An Analysis of Reinfection Pneumonia Model with Carrier State

Naga soundarya lakshmi V.S.V[a], and A.Sabarmathi[b]
[a] Research Scholar, Department of Mathematics, Auxilium College, Vellore
[b] Assistant professor, Department of Mathematics, Auxilium College, Vellore

Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 28 April 2021

Abstract: A reinfection model with Carrier state for pneumonia was formulated. The boundedness and positiveness of the state variables were verified. The local and global stability of the model was established. By the equilibrium analysis the optimal values of Susceptible, Infectious, Carrier and Recovery were found. Through the numerical simulations, the flow of S,I,C,R and the flow of variables for different set of parameters were studied.

Keywords: Pneumonia, reinfection model, Carrier state, Stability..

1. Introduction

Mathematicians use different models to analyse the spread of infectious disease. The SICR model was developed from the basic SIR model with a carrier state. In the carrier state, the infectious person can spread the disease to others without any symptoms. The reinfection model defines as the recovery individuals can be affected by the infection again. So the transition of disease passes from the recovery state to the susceptible state.

Pneumonia is one of the respiratory disease which leads to the limitation of the oxygen and cause the breathing difficulty. Pneumonia was being the biggest killer disease among the acute respiratory infection in 2018, on the report of National Health Profile (NHP) India. According to UNICEF, in 2018 India is in the second rank in the deaths of children under the age of five due to pneumonia.

Talawar [8] has analysed the stability of SIS epidemic model with vaccination. Li-Ming Cai [5] has studied the malaria model with partial immunity to reinfection. Wang [3] has developed an SIS epidemic model with saturated and incidence rate. Cyrus G.Ngari [1] has formulated the SI model with the class treatment among children for Pneumonia. Fulgensia Kamugisha Mbabazi [2] has investigated the SVECI model with carrier and vaccination states for pneumonia. Victor Okhuese [9] has established an reinfection endemic model SEIRUS for covid-19. In this paper we formulated the reinfection model with carrier state for pneumonia and analysed the model with the different values of parameters as in [4, 6, 7].

2. Formulation of the Model

The SICR reinfection model of Pneumonia is represented by the following system of four ordinary differential equations

$$\begin{aligned}
\frac{dS}{dt} &= \omega\, N(t) + \alpha\, R(t) - (\beta + \mu) S(t) \\
\frac{dI}{dt} &= (1-\sigma)\,\beta\, S(t) - (\gamma + \mu + d) I(t) \\
\frac{dC}{dt} &= \sigma\,\beta\, S(t) - (\delta + \mu)\, C(t) \\
\frac{dR}{dt} &= \gamma\, I(t) + \delta\, C(t) - (\alpha + \mu)\, R(t)
\end{aligned} \tag{1}$$

With the initial conditions $S(t), I(t), C(t), R(t) \geq 0$. Also $\sigma, \beta, \gamma, \mu, d, \omega, \alpha, \delta > 0$,

$$N(t)=S(t)+I(t)+C(t)+R(t) \tag{2}$$

The following figure shows the SICR reinfection model for pneumonia.

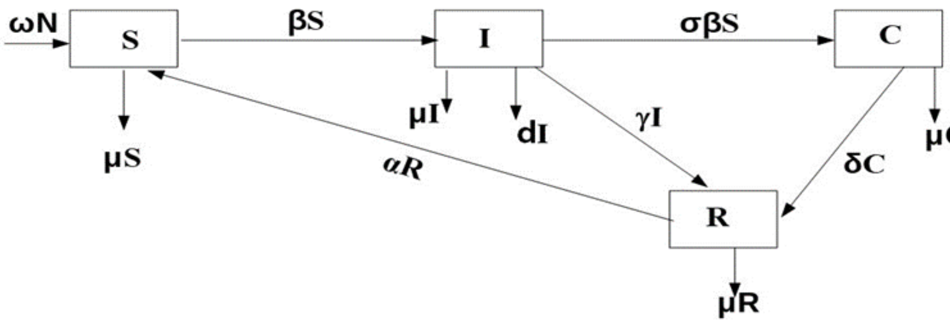

Figure 1. Reinfection model for pneumonia

Where S(t),I(t),C(t),R(t) are the Susceptible, Infectious, Carrier and Recovery state respectively and ω - Average birth rate, μ - Average death rate, β - Infectious rate, σ - Factor of new infection, γ - Recovery rate , α - Reinfectious rate, δ - Immunity rate, d - disease induced death rate, N(t) – Total Population.

