A Comparative Analysis of Quadratics Unit in Singaporean, Turkish and IBDP Mathematics Textbooks[1]

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Abstract

The purpose of this study was to analyze and compare the contents of the chapters on quadratics in three mathematics textbooks selected from Turkey, Singapore, and the International Baccalaureate Diploma Program (IBDP) through content analysis. The analysis of mathematical content showed that the three textbooks have different approaches and priorities in terms of the positions of chapters and weights of the quadratics units, and the time allocated to them within the respective curricular programs. It was also found that the Turkish textbook covers a greater number of learning outcomes targeted for quadratics among the three mathematics syllabi, showing a detailed treatment of the topic compared to the other two textbooks.

Key Words: Content analysis, international comparative studies, mathematics textbooks

Özet

Bu çalışmanın amacı ikinci dereceden denklemler ünitesinin farklı ülkelerde nasıl ele alındığını karşılaştırmalı olarak analiz etmektir. Bu amaçla Türkiye, Singapur ve Bakalorya Diploması Programının matematik ders kitapları doküman analizi yöntemiyle incelendi. Türkiye'deki matematik ders kitaplarında diğerlerine göre ikinci dereceden denklemler konusuyla ilgili daha fazla sayıda kazanımın olduğu belirlendi. Bu farklılıkların yanında benzerliklere de rastlanmıştır. Örneğin, Singapur'daki kitaplarda ikinci dereceden fonksiyon ifadesi doğrudan kullanılmasa da her üç programın kitaplarında ikinci dereceden eşitsizlikler ve fonksiyonlar daha fazla yer almaktadır. Bu bulgular incelenen ders kitaplarında ikinci dereceden denklemler konusuna farklı yaklaşıldığını, önceliklerin ve kapsamının değiştiğini göstermektedir.

Anahtar Kelimeler: Doküman analizi, uluslararası karşılaştırmalı çalışmalar, matematik ders kitapları

1. Introduction

Textbooks serve universally as a basic source for teaching to facilitate student learning and teacher instruction (Semerci, 2004). They give teachers ideas about the mathematical

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content to be taught and the methods and time for teaching mathematical topics. While planning homework, teachers use textbooks as the primary source of reference as well (Apple, 1992; Ben-Peretz, 1990; Freeman & Porter, 1989; Haggarty & Pepin, 2001; Nicol & Crespo, 2006; Russell, Schifter, Bastable, Yaffee, Lester, & Cohen, 1995; Schmidt, McKnight, & Raizen, 1997; Sosniak & Perlman, 1990). As Rock (1992) states succinctly, "it appears in most mathematics classes in most schools, that the curriculum for mathematical knowledge and learning is defined by the curricular materials, primarily by the mathematics textbook" (p. 30). Therefore, textbooks are important indicators of how much opportunity is given to students in terms of learning a mathematical topic (Törnroos, 2004). In summary, teachers' mathematical textbooks mediate students' opportunities to learn mathematics (St. George, 2001).

Regarding the roles of textbooks in classrooms and in educational systems, Foxman (1999) points out that textbooks are indicators of the first two of the three levels of curriculum model used by TIMSS (Trends in International Mathematics and Science Study). According to this model, there are three levels of curriculum; the *intended curriculum*, the one stipulated in official documents, the *implemented curriculum*, what is fulfilled by students and teachers in classrooms, and the *attained curriculum*, acquired knowledge, skills, understanding, and attitudes by students (Travers & Westbury, 1989). Similarly, Schmidt, McKnight, Valverde, Houang, and Wiley (1997) suggest that "textbooks serve as intermediaries in turning intention to implementation" (p. 178) and hence connect the intended and implemented curricula. In fact, textbooks are included as the fourth level of the curriculum model above, *potentially implemented curriculum*, which was integrated into the TIMSS model later (Johansson, 2003; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002).

