Research Article

Boundary layer and heat transfer Williamson fluid flow over a stretching sheet with Newtonian heating

B.Shankar Goud[a], Mohammed Fareeduddin[b], and Pudhari Srilatha[c]

[a] Department of Mathematics, JNTUH College of Engineering Hyderabad,
Kukatpally, Telangana-500085, India.
[b] Lecturer, UTAS-Al Musannah Sultanate of Oman.
[c] Department of Mathematics, Institute of Aeronautical Engineering, Hyderabad.

Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 28 April 2021

Abstract: The Aligned magnetic field with Williamson fluid has been analyzed using a stretching sheet with Newtonian heating. The governing partial differential equations are transformed to the nonlinear ordinary differential equation by employing the similarity transformations and then solved by using the MATLAB inbuilt solver bvp4c. The influence of various parameters on dimensionless velocity and temperature was graphically explored. Comparisons of all conditions for a particular situation have been made and a very effective agreement has been reached.

Keywords: Newtonian Heating, magnetic field, Aligned angle, Williamson fluid, bvp4c.

1. Introduction

Fluid dynamics caused by a stretching sheet play an important role in extrusion processes. Recent research has shown that the non-Newtonian fluid has advanced significantly. The specific character of the fluid used in diverse engineering and manufacturing applications such as polymer sheet industrial, glass blowing, paper making, and aerodynamic extrusion of plastic sheets can be used to track this growth. Kumaran and Ramanaiah [1] studied on a note on the flow over a stretching sheet. Elbashbeshy[2] examined the flow and heat transfer of viscous fluids using the exponential stretching sheet. Kumar[3] investigated the impact of MHD boundary layer flow on heat and mass transfer over a stretching sheet with slip. Salleh et al.[4] inspected boundary layer flow and heat transfer over a newtonian heating stretch sheet. S.Nadeem and Hussain[5] address heat transfer effects on the Williamson fluid over an elastic exponentially stretching surface, as well as fluid model boundary layer equations for two-dimensional flow with heat transfer. Hasmawani et al. [6] looked into the impact of thermal radiation on the MHD stagnation point flow of Williamson fluid across a stretching surface. Salahuddin et al.[7] have considered Williamson fluid with slip conditions over an extended cylinder on the mixed convection boundary layer flow. Srinivasulu and Goud[8] studied the effect on a stretching surface with a convective boundary conditions, of aligned magnetic field on Williamson's nanofluid. Arifin et al.[9] investigated aligned magnetic field on a dusty Casson fluid with Newtonian heating through a stretch sheet. Several authors studied in different fields like ref [10-18].

This work was carried out with the aim of investigating fluid flow based on the above literature, non-Newtonian Williamson fluid, with the corresponding Newtonian heating boundary condition and aligned magnetic field.

Mathematical Formulation

A steady 2-D incompressible Williamson fluid over a vertical stretching sheet with x- axis is

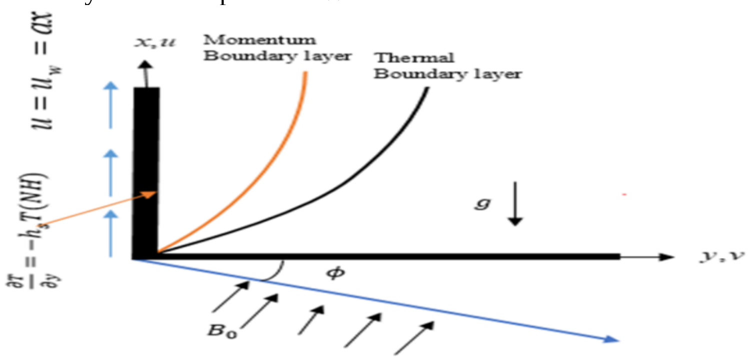

Fig.1. Schematic diagram.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\sqrt{2}\nu\,\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+\sigma u\beta_0^2\sin^2\theta \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial u}{\partial y}=\alpha\frac{\partial^2 u}{\partial y^2} \tag{3}$$

The boundary conditions are

$$\left.\begin{array}{ll} u = u_w(x),\ \ v = 0, \frac{\partial T}{\partial y} = -h_s T & at\ y = 0 \\ u = 0, \quad T \to T_\infty & as\ y \to \infty \end{array}\right\} \tag{4}$$

Here the velocity factors u and v are along the x, y axis directions, respectively. Additionally, ν is the kinematics viscosity, Γ refers to the time constant, β_0 indicates the magnetics field strength, T is the fluid temperature in the boundary layer, and h_s refers to the heat transfer coefficient, α is the thermal diffusivity.