3. Boundedness and Positiveness

First we prove the Positiveness of the variables,

From (1),

$$\frac{dS}{dt} \geq -(\beta+\mu)\, S \tag{3}$$

$$\frac{dI}{dt} \geq -(\gamma+\mu+d)\, I \tag{4}$$

$$\frac{dC}{dt} \geq -(\delta+\mu)\, C \tag{5}$$

$$\frac{dR}{dt} \geq -(\alpha+\mu)\, R \tag{6}$$

From (3),

$$\frac{1}{S}\, dS \geq -(\beta+\mu)\, dt$$

Integrating on both the sides with respect to t,

$$\int_0^t \frac{1}{S}\, dS \geq -(\beta+\mu)\int_0^t dt$$

$$S(t) \geq S(0)e^{-(\beta+\mu)t}$$

$$S(t) \geq 0, as\ S(0) \geq 0$$

From (4),

$$\frac{1}{I}\, dI \geq -(\gamma+\mu+d)\, dt$$

Integrating on both the sides with respect to t,

$$\int_0^t \frac{1}{I}\, dI \geq -(\gamma+\mu+d)\int_0^t dt$$

$$I(t) \geq I(0)e^{-(\gamma+\mu+d)t}$$

$$I(t) \geq 0, as\ I(0) \geq 0$$

From (5),

$$\frac{1}{C}\, dC \geq -(\delta+\mu)\, dt$$

Integrating on both the sides with respect to t,

$$\int_0^t \frac{1}{C}\, dC \geq -(\delta+\mu)\int_0^t dt$$

$$C(t) \geq C(0)e^{-(\delta+\mu)t}$$

$$C(t) \geq 0, as\ C(0) \geq 0$$

From (6),

$$\frac{1}{R}\,dR \;\geq\; -(\alpha+\mu)\,dt$$

Integrating on both the sides with respect to t,

$$\int_0^t \frac{1}{R}\,dR \;\geq\; -(\alpha+\mu)\int_0^t dt$$

$$R(t) \geq R(0)e^{-(\alpha+\mu)t}$$

$$R(t) \geq 0, as\ R(0) \geq 0$$

Hence we proved the positiveness of the state variables S, I, C and R.
Let us proceed with the boundedness of the variables,
From (2),

$$N = S + I + C + R$$

Differentiating on both the sides with respect to t,

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dC}{dt} + \frac{dR}{dt}$$

$$\frac{dN}{dt} \leq \omega N - \mu N$$

$$\frac{1}{N}\,dN \;\leq (\omega-\mu)\,dt$$

Integrating on both the sides with respect to t,

$$\int_0^t \frac{1}{N}\,dN \;\leq (\omega-\mu)\int_0^t dt$$

$$N(t) \;\leq N(0)e^{(\omega-\mu)t}$$

Hence, $N(t)$ is bounded by a positive integer.
As S, I, C, R are positive and $N = S + I + C + R$, we conclude that $S, I.C.R$ are bounded.

4. Equilibrium Analysis

The steady states are $G_0(0,0,0,0)$, $G_1(\bar{S},\bar{I},0,0)$, $G_2(\bar{S},0,\bar{C},0)$, $G_3(\bar{S},0,0,\bar{R})$, $G_4(S',I',C',0)$ and $G_5(S^*,I^*,C^*,R^*)$

Case 1: Trivial steady state $G_0(0,0,0,0)$ exists always

Case 2: (i) For $G_1(\bar{S},\bar{I},0,0)$

Let $\bar{S},\bar{I}$ be the positive solutions of $\frac{dS}{dt} = 0\ and\ \frac{dI}{dt} = 0$

From (1),

$$\bar{S} = \frac{\omega\, N}{(\beta+\mu)} \tag{7}$$

From (1),

$$(\gamma+\mu+d)\bar{I} = (1-\sigma)\,\beta\bar{S}$$

Using (7), we have

$$\bar{I} = \frac{(1-\sigma)\beta\omega N}{(\beta+\mu)(\gamma+\mu+d)}$$

$$\therefore G_1(\bar{S},\bar{I},0,0) = G_1\left(\frac{\omega N}{(\beta+\mu)}, \frac{(1-\sigma)\beta\omega N}{(\beta+\mu)(\gamma+\mu+d)}, 0,0\right)$$