Countries and educational programs use textbooks in order to meet the goals of their educational systems. Philosophies and needs of education in countries and programs shape the way textbooks are used (Howson, 1993; St. George, 2001). It is obvious that mathematics textbooks—with their distinctive preferences—are expected to reflect intentions and values of mathematics education within the countries they are developed in and used (Schmidt et al., 1997). Therefore for the purpose of this study, the cultural contexts, educational systems and assumptions in Turkey, Singapore, and the IBDP have to be taken into account in the process of analyzing and comparing the mathematics textbooks. It is therefore necessary to consider the intended learning outcomes in these three mathematics education systems, what skills they aim for students to gain, and what they value in mathematics education for the quadratics unit.

Mathematics curriculum in Turkey has undergone reforms starting from 2005 to better meet the needs of students. Thus in 2008; a new curriculum for secondary education has been put into effect (Erbaş, Alacaci, & Bulut, 2012). This improvement has the approach that "every student can learn mathematics" and advocates the importance of students' making connections between prior and new knowledge. In addition, this student-centered

approach has provisioned that students need to approach to learning mathematics with seven key components constituting the learning cycle *problem*, *discovery*, *hypothesizing*, *verification*, *generalization*, *making connection*s, and *reasoning* (MoNE, 2011). As a result of this transition in secondary education, the textbooks used in Turkish classrooms have changed accordingly to be consistent with the new curricular goals (Erbaş et al., 2012).

Singaporean mathematics textbooks are interesting for a few reasons. Singaporean students have been successful in international mathematics exams such as TIMSS and PISA (Programme for International Student Assessment) lately (Mullis, Gonzalez, Gregory, Garden, O'Connor, Chrostowski, & Smith, 2000; Mullis, Martin, Gonzalez, & Chrostowski, 2004; Zhu & Fan, 2004). In addition, there is a widely-held view that Singapore's textbooks' are distinctive for setting high standards, containing both ordinary and extraordinary problems, using a unique pedagogical approach, and being logically structured and focused on the essential skills of mathematics (Ahuja, 2005; Hoven & Garelick, 2007). When the structure of secondary school mathematics education in Singapore is analyzed, it is clear that mathematical problem solving skill is an important learning outcome and it is supported by the following five crucial components of student learning: *concepts*, *skills*, *processes*, *attitudes*, and *metacognition* (Singapore Ministry of Education, 2006).

The IBDP is a two-year, demanding, pre-university course of studies which aims to meet the needs of highly motivated secondary school students between the ages of 16 and 19 (International Baccalaureate, 2012a; Van Tassel-Baska, 2004). The program is being used in many private and state schools all around the world—including Turkey (within at least twenty-six schools) and Singapore—with its own assessment system (Hayden, 2006; International Baccalaureate, 2012b; Paris, 2003). In IBDP mathematics, it is aimed to help students appreciate the power and usefulness of topics together with its multicultural and historical perspectives, strengthen creative and critical thinking skills, have an understanding of the principles of mathematics, and develop persistence in problem solving (IBDP, 2006; Tilke, 2011).

Quadratics unit is a pivotal topic in high school mathematics curriculum linking elementary algebra to calculus. These types of functions can be used to model a multitude of real life phenomena from free fall, projectiles, to maxima, minima and optimization problems. Quadratic equations contain a multifaceted web of concepts that connect algebraic and graphical representations with such components as vertex, intercepts, real and imaginary roots. As relatively simpler forms of polynomials, quadratic equations lend themselves to various important ways of algebraic manipulation. For example, students learn about factorization, completing the square and discriminant formula to solve quadratic equations (Cooney, Beckman, Lloyd, Wilson, & Zbiek, 2010). In this study, we selected quadratics unit to compare textbooks, because we thought it represents high school mathematics curriculum fairly well with its algebraic and visual contents and components.

2. Method

This study used content analysis method to compare the contents of the chapters from the three textbooks on quadratics. The study sought to answer the following research question:

How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of content within the general structure of the textbooks, position and weight of quadratics unit within the textbooks, and in terms of the learning outcomes addressed and styles of presentation?

2.1. Research Design

Content analysis was the research method used in this study. Data from the study were mainly qualitative, as qualitative methods enabled to address the issues related to research question comprehensively and deeply (Patton, 1990).

2.2. Context

The three units on *quadratics* were analyzed in the textbooks displayed in Table 1 below.