By employing the similarity transformations to the Eqs.(1)-(4) are as follows [4]

$$\eta = \sqrt{\frac{a}{v}}y, \psi = \sqrt{av}xf(\eta), \theta(\eta) = \frac{T-T_\infty}{T_\infty} \tag{5}$$

Where η and $\theta(\eta)$ is the dimensionaless parameter, while ψ is the stream function & with this velocity components can be defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, these components satisfies the Eqn.(1). By solving u and v obtained as $u = axf'(\eta)$, $v = -\sqrt{av}f(\eta)$ (6)

By substituting Eqn.(5) and (6) in Eqn.(2)–(3), the following are obtain:

$$(1 + \lambda f'')f''' + ff'' - (f')^2 - (M sin^2\phi)f' = 0 \tag{7}$$

$$\theta'' + Prf\theta' = 0 \tag{8}$$

The flowing appropriate boundary conditions are

$$\left.\begin{array}{ll} f(\eta) = 0, f'(\eta) = 1,\ \theta'(\eta) = \gamma(1 + \theta(\eta)) & at\ \eta = 0 \\ f'(\eta) \longrightarrow 0, \quad \theta(\eta) \to 0 & as\ \eta \to \infty \end{array}\right\} \tag{9}$$

Where $'$ denote the differentiation with respect to η, $\lambda = x\Gamma\sqrt{\frac{2c^3}{v}}$ is the parameter of the non-Newtonian Williamson fluid, $M = \frac{\sigma\beta_0^2}{a}$ is Magnetic field parameter, and $Pr = \frac{v}{\alpha}$ is Prandtl number, $\gamma = h_s\left(\frac{v}{a}\right)^{1/2}$ is the conjugate factor for Newtonian heating. Noted that $\gamma = 0$ is for the insulated plate and also $\gamma \to \infty$ is the surface temperature does not change i.e, remains constant..

The skin friction coefficient C_f which is provided by physical quantities of interest is $C_f = \frac{\tau_w}{\rho u_w^2}$,

and the Nusselt number is defined as $Nu_x = \frac{xq_w}{k}$.

Where ρ is the density of the fluid density, the surface shear stress τ_w is given by $\tau_w = \mu\frac{\partial u}{\partial y}\left[1 + \Gamma\sqrt{\frac{1}{2}}\frac{\partial u}{\partial y}\right]$ with the dynamic viscosity $\mu = \rho\upsilon$ with the help of the similarity transformation (5) give

$C_f Re_x^{-1/2} = f''(0) + \frac{\lambda}{2}\left(f''(0)\right)^2$, and the Nusselt number coefficient is defined as $Nu_x Re_x^{-1/2} = -\theta'(0)$.

Where $Re_x = \frac{u_w x}{v}$ is the Reynolds number.

2. Solution of the Problem

The MATLAB tool bvp5c is used to implement the list of nonlinear ODE's (8-9) as well as the boundary conditions (10). In order to do this, the set of ODE's are first modified to ODE's of first order. The following substitutes are included

$$g(1) = f(\eta), g(2) = f'(\eta), g(3) = f''(\eta)$$
$$g(4) = \theta(\eta), g(5) = \theta'(\eta)$$

Now the first ODE get the following ways.

$$\begin{pmatrix} g'(1) \\ g'(2) \\ g'(3) \\ \mathcal{F}'(4) \\ \mathcal{F}'(5) \end{pmatrix} = \begin{pmatrix} g(2) \\ g(3) \\ \left(\left(g(2) * g(2)\right) - g(1) * g(3) + (M sin^2\phi) * g(2)\right)/((1 + \lambda) * g(3)) \\ g(5) \\ -Pr * g(1) * g(5) \end{pmatrix}$$

The appropriate initial conditions are

$$\begin{pmatrix} g_a(1) \\ g_a(2) \\ g_a(5) \\ g_b(2) \\ g_b(4) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -\gamma\left(1 + g_a(4)\right) \\ 0 \\ 0 \end{pmatrix}$$

The boundary was stable to 10^{-5}.In this approach, the choice of $\eta_\infty = 3$, confirms that each numerical outcome approach asymptotic assets exactly.