(ii) For $G_2(\bar{S},0,\bar{C},0)$

Let $\bar{S},\bar{C}$ be the positive solutions of $\frac{dS}{dt} = 0\ and\ \frac{dC}{dt} = 0$

From (1),

$$\bar{S} = \frac{\omega\, N}{(\beta+\mu)} \tag{8}$$

From (1),

$$\bar{C} = \frac{\sigma\beta\bar{S}}{(\mu+\delta)}$$

Using (8), we have

$$\bar{C} = \frac{\sigma\beta\omega N}{(\beta+\mu)(\mu+\delta)}$$

$\therefore G_2(\bar{S},0,\bar{C},0) = G_2(\frac{\omega N}{(\beta+\mu)},0,\frac{\sigma\beta\omega N}{(\beta+\mu)(\mu+\delta)},0)$

(iii) For $G_3(\bar{S},0,0,\bar{R})$

Let $\bar{S},\bar{R}$ be the positive solutions of $\frac{dS}{dt}=0$ *and* $\frac{dR}{dt}=0$

From (1),

$$\bar{R}=0 \quad (9)$$

From (1),

$$\frac{dS}{dt}=0 \Rightarrow \omega N+\alpha\bar{R}-(\beta+\mu)\bar{S}=0$$

Using (9), we have

$$\bar{S}=\frac{\omega N}{(\beta+\mu)}$$

$\therefore G_3(\bar{S},0,0,\bar{R}) = G_3(\frac{\omega N}{(\beta+\mu)},0,0,0)$

Case 3: For $G_4(S',I',C',0)$

Let $\bar{S},\bar{I},\bar{C}$ be the positive solutions of $\frac{dS}{dt}=0,\frac{dI}{dt}=0$ *and* $\frac{dC}{dt}=0$

From (1),

From (1),

$$\bar{S}=\frac{\omega N}{(\beta+\mu)} \quad (10)$$

$$I'=\frac{(1-\sigma)\beta S'}{(\gamma+\mu+d)}$$

Using (10), we have

$$I'=\frac{(1-\sigma)\beta\omega N}{(\beta+\mu)(\gamma+\mu+d)}$$

From (1),

$$C'=\frac{\sigma\beta S'}{(\delta+\mu)}$$

Using (10), we have

$$C'=\frac{\sigma\beta\omega N}{(\delta+\mu)(\beta+\mu)} \quad (11)$$

$\therefore G_4(S',I',C',0) = G_4(\frac{\omega N}{(\beta+\mu)},\frac{(1-\sigma)\beta\omega N}{(\beta+\mu)(\gamma+\mu+d)},\frac{\sigma\beta\omega N}{(\delta+\mu)(\beta+\mu)},0)$

Case 4:For $G_5(S^*,I^*,C^*,R^*)$

Let S^*,I^*,C^*,R^* be the positive solutions of $\frac{dS}{dt}=0,\frac{dI}{dt}=0,\frac{dC}{dt}=0$ *and* $\frac{dR}{dt}=0$

From (1),

$$R^*=\frac{(\beta+\mu)S^*-\omega N}{\alpha} \quad (12)$$

$$I^*=\frac{(1-\sigma)\beta S^*}{(\gamma+\mu+d)} \quad (13)$$

$$\gamma I^*+\delta C^*-(\alpha+\mu)R^*=0$$

$$C^*=\frac{\sigma\beta S^*}{(\delta+\mu)} \quad (14)$$

Using (12), (13), (14) we have

$$S^*=\omega N\left[\frac{1}{(\beta+\mu)}-\frac{(\alpha+\mu)(\gamma+\mu+d)}{(1-\sigma)\gamma\beta\alpha}-\frac{(\alpha+\mu)(\mu+\delta)}{\alpha\sigma\beta\delta}\right] \quad (15)$$

Using (15) in (13)

$$I^*=\omega N\left[\frac{(1-\sigma)\beta}{(\gamma+\mu+d)(\beta+\mu)}-\frac{(\alpha+\mu)}{\gamma\alpha}-\frac{(\alpha+\mu)(\mu+\delta)(1-\sigma)}{\alpha\sigma\delta(\gamma+\mu+d)}\right]$$

Using (15) in (14)

$$C^* = \sigma\beta\omega N\left[\frac{1}{(\delta+\mu)(\beta+\mu)} - \frac{(\alpha+\mu)(\gamma+\mu+d)}{(1-\sigma)(\delta+\mu)\gamma\beta\alpha} - \frac{(\alpha+\mu)}{\alpha\sigma\beta\delta}\right]$$