The Turkish textbook used in this study was the one approved by the Turkish Ministry of National Education and has the potential of both reaching most Turkish high school students and being representative of Turkish mathematics textbooks.

Table 1: Textbooks Analyzed in the Study

Turkey	Kaplan, E. (2008). *Ortaöğretim Matematik 10 Ders Kitabı* (pp. 55-121). Ankara, Turkey: Paşa Yayıncılık Ltd.
Singapore	Thong, H. S., &Hiong, K. N. (2006). *New Additional Mathematics* (pp. 61-80). Singapore: SNP Panpac Publishing House.
IBDP-SL	Owe Owen, J., Haese, R., Haese, S., & Bruce, M. (2008). *Mathematics for the International Student Mathematics SL* (pp. 149-194). West Beach, SA: Haese & Harris Publications.

The Singaporean mathematics textbook, on the other hand, is written for students who are preparing for the GCE O Level Additional Mathematics Exam, which is conducted annually in Singapore and authorized by the Singapore Ministry of Education and the University of Cambridge International Examinations (CIE). Hence, the textbook being used in this study is approved by the Singapore Ministry of Education (Singapore Examinations and Assessment Board, 2011; Singapore Ministry of Education, 2006). Lastly, the IBDP-SL mathematics textbook used in this study is selected because its being a representative textbook that reflects the philosophy of IBDP mathematics.

In this study, the unit *quadratics* was selected for analysis and comparison within the three textbooks due to its serving as an important bridge between basic mathematics topics

such as linear functions and polynomials, and higher mathematics topics such as differentiation and integration. Moreover, the topic has both algebraic and geometric features as discussed earlier, which makes it a relatively good topic representative of the school mathematics in general.

2.3. Method of data coding and analysis

The chapters on quadratics within the three mathematics textbooks were analyzed in terms of their mathematical content. Findings of this analysis were also used to make inferences and reach generalizations about philosophies of the Turkish, Singaporean, and IBDP-SL mathematics curricula.

Inter-rater reliability was ensured by coding the data by two researchers. Another reviewer, an MA student in the educational sciences was trained to conduct the analysis of a significant subset of material from the textbooks. After comparing results of the reviewer and the researcher, a similarity above 80% was regarded as an acceptable level of agreement. When this is not the case, deliberations and further training were held to resolve disagreements.

2.4. External positioning of the quadratics chapters

In this section, to describe position and weight of the quadratics unit in the three mathematics textbooks, the chapters coming before and after each quadratics unit were recorded first by creating tables of unit titles for each textbook to compare their positions. Then, the space occupied by quadratics within the totality of each textbook was computed by calculating the percentage of the number of pages allocated to the unit over the total number of pages in each textbook. Thus, a table—including number of units, total number of pages, and the number of pages allocated to quadratics and the percent weights of the quadratics unit over the whole textbooks—was constructed in order to see the differences and similarities among the textbooks. Moreover, the time allocated to the unit in the Turkish, Singaporean, and IBDP-SL mathematics curricula and the grade when the unit is taught were explored by using the related parts of the Turkish Ministry of National Education's Grade 10 mathematics syllabus (MoNE, 2011), Singapore's Secondary school mathematics syllabus (Singapore Ministry of Education, 2006), and the IBDP-SL mathematics syllabus (IBDP, 2006).

2.5. Analyzing internal content

While analyzing the quadratics unit in the three textbooks with regard to the unit's internal content, the learning outcomes for each of the quadratics chapters in the Turkish, Singaporean and IBDP-SL mathematics curricula were listed initially through the use of the three syllabi mentioned above. As the Turkish Ministry of National Education's Grade 10 mathematics syllabus (MoNE, 2011) was used as a starting point and then extra learning outcomes within the Singaporean and IBDP-SL mathematics syllabi were added to this list when they were not observed in the Turkish syllabi. Thus, three all-inclusive tables (for

quadratic equations, *inequalities*, and *functions* separately) containing all of the learning outcomes were created. Afterwards, all of the pages included within the three chapters on quadratics were scanned, and a check mark was placed next to each learning outcome in all-inclusive tables when the learning outcome is covered (or not covered) in the corresponding mathematics textbook. Thus, the number of the learning outcomes met in each of the three textbooks was counted. Furthermore, outcomes of internal content analysis were interpreted by discussing the whole data in a general resultant table containing the total number of learning outcomes addressed within each quadratics unit as well as the textbooks' percentage of covering these outcomes. Finally, the reasons which affected the outcomes of this analysis were explained for each of the Turkish, Singaporean and IBDP-SL mathematics curriculum.