3. Results and Discussion

The observed numerical results are checked using $-\theta'(0)$ reference values. Table 1 shows the relation with [4],[6] and [9] and table 2 with [6]. Both have been numerically resolved using MATLAB inbuilt software's tool bvp4c. The numerical figures were well agreed, so we know the accuracy of the results. From the Table 1 with an increase of Pr the values of $-\theta(0)$ decreases. From the table 2 with an increase of aligned parameter the results in $C_f Re_x^{1/2}$ and $-\theta(0)$ is also increases. The influence on temperature profiles for different values of Prandtl is shown in Figure 2. When Pr values rise and reduce the temperature distribution due to a high Pr-value, extremely viscous fluid is present with low thermal conductivity. Physically, the Prandtl number is called the association between the momentum and thermal diffusivities, the higher values of Pr have low conduction and the thermal conductivity of lower Pr values. Figure 3 depicts the temperature curves for many values of the conjugate parameter γ. As γ increases, so does the thickness of the boundary sheet. Furthermore, the thermal transfer coefficients as well as the wall temperature often increase due to boundary conditions (9). Figure 4 shows the velocity curves for various magnetic field parameter parameters M values. The rise in M effects is observed to decrease the velocity profile. Physically, the rise in M decreased the boundary layer width, which meant that the velocity gradient was increasingly magnitude, thereby enhancing the declines the skin friction factor.

For changed values aligned angle parameter(ϕ) on the velocity profile is depicted in figure 5 presents the velocity profile. The relevant results are the effects of alignment which may vary from 0 to 90 and where the magnetic influence is considered absent by $\phi = 0$. The rising angle has been shown to decrease the velocity gradient. The increased physical density of ϕ improves magnetic field strength and thus produces identical results to Figure 4. Figure 6 show that the deviation of the Nusselt number$\left(Nu_x Re_x^{-1/2}\right)$ with λ for changed values of ϕ when the remaining values are fixed. The results found that from figure Nusselt number enhances with the increase the values of ϕ, while Nusselt number enhances with increase in λ for constant values of ϕ. Similar results achieved in figure 7, Nusselt number increases with enhances values of M, while Nusselt number enhance with rise in λ for fixed values of M

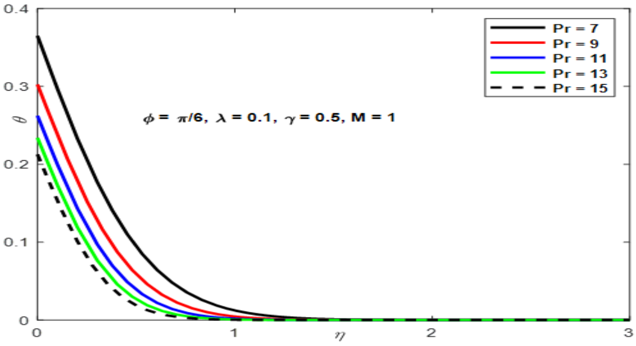

Fig.2: Temperature v/s Pr.

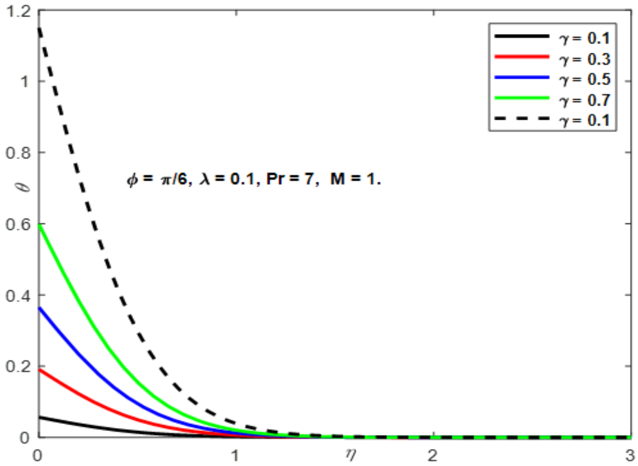

Fig.3: Temperature v/s γ.