Using (15) in (12)

$$R^* = -\frac{\omega N(\alpha+\mu)(\beta+\mu)}{\alpha^2\beta}\left[\frac{(\gamma+\mu+d)}{(1-\sigma)\gamma} + \frac{(\mu+\delta)}{\sigma\delta}\right]$$

Hence, the endemic equilibrium is

$$G_5(S^*, I^*, C^*, R^*) = (\omega N\left[\frac{1}{(\beta+\mu)} - \frac{(\alpha+\mu)(\gamma+\mu+d)}{(1-\sigma)\gamma\beta\alpha} - \frac{(\alpha+\mu)(\mu+\delta)}{\alpha\sigma\beta\delta}\right],$$
$$\omega N\left[\frac{(1-\sigma)\beta}{(\gamma+\mu+d)(\beta+\mu)} - \frac{(\alpha+\mu)}{\gamma\alpha} - \frac{(\alpha+\mu)(\mu+\delta)(1-\sigma)}{\alpha\sigma\delta(\gamma+\mu+d)}\right], \sigma\beta\omega N\left[\frac{1}{(\delta+\mu)(\beta+\mu)} - \frac{(\alpha+\mu)(\gamma+\mu+d)}{(1-\sigma)(\delta+\mu)\gamma\beta\alpha} - \frac{(\alpha+\mu)}{\alpha\sigma\beta\delta}\right],$$
$$-\frac{\omega N(\alpha+\mu)(\beta+\mu)}{\alpha^2\beta}\left[\frac{(\gamma+\mu+d)}{(1-\sigma)\gamma} + \frac{(\mu+\delta)}{\sigma\delta}\right])$$

5. Local Stability

By the Routh-Hurwitz criteria, we find the local stability of (1).
The Jacobian matrix for the system (1) is

$$\begin{pmatrix} -(\beta+\mu) & 0 & 0 & \alpha \\ (1-\sigma)\beta & -(\gamma+\mu+d) & 0 & 0 \\ \sigma\beta & 0 & -(\delta+\mu) & 0 \\ 0 & \gamma & \delta & -(\alpha+\mu) \end{pmatrix} \qquad (16)$$

When $\frac{dS}{dt} = 0$,

$$-(\beta+\mu) = -\frac{\omega N + \alpha R}{S} \qquad (17)$$

When $\frac{dI}{dt} = 0$,

$$-(\gamma+\mu+d) = -\frac{(1-\sigma)\beta S}{I} \qquad (18)$$

When $\frac{dC}{dt} = 0$

$$-(\delta+\mu) = -\frac{\sigma\beta S}{C} \qquad (19)$$

When $\frac{dR}{dt} = 0$

$$-(\alpha+\mu) = -\frac{\gamma I + \delta C}{R} \qquad (20)$$

At the interior equlilibrium (16) becomes

$$\begin{pmatrix} -\frac{(\omega N+\alpha R)}{S} & 0 & 0 & \alpha \\ (1-\sigma)\beta & \frac{-(1-\sigma)\beta S}{I} & 0 & 0 \\ \sigma\beta & 0 & \frac{-(\sigma\beta S)}{C} & 0 \\ 0 & \gamma & \delta & \frac{-(\gamma I+\delta C)}{R} \end{pmatrix} \qquad (21)$$