3. Results

The contents related to quadratics from each of the three textbooks were analyzed from two perspectives: externally and internally.

3.1. External positioning of the quadratics chapters

In this section, a report of the position of the units on quadratics within each of the three mathematics textbooks was presented. To do this, the following were reported: a general structure of the three textbooks, the position and weights of the units on quadratics within the totality of textbooks, and the time allocated to the units during the academic year based on respective curriculum frameworks and to which grades the units are taught. Table 2, 3, and 4 display and compare the unit titles in the Turkish, Singaporean and IBDP-SL mathematics textbooks.

Table 2: Unit Titles in the Turkish Mathematics Textbook

1. Polynomials
2. Quadratic equations, functions with one unknown, and inequalities
3. Permutations, combinations, binomials, and probability
4. Trigonometry

As displayed in Table 2, in the Turkish textbook the unit *quadratic equations, functions with one unknown, and inequalities* is the second one following the polynomials unit and before the unit permutations, combinations, binomials, and probability. As the Turkish textbook is approved by the Turkish Ministry of National Education for the 10^{th} grade, the quadratics unit has the same order and a similar weight in the national curriculum. Since the textbook contains 256 pages in total and 70 (pp. 55-124) pages were reserved for the quadratics unit, the unit constitutes 27.3% of the whole textbook. Moreover, the quadratic equations, functions with one unknown, and inequalities unit occupies 30% of national mathematics curriculum of the 10^{th} grade, and 44 lesson hours (one lesson hour means forty-five minutes) out of 144 in total are allocated to teach quadratics in the Turkish context (MoNE, 2011).

Table 3 shows that in the Singaporean mathematics textbook, the unit *quadratic expressions and equations* is the fourth one coming after the unit indices, surds, and logarithms and before the unit remainder and factor theorems. The Singaporean mathematics textbook has 623 pages, 22 of which (pp. 61- 82) are devoted to the quadratics unit and nearly 5 of them are used in the quadratic function sections in unit 9 (functions). So, in total 27 pages belong to quadratics, which means 4.3% of the whole textbook is occupied by quadratics topics. On the other hand, in the GCE O level additional mathematics curriculum, the unit quadratic expressions and equations is one of 11 topics in total and it is taught in the last two years of secondary school. Different from the Turkish Grade 10 mathematics syllabus, Singapore's GCE O level additional mathematics syllabus does not give a specified time allocation for the quadratics unit and, hence, a percentage value for the weight of the unit in the curriculum could not be computed (Singapore Ministry of Education, 2006).

Table 3: Unit Titles in the Singaporean Mathematics Textbook

1. Sets	13. Permutations and combinations
2. Simultaneous equations	14. Binomial theorem
3. Indices, surds, and logarithms	15. Differentiation and its technique
4. Quadratic expressions and equations	16. Rates of change
5. Remainder and factor theorems	17. Higher derivatives and applications
6. Matrices	18. Derivatives of trigonometric functions
7. Coordinate geometry	19. Exponential and logarithmic functions
8. Linear law	20. Integration
9. Functions	21. Applications of integration
10. Trigonometric functions	22. Kinematics
11. Simple trigonometric identities and equations	23.Vectors
	24. Relative velocity
12. Circular measure	

Table 4 shows that in the IBDP-SL mathematics textbook, the unit *quadratic equations and functions* is the eighth unit coming after coordinate geometry and before binomial theorem units. The textbook is composed of 812 pages and 46 of them are devoted to the quadratics unit, which means quadratics unit covers 5.7% of the whole textbook.