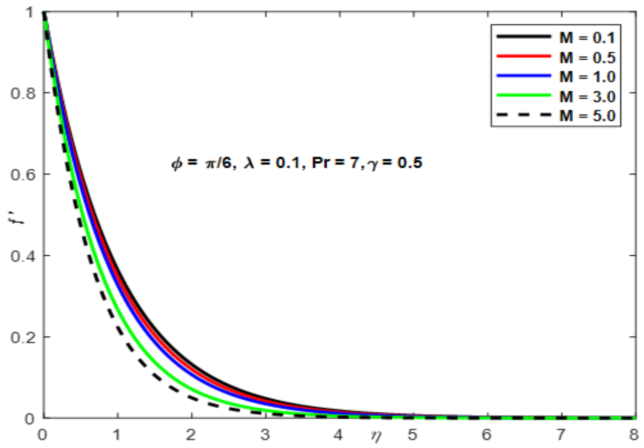

Fig.4: Velocity v/s M.

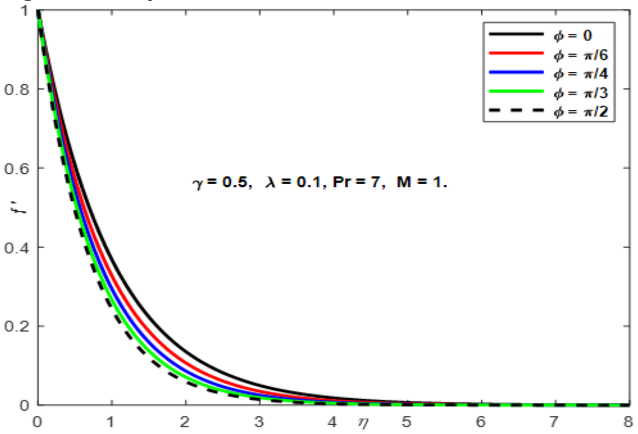

Fig.5: Velocity v/s ϕ.

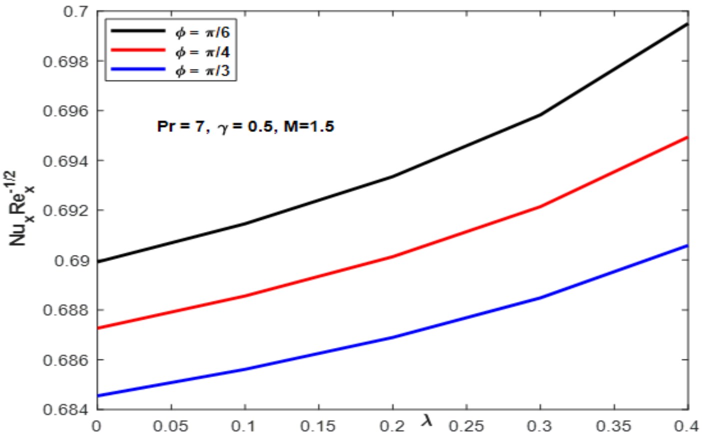

Fig.6: Variations of Nu v/s λ with ϕ.

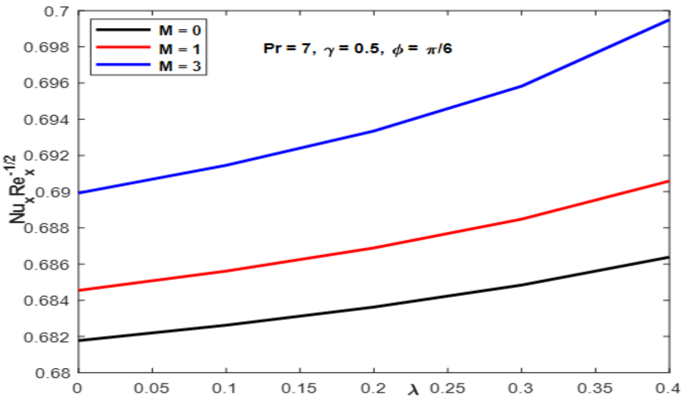

Fig.7: Variations of Nu v/s λ with M.