The characteristic equation of (21) is given by

$$\begin{vmatrix} -\frac{(\omega N+\alpha R)}{S}-\lambda & 0 & 0 & \alpha \\ (1-\sigma)\beta & \frac{-(1-\sigma)\beta S}{I}-\lambda & 0 & 0 \\ \sigma\beta & 0 & \frac{-(\sigma\beta S)}{C}-\lambda & 0 \\ 0 & \gamma & \delta & \frac{-(\gamma I+\delta C)}{R}-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^4 + \left(\frac{\omega N}{S} + \frac{(1-\sigma)\beta S}{I} + \frac{\gamma I}{R} + \frac{\delta C}{R} + \frac{\sigma\beta S}{C}\right)\lambda^3 + \left(\frac{(1-\sigma)\beta\omega N}{I} + \frac{\gamma I\omega N}{SR} + \frac{\delta C\omega N}{SR} + \frac{\sigma\beta\omega N}{C} + \frac{(1-\sigma)\beta\gamma S}{R} + \frac{(1-\sigma)\delta\beta SC}{IR} + \frac{(1-\sigma)\sigma\beta^2 S^2}{IC} + \frac{\sigma\beta\gamma SI}{CR} + \frac{\sigma\beta\delta S}{R}\right)\lambda^2 + \left((1-\sigma)\beta\gamma\omega N + \frac{(1-\sigma)\delta\beta C\omega N}{IR} + \frac{(1-\sigma)\sigma\beta^2 S\omega N}{IC} + \frac{\sigma\beta\gamma I\omega N}{CR} + \frac{\sigma\beta\delta\omega N}{R} + \frac{(1-\sigma)\sigma\beta^2 S^2\gamma}{CR} + \frac{(1-\sigma)\sigma\beta^2 S^2\delta}{IR} - (1-\sigma)\beta\alpha\gamma + \alpha\delta\sigma\beta\right)\lambda + \frac{(1-\sigma)\sigma\beta^2\gamma S\omega N}{CR} + \frac{(1-\sigma)\sigma\beta^2 S\delta\omega N}{IR} + \frac{(1-\sigma)\alpha\sigma\beta^2\gamma S}{C} + \frac{(1-\sigma)\sigma\beta^2\delta\alpha S}{I} = 0 \quad (22)$$

Comparing (22) with $S^4 + AS^3 + BS^2 + CS + D = 0$,

Where $A = \frac{\omega N}{S} + \frac{(1-\sigma)\beta S}{I} + \frac{\gamma I}{R} + \frac{\delta C}{R} + \frac{\sigma\beta S}{C}$

$$B = \frac{(1-\sigma)\beta\omega N}{I} + \frac{\gamma I\omega N}{SR} + \frac{\delta C\omega N}{SR} + \frac{\sigma\beta\omega N}{C} + \frac{(1-\sigma)\beta\gamma S}{R} + \frac{(1-\sigma)\delta\beta SC}{IR} + \frac{(1-\sigma)\sigma\beta^2 S^2}{IC} + \frac{\sigma\beta\gamma SI}{CR} + \frac{\sigma\beta\delta S}{R}$$

$$C = (1-\sigma)\beta\gamma\omega N + \frac{(1-\sigma)\delta\beta C\omega N}{IR} + \frac{(1-\sigma)\sigma\beta^2 S\omega N}{IC} + \frac{\sigma\beta\gamma I\omega N}{CR} + \frac{\sigma\beta\delta\omega N}{R} + \frac{(1-\sigma)\sigma\beta^2 S^2\gamma}{CR} + \frac{(1-\sigma)\sigma\beta^2 S^2\delta}{IR} - (1-\sigma)\beta\alpha\gamma + \alpha\delta\sigma\beta$$

$$D = \frac{(1-\sigma)\sigma\beta^2\gamma S\omega N}{CR} + \frac{(1-\sigma)\sigma\beta^2 S\delta\omega N}{IR} + \frac{(1-\sigma)\alpha\sigma\beta^2\gamma S}{C} + \frac{(1-\sigma)\sigma\beta^2\delta\alpha S}{I}$$

By the Routh-Hurwitz criteria, the system is locally stable for $S^4 + AS^3 + BS^2 + CS + D = 0$ if and only if $A > 0, D > 0, AB - C > 0, C(AB - C) - A^2D > 0$

Here $A > 0; D > 0; AB - C > 0;\ C(AB - C) - A^2D > 0$ as $(1-\sigma)$ is positive.

Hence the system (1) is locally stable.

6. Global Stability

To find the global stability at (S^*, I^*, C^*, R^*), we construct the following Lyapunov function.

$$V(S,I,C,R) = \left[(S-S^*) - S^*\ln\frac{S}{S^*}\right] + l_1\left[(I-I^*) - I^*\ln\frac{I}{I^*}\right] + l_2\left[(C-C^*) - C^*\ln\frac{C}{C^*}\right] + l_3\left[(R-R^*) - R^*\ln\frac{R}{R^*}\right] \quad (23)$$

Differentiate (23) with respect to t,

$$\frac{dV}{dt} = \left(\frac{S-S^*}{S}\right)\frac{dS}{dt} + l_1\left(\frac{I-I^*}{I}\right)\frac{dI}{dt} + l_2\left(\frac{C-C^*}{C}\right)\frac{dC}{dt} + l_3\left(\frac{R-R^*}{R}\right)\frac{dR}{dt}$$