Regarding the position and weight of the unit in the IBDP-SL mathematics curriculum, the unit quadratic equations and functions is included in topic 2 (*functions and equations)*, which is one of the 7 main SL mathematics topics. Additionally, in the IBDP-SL mathematics curriculum, 24 teaching hours out of 140 hours in total are allocated to topic 2. When the review sets are excluded, learning outcomes of topic 2 are covered within approximately 90 pages and those of quadratics are met by using 42 pages.

Table 4: Unit Titles in the IBDP-SL Mathematics Textbook

1. Functions	16. Vectors in 3-dimensions
2. Sequences and series	17. Lines in the plane and in space
3. Exponents	18. Descriptive statistics
4. Logarithms	19. Probability
5. Natural logarithms	20. Introduction to calculus
6. Graphing and transforming functions	21. Differential calculus
7. Coordinate geometry	22. Applications of differential calculus
8. Quadratic equations and functions	23. Derivatives of exponential and logarithmic functions
9. The binomial theorem	
10. Practical trigonometry with right angled triangles	24. Derivatives of trigonometric functions
11. The unit circle	25. Areas within curved boundaries
12. Non right angled triangle trigonometry	26. Integration
13. Periodic phenomena	27. Trigonometric integration
14. Matrices	28. Volumes of revolution
15. Vectors in 2-dimensions	29. Statistical distributions

Therefore, the time allocated to quadratics in the IBDP-SL mathematics curriculum is about 11 teaching hours, which corresponds to approximately 15 lesson hours in the Turkish secondary school context (in the IBDP a teaching hour means 60 minutes whereas a lesson hour in Turkish high schools represents 45 minutes). Moreover, IBDP is taught in the last two years of secondary school including the quadratics unit (IBDP, 2006).

Since both the Singaporean and IBDP-SL mathematics textbooks cover a two-year mathematics curricula, they include many topics while the Turkish mathematics textbook covers only 10^{th} grade topics. The weights of the quadratics unit in respective textbook are shown in Table 5.

Table 5: Weight of the Quadratics Units in the Three Mathematics Textbooks

Textbook	Number of units	Total number of pages	Number of pages allocated to quadratics	Percent weights of quadratics unit over whole textbook
Turkey	4	256	70	27.3%
Singapore	24	623	27	4.3%
IBDP-SL	29	812	46	5.7%

Table 5 shows that the Turkish mathematics textbook is the one allocating the largest space to the quadratics unit. However, since the Turkish mathematics textbook covers the topics of only a one-year curriculum and the Singaporean and IBDP-SL mathematics textbooks represent two-year curricula, the percent weights of the quadratics unit over curricula in Singapore and the IBDP-SL can be multiplied by 2 (which gives 8.6 % and

11.4% respectively) in order to obtain a more accurate comparison.

3.1. Analyzing internal content

In this section, the three chapters on quadratics were analyzed in terms of their internal contents through the use of learning outcomes stated in the Turkish, Singaporean, and IBDP-SL mathematics curricula. Table 6, 7 and 8 below are arranged according to the three subtopics of quadratics; quadratic equations, inequalities, and quadratic functions respectively. They represent the all-inclusive list of learning outcomes, and whether the learning outcomes are covered or not in the three textbooks.

Table 6: Comparison of Internal Content among the Three Quadratics Units According to Learning Outcomes of Quadratic Equations (T: the Turkish mathematics textbook, S: the Singaporean mathematics textbook, IB: the IBDP-SL mathematics textbook)