Table 1: The comparison between the previous and existing results of $\theta(0) and\ \theta'(0)$ for changed values of Pr when $\phi = \lambda = M = 0$ and $\gamma = 1$.

Pr	Salleh et al.[4]	Arifin et l.[9]	Hasmawani et al.[6]	Present study	Salleh et al.[4]	Present study
	$-\theta(0)$	$-\theta(0)$	$-\theta(0)$	$-\theta(0)$	$-\theta'(0)$	$-\theta'(0)$
3	6.0258	6.0513	6.05159	6.051715	7.02577	7.051715
5	1.7659	1.7604	1.76039	1.760392	2.76594	2.760392
7	1.1351	1.1168	1.11681	1.116814	2.13511	2.116814
10	0.7653	0.7645	0.76452	0.764524	1.76531	1.764524
100	0.1612	0.1478	0.14782	0.147801	1.16115	1.147801

Table 2: The comparison between the previous and existing results of $C_f Re_x^{1/2}$ and $-\theta(0)$ for different values of λ when $\mathrm{Pr} = 7, \phi = \frac{\pi}{2}, \gamma = 1\ and\ M = 1.5$.

	Hasmawani[6]		Present Study	
λ	$C_f Re_x^{1/2}$	$-\theta(0)$	$C_f Re_x^{1/2}$	$-\theta(0)$
0.1	-1.14957	2.17873	-1.147283	2.177361
0.15	-1.13578	2.18523	-1.133595	2.183812
0.2	-1.12127	2.19243	-1.119184	2.190956
0.25	-1.10589	2.20049	-1.103918	2.198952
0.3	-1.08947	2.20965	-1.087610	2.208029

4. Conclusion

The current research investigated the aligned magnetic field with Newtonian heating on a stretched sheet of Williamson fluid. Skin friction values as well as velocity and temperature curves, influenced by non-Newtonian Williamson fluid parameter(γ), Magnetic parameter(M), Prandtl number(Pr), Aligned angle parameter(λ), numerically analyzed. The following are significant observations.

- Increasing the Williamson fluid parameter has led to a negative value $C_f Re_x^{1/2}$ because of the opposite direction of fluid movement with the stretching plate. Note that the Fluid Parameter Williamson gives no impact on the distribution of temperature and velocity.
- Increased Pr values lead to a reduction in the temperature profile. Increased the thickness of the boundary laying in γ by taking the conjugate parameter into account.
- Increasing M values results in to decline in the velocity profile while increasing ϕ causes a reduction in the velocity gradient for the aligned angle parameter.

References

1. Kumaran, V. and Ramanaiah, G. A note on the flow over a stretching sheet. Acta Mechanica 116(1-4), 229-233 (1996).
2. Elbashbeshy, E. M. A. Heat transfers over an exponentially stretching continuous surface with suction. Archives of Mechanics, 53(6),643-651(2001).
3. R. Kumar, "MHD boundary layer flow on heat and mass transfer over a stretching sheet with slip effect," Journal of Naval Architecture and Marine Engineering,10(2),109-118, 2013.
4. M.Z. Salleh, R. Nazar, I. Pop "Boundary layer flow and heat transfer over a stretching sheet with Newtonian heating", Journal of the Taiwan Institute of Chemical Engineers 41,651-655(2010).
5. S. Nadeem,S. T. Hussain "Heat transfer analysis of Williamson fluid over exponentially stretching surface Appl.Math.Mech.-Engl. Ed., 35(4), pp. 489-502, 2014.
6. Hasmawani Hashim, Muhammad Khairul Anuar Mohamed, Nazila Ishak, Norhafizah Md Sarif, Mohd Zuki Salleh "Thermal radiation effect on MHD Stagnation Point flow of Williamson fluid over a stretching Surface" Journal of Physics: Conference Series 1366,1-12(2019) 012011.
7. Salahuddin, T., Malik, M. Y., Hussain, A., Awais, M., & Bilal, S.Mixed Convection Boundary layer flow of Williamson fluid with slip conditions over a stretching cylinder by using Keller-Box method. International Journal of Nonlinear Sciences and Numerical Simulation, 18(1),9-17, 2017. doi:10.1515/ijnsns-2015-0090.

