz) \leq \vee\{\bigvee_{i\in I} \lambda_{A_i}(x), \bigvee_{i\in I} \lambda_{A_i}(z)\}$

Hence intersection of a non-empty collection of hybrid fuzzy bi-ideal is also a hybrid fuzzy bi-ideal of N.

4. Homomorphism of a hybrid structure

Definition:4.1[3]Let $g: L \to M$ be a mapping from a set L to set an M for a hybrid structure $\widetilde{f_\lambda}$ in M over U. Consider a hybrid structure

$g^{-1}(\tilde{f_\lambda}) := (g^{-1}(\tilde{f}), g^{-1}(\lambda))$ in L over U. Where $g^{-1}\left(\tilde{f}(x)\right) = \tilde{g}(f(x))$ and $g^{-1}(\lambda)(x)) = \lambda(g(x)) \forall\, x \in L$. Say that $g^{-1}(\tilde{f_\lambda})$ is the hybrid pre-image of $\tilde{f_\lambda}$ under g

For a hybrid structure $\tilde{f_\lambda}$ in L over U. The hybrid image of $\tilde{f_\lambda}$ under g is defined in M over U where for every k $\in M$

$$g\left(\tilde{f}(k)\right) = \begin{cases} \bigcup_{x \in g^{-1}(k)} \tilde{f}(x), & if\ g^{-1}(k) \neq \phi \\ \phi, & otherwise \end{cases}$$

$$g(\lambda(k)) = \begin{cases} \bigwedge_{x \in g^{-1}(k} \lambda(x), & if\ g^{-1}(k) \neq \phi \\ 1, & otherwise \end{cases}$$

Definition:4.2[3]Let N_1 and N_2 be two near-rings. Let $g: N_1 \rightarrow N_2$ is called a near- ring homomorphism if $g(x+y) = g(x) + g(y)$ and $g(xy) = g(x)g(y)$ for any $x, y \in N_1$

Theorem:4.3

Every Homomorphic hybrid pre-image of a hybrid fuzzy bi-ideal is also a hybrid fuzzy bi-ideal in N_1

Proof:

Let $g: N_1 \rightarrow N_2$ be a near- ring homomorphism and $\tilde{f_\lambda}$ be a hybrid fuzzy bi-ideal of N_2 over U and let $x, y, z \in N_1$ then

$(i) g^{-1}(\tilde{f})(x+y) = \tilde{f}(g(x+y)) = \tilde{f}(g(x) + g(y)) \supseteq \cap \{\tilde{f}(g(x)), \tilde{f}(g(y))\}$

Therefore $g^{-1}(\tilde{f})(x+y) \quad \supseteq \cap \{g^{-1}\left(\tilde{f}(x)\right), g^{-1}(\tilde{f}(y))\}$

$(ii) g^{-1}(\lambda)(x+y) = \lambda(g(x+y)) = \lambda(g(x) + g(y)) \leq \vee \{\lambda(g(x)), \lambda(g(y))\}$

Therefore $g^{-1}(\lambda)(x+y) \leq \vee \{g^{-1}(\lambda(x)), g^{-1}(\lambda(y))\}$

$(iii) g^{-1}(\tilde{f})(xyz) = \tilde{f}(g(xyz)) = \tilde{f}(g(x)g(y)g(z)) \supseteq \cap \{\tilde{f}(g(x)), \tilde{f}(g(z))\}$

Therefore $g^{-1}(\tilde{f})(xyz) \quad \supseteq \cap \{g^{-1}\left(\tilde{f}(x)\right), g^{-1}(\tilde{f}(z))\}$

$(iv) g^{-1}(\lambda)(xyz) = \lambda(g(xyz)) = \lambda(g(x)g(y)g(z)) \leq \vee \{\lambda(g(x)), \lambda(g(z))\}$

Therefore $g^{-1}(\lambda)(xyz) \quad \leq \vee \{g^{-1}(\lambda(x)), g^{-1}(\lambda(z))\}$

Therefore $g^{-1}(\tilde{f_\lambda})$ is a hybrid fuzzy bi-ideal in N_1

Theorem:4.4

Let $g: N_1 \rightarrow N_2$ be an onto homomorphism of near-rings let $g^{-1}(\tilde{f_\lambda}) = (g^{-1}(\tilde{f}), g^{-1}(\lambda))$ be a hybrid fuzzy bi-ideal of N_1 over U where $\tilde{f_\lambda}$ is a hybrid structure in N_2 over U.

Proof:

Let $x_1, y_1, z_1 \in N_2$ then $g(x_2) = x_1$, $g(y_2) = y_1$, $g(z_2) = y_1$ for some $x_2, y_2, z_2 \in N_1$

Now,$(i) \tilde{f}(x_1 + y_1) = \tilde{f}(g(x_2) + g(y_2)) = \tilde{f}(g(x_2 + y_2)) = g^{-1}(\tilde{f})(x_2 + y_2)$

$\supseteq \cap \{g^{-1}(\tilde{f})(x_2), g^{-1}(\tilde{f})(y_2)$

Therefore $\tilde{f}(x_1 + y_1) \supseteq \cap \{\tilde{f}(x_1), \tilde{f}(y_1)\}$

(ii) $\lambda(x_1 + y_1) = \lambda(g(x_2) + g(y_2)) = \lambda(g(x_2 + y_2)) = g^{-1}(\lambda)(x_2 + y_2)$

$\leq \vee \{g^{-1}(\lambda)(x_2), g^{-1}(\lambda)(y_2)$

Therefore $\lambda(x_1 + y_1) = \vee \{\lambda(x), \lambda(y)\}$

(iii) $\tilde{f}(x_1 y_1 z_1) = \tilde{f}(g(x_2)g(y_2)g(z_2)) = \tilde{f}(g(x_2 y_2 z_2) \quad \supseteq \cap \{\tilde{f}(g(x_2)), \tilde{f}(g(z_2))$

Therefore $\tilde{f}(x_1 y_1 z_1) = \cap \{\tilde{f}(x_1), \tilde{f}(z_1)\}$

(iv) $\lambda(x_1 y_1 z_1) = \lambda(g(x_2)g(y_2)g(z_2)) = \lambda(g(x_2 y_2 z_2)) \leq \vee \{\lambda(g(x_2)), \lambda(g(z_2))$

Therefore $\lambda(x_1 y_1 z_1) \quad \leq \vee \{\lambda(x_1), \lambda(z_1)\}$

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