Using the model equations (1),

$$\frac{dV}{dt} = \left(\frac{S-S^*}{S}\right)[\omega N + \alpha R - (\beta+\mu)S] + l_1\left(\frac{I-I^*}{I}\right)[(1-\sigma)\beta S - (\gamma+\mu+d)I] + l_2\left(\frac{C-C^*}{C}\right)[\sigma\beta S - (\delta+\mu)C] + l_3\left(\frac{R-R^*}{R}\right)[\gamma I + \delta C - (\alpha+\mu)R]$$

$$= (S-S^*)\left[\frac{\omega N + \alpha R}{S} - (\beta+\mu)\right] + l_1(I-I^*)\left[\frac{(1-\sigma)\beta S}{I} - (\gamma+\mu+d)\right]$$

$$+l_2(C-C^*)\left[\frac{\sigma\beta S}{C}-(\delta+\mu)\right]+l_3(R-R^*)\left[\frac{\gamma I+\delta C}{R}-(\alpha+\mu)\right]$$

At (S^*, I^*, C^*, R^*), we have

$$\frac{dV}{dt}=(S-S^*)\left[\frac{\omega N+\alpha R}{S}-\left(\frac{\omega N+\alpha R^*}{S^*}\right)\right]+l_1(I-I^*)\left[\frac{(1-\sigma)\beta S}{I}-\frac{(1-\sigma)\beta S^*}{I^*}\right]$$

$$+l_2(C-C^*)\left[\frac{\sigma\beta S}{C}-\frac{\sigma\beta S^*}{C^*}\right]+l_3(R-R^*)\left[\frac{\gamma I+\delta C}{R}-\frac{\gamma I^*+\delta C^*}{R^*}\right]$$

Choosing $l_1=\frac{1}{(1-\sigma)\beta}$, $l_2=\frac{1}{\sigma\beta}$, $l_3=\frac{1}{\gamma\delta}$

$$=(S-S^*)\omega N\left[\frac{1}{S}-\frac{1}{S^*}\right]+(S-S^*)\,\alpha\left[\frac{R}{S}-\frac{R^*}{S^*}\right]+(I-I^*)\frac{(1-\sigma)\beta}{(1-\sigma)\beta}\left[\frac{S}{I}-\frac{S^*}{I^*}\right]+(C-C^*)\frac{\sigma\beta}{\sigma\beta}\left[\frac{S}{C}-\frac{S^*}{C^*}\right]$$
$$+\frac{(R-R^*)\gamma}{\gamma\delta}\left[\frac{I}{R}-\frac{I^*}{R^*}\right]+\frac{(R-R^*)\delta}{\gamma\delta}\left[\frac{C}{R}-\frac{C^*}{R^*}\right]$$

$$=(S-S^*)\omega N\left[\frac{S^*-S}{SS^*}\right]+(S-S^*)\,\alpha\left[\frac{RS^*-SR^*}{SS^*}\right]+(I-I^*)\left[\frac{SI^*-IS^*}{II^*}\right]$$

$$+(C-C^*)\left[\frac{SC^*-CS^*}{CC^*}\right]+(R-R^*)\left[\frac{IR^*-RI^*}{RR^*}\right]+(R-R^*)\left[\frac{CR^*-RC^*}{RR^*}\right]$$

$$=-\frac{\omega N(S-S^*)^2}{SS^*}+\frac{\alpha}{SS^*}(RSS^*-S^2R^*-RS^{*2}+SS^*R^*)+\frac{1}{II^*}(SII^*-S^*I^2-SI^{*2}+S^*II^*)$$
$$+\frac{1}{CC^*}(SCC^*-S^*C^2+SC^{*2}+S^*CC^*)+\frac{1}{\delta RR^*}(IRR^*-I^*R^2-IR^{*2}+I^*RR^*)$$
$$+\frac{1}{\gamma RR^*}(CRR^*-C^*R^2+CR^{*2}+C^*RR^*)$$

$$=-\frac{\omega N(S-S^*)^2}{SS^*}+\alpha\left(R-\left(\frac{RS^*}{S}+\frac{R^*S}{S^*}\right)+R^*\right)+\left(S-\frac{S^*I}{I^*}-\frac{SI^*}{I}+S^*\right)+\left(S-\frac{S^*C}{C^*}-\frac{SC^*}{C}+S^*\right)$$

$$+\frac{1}{\delta}\left(I-\frac{I^*R}{R^*}-\frac{IR^*}{R}+I^*\right)+\frac{1}{\gamma}\left(C-\frac{C^*R}{R^*}-\frac{CR^*}{R}+C^*\right)$$