8. Thadakamalla Srinivasulu and B Shankar Goud "Effect of inclined magnetic field on flow, heat and mass transfer of Williamson nanofluid over a stretching sheet", Case Studies in Thermal Engineering 23,100819, 1-13, (2021).
9. Arifin, N. S., Zokri, S. M., Kasim, A. R. M., Salleh, M. Z., Yusoff, W. N. S. W., Mohammad, N. F. and Shafie, S. "Aligned magnetic field on dusty Casson fluid over a stretching sheet with Newtonian heating". Malaysian Journal of Fundamental and Applied Sciences, 13(3), pp 245-248(2017).
10. B. Shankar Goud "Heat Generation/Absorption influence on steady stretched permeable surface on MHD flow of a micropolar fluid through a porous medium in the presence of variable suction/injection", International Journal of Thermofluids, vol.7-8 (2020) 100044. https://doi.org/10.1016/j.ijft.2020.100044.
11. B. Shankar Goud "Thermal radiation influences on MHD stagnation point stream over a stretching sheet with slip boundary conditions", International Journal of Thermofluid Science and Technology (2020), 7(2), Paper No.070201.
12. B. Shankar Goud ,Dharmendar Reddy Yanala "Heat source fffect on MHD fluid flow over a moving vertical plate in the presence of chemical reaction with convective surface boundary conditions" Journal of Engineering, Computing and Architecture, 10(1), Page No: 38-46(2020).
13. Thadakamalla Srinivasulu, B.ShankarGoud " Effect of inclined magnetic field on flow, heat and mass transfer of Williamson nanofluid over a stretching sheet", Case Studies in Thermal Engineering,23, 2021,100819 , https://doi.org/10.1016/j.csite.2020.100819.
14. B.ShankarGoud, P Bindu, Pudhari Srilatha,Y. Hari Krishna " The Joule heating effect on MHD natural convective fluid flow in a permeable medium over a semi-infinite inclined vertical plate in the presence of the chemical reaction", IOP Conf. Series: Materials Science and Engineering 993 (2020) 012111,doi:10.1088/1757-899X/993/1/012111.
15. B. Shankar Goud, Y. Dharmendar Reddy, V. Srinivasa Rao, Zafar Hayat Khan "Thermal radiation and joule heating effects on a magnetohydrodynamic Casson nanofluid flow in the presence of chemical reaction through a non-linear inclined porous stretching sheet", Journal of Naval Architecture and Marine Engineering, 7(2),143-164(2020). http://dx.doi.org/10.3329/jname.v17i2.49978.
16. M. Anil Kumar, Y. Dharmendar Reddy, V. Srinivasa Rao, B. Shankar Goud " Thermal radiation impact on MHD heat transfer natural convective nano fluid flow over an impulsively started vertical plate", Case Studies in Thermal Engineering 24(2021) 100826, https://doi.org/10.1016/j.csite.2020.100826
17. B. Shankar Goud, Pudhari Srilatha, MN Raja Shekar "Effects of mass suction on MHD boundary layer flow and heat transfer over a porous shrinking sheet with heat source/sink" Int.Journal of Innovative Technology and Exp. Engineering, 8(10),263-266(2019).
18. B. Shankar Goud "Numerical solution of an unsteady flow past on moving vertical plate with variable temperature and heat source in the presence of inclined magnetic field and viscous dissipation through porous media". International Journal of Research, Vol.7, Issue X, 2018, 1345-1351(2018). doi:16.10089.ijr.2018.v7i10.285311.003441.