$$=-\frac{\omega N(S-S^*)^2}{SS^*}+\alpha\left((R+R^*)-\left(\frac{RS^*}{S}+\frac{R^*S}{S^*}\right)\right)+(S+S^*)-\left(\frac{SI^*}{I}+\frac{S^*I}{I^*}\right)$$

$$+(S+S^*)-\left(\frac{S^*C}{C^*}+\frac{SC^*}{C}\right)+\frac{1}{\delta}\left((I+I^*)-\left(\frac{I^*R}{R^*}+\frac{IR^*}{R}\right)\right)+\frac{1}{\gamma}\left((C+C^*)-(\frac{C^*R}{R^*}+\frac{CR^*}{R})\right)$$

Hence, $\frac{dV}{dt}<0$, as all the terms in R.H.S. are negative.

From, Lyapunov theorem the system (1) is globally asymptotically stable.

7. Numerical Analysis

For numerical simulations we consider the values of the parameters as:

$$\sigma=0.338, \beta=0.16, \gamma=0.0238, \mu=0.0199, d=0.33, \omega=0.0488, \alpha=0.2, \delta=0.025$$

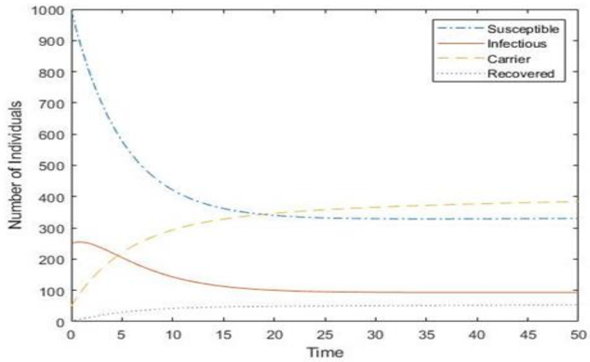

Figure 2. Flow of variables with respect to time t

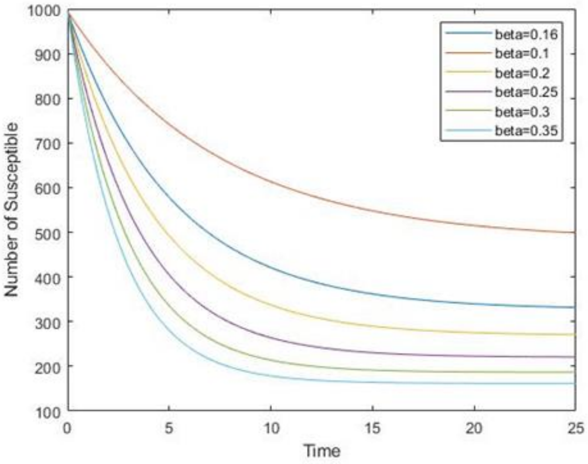

Figure 3. Susceptible class for different values of β

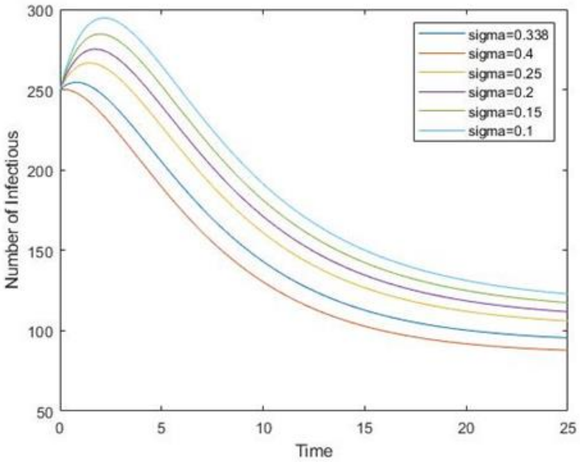

Figure 4. Infectious class for different values of σ

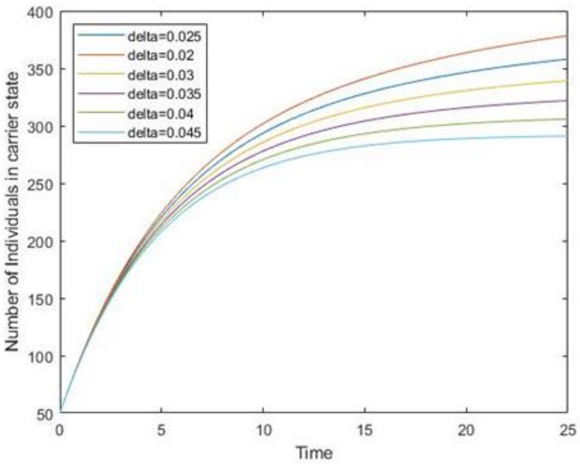

Figure 5. Carrier state for different values of δ

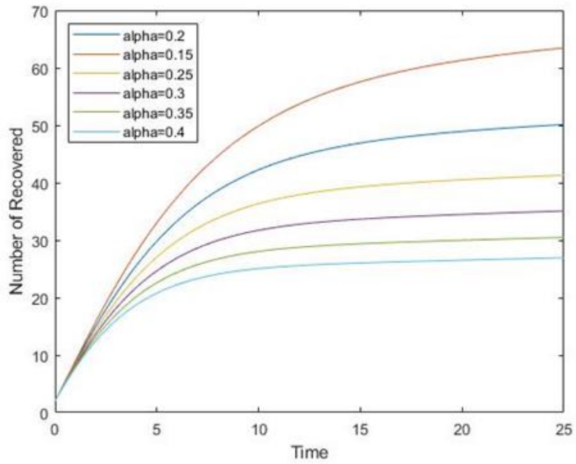

Figure 6. Recovered state for different values of α

Figure (2) shows the flow of Susceptible, Infectious, Carrier and Recovery class with respect to time for the reinfection Pneumonia model. From figure (3) ,the susceptible individuals decreases whenever the infectious rate increases and figure (4) clears that the infectious individuals increases whenever there is decrease in the factor of new infection. It is clear from figure (5) that as the immunity rate increases, the individuals in carrier state decreases and from figure (6) as the reinfection rate increases, the recovered individuals decreases.

8. Conclusion

A reinfection model with carrier state for pneumonia was formulated. The boundedness and positiveness of the state variables were verified. The optimal values of S, I, C, R were derived by equlibrium analysis. The model exhibits the local stability an global asymptotic stability behaviour under the suitable conditions. By the numerical simulations, it is clear that the flow of variables is stable for selected set of parameters.

References

1. Cyrus G Ngari, David M Malonza and Grace G Muthuri. (2014). A Model for Childhood Pneumonia Dynamics, Journal of Life Sciences Research, 1(2), 31-40.
2. Fulgensia Kamugisha Mbabazi, Joseph Y. T. Mugisha and Mark Kimathi.(2019). Hopf-Bifurcation Analysis of Pneumococcal Pneumonia with Time Delays, Abstract and Applied Analysis, 2019, Article ID 3757036, 21 pages, https://doi.org/10.1155/2019/3757036
3. Wang and Jiang.(2014). Analysis of an SIS epidemic model with treatment, Advances in Difference Equations 2014:246, doi:10.1186/1687-1847-2014-246
4. Jinde Cao, Yi Wang, Abdulaziz Alofi, Abdullah Al-Mazrooei and Ahmed Elaiw. (2015). Global stability of an epidemic model with carrier state in heterogeneous networks, IMA Journal of Applied Mathematics, 80, 1025–1048.
5. Li-Ming Cai, Abid Ali Lashari, Il Hyo Jung , Kazeem Oare Okosun, and Young Il Seo.(2013) . Mathematical Analysis of a Malaria Model with Partial Immunity to Reinfection, Abstract and Applied Analysis, 2013, Article ID 405258, 17 pages, http://dx.doi.org/10.1155/2013/405258
6. Mohammed Kizito and Julius Tumwiine.(2018). A Mathematical Model of Treatment and Vaccination Interventions of Pneumococcal Pneumonia Infection Dynamics, Journal of Applied Mathematics, 2018, 1-16.
7. Ong'ala Jacob Otieno, Mugisha Joseph and Oleche Paul.(2013). Mathematical Model for Pneumonia Dynamics with Carriers, Int. Journal of Math. Analysis, 7(50), 2457 – 2473.
8. Talawar A. S and U. R. Aundhakar.(2011), Stability Analysis of SIS Epidemic Model, Global Journal Engineering and Applied Sciences, 1(3), 126-128.
9. Victor Okhuese A. (2020). Estimation of the Probability of Reinfection With COVID-19 by the Susceptible-Exposed-Infectious-Removed-Undetectable-Susceptible Model, JMIR Public Health Surveill , 6(2):e19097, doi: 10.2196/19097