Learning outcomes of *quadratic equations*	T*	S	IB
1. Explains a quadratic equation with one unknown.	√	√	√
2. Identifies roots and solution set of a quadratic equation with one unknown by factorization, completing the square, and using quadratic formula.	√	√	√
3. Identifies roots and solution set of a quadratic equation with one unknown by using technology.	**x**	**x**	√
4. Identifies presence and number of roots of a quadratic equation, as well as their being reel or not and their possible positions on curve according to the sign of *discriminant*.	√	√	√
5. Shows the relationships between roots and coefficients in a quadratic equation.	√	√	√
6. Given a quadratic equation with one unknown including a parameter, finds out the parameter according to desired restrictions.	√	√	√
7. Writes the quadratic equation whose roots are given.	√	**x**	**x**
8. Identifies solution set of an equation which can be turned into a quadratic equation with one unknown.	√	**x**	**x**
9. Explains systems of quadratic equations with two unknowns, and identifies solution set of systems of quadratic equations with two unknowns which can be turned into a quadratic equation with one unknown.	√	**x**	**x**
10. Formulates a pair of linear equations (a linear equation system) and turns them into a quadratic equation with one unknown according to given restrictions, and finds roots of obtained quadratic equation in problem solving.	√	**x**	**x**
11. Formulates a quadratic equation in one unknown according to given restrictions and finds its roots in problem solving.	√	**x**	√
Total number of coverage of the learning outcomes	**10**	**5**	**7**

* T: Turkish, S: Singapore, IB: International Baccalaurate

Table 6 gives a comparison of the internal contents of the three quadratics units based on the learning outcomes covered for the subtopic quadratic equations. The table shows that while all the three textbooks address quadratic equations as a part of the quadratics chapter; they differ in the number of learning outcomes addressed. That is, the Turkish textbook covers a greater number of learning outcomes for quadratic equations compared to the other two textbooks.

Table 7: Comparison of Internal Content among the Three Quadratics Units According to Learning Outcomes of Inequalities

Learning outcomes of *inequalities*	T	S	IB
1. Analyzes sign of a binomial $ax+b$ by showing it in table, and identifies solution set of a linear inequality with one unknown.	√	**x**	**x**
2. Analyzes sign of a trinomial ax^2+b+c.	√	√	**x**
3. Identifies solution set of a quadratic inequality with one unknown.	√	√	**x**
4. Identifies solution set of an inequality which is given as product of polynomials with first degree.	√	√	**x**
5. Identifies solution set of an inequality which is given as fraction of polynomials with first and second degree.	√	**x**	**x**
6. Identifies solution set of systems of linear or quadratic inequalities.	√	**x**	**x**
7. Identifies presence of roots in a quadratic equation with one unknown, without solving the equation.	√	√	√
8. Identifies signs of roots in a quadratic equation with one unknown, without solving the equation.	√	**x**	**x**
9. Given a quadratic equation with one unknown including a parameter, identifies presence of roots according to the values which the parameter gets.	√	√	√
10. Given a quadratic equation with one unknown including a parameter, identifies signs of roots on table according to the values which the parameter gets.	√	**x**	**x**
11. Solves a quadratic inequality by using the corresponding quadratic curve.	**x**	√	**x**
12. Shows solution set of a quadratic inequality and solution set of systems of quadratic inequalities on the graph.	√	**x**	**x**
Total number of coverage of the learning outcomes	**11**	**6**	**2**

Table 7 presents a comparison of the internal contents of the three chapters on the subtopic inequalities. It shows that although the IBDP-SL mathematics textbook does not include an inequalities section in the quadratics unit, it addresses two of the learning outcomes, which are related to presence of roots in a quadratic equation. On the other hand, the Turkish textbook is the one that presents the section on inequalities in a relatively more detailed way.

Table 8 below shows that the quadratic functions section is the most comprehensive one being covered by all three textbooks. For this section, the Turkish mathematics textbook

covers all of the learning outcomes, and the IBDP-SL mathematics textbook transmits *nearly all* of them. In addition, although the Singaporean mathematics textbook never uses the expression *quadratic function* throughout the chapter, it addresses more than half of the learning outcomes in this part.

Table 8: Comparison of Internal Content among the Three Quadratics Units According to Learning Outcomes of Quadratic Functions

Learning outcomes of *quadratic functions*	T	S	IB
1. Explains a quadratic function, and finds out its maximum and minimum points.	√	√	√
2. Draws graph of a quadratic function (parabola) given in one of the forms: $y= ax^2+bx+c$, $y= a\ (x-h)^2+k$, $y= a\ (x-a)(x-b)$; and finds out vertex, x and y intercepts, and axis of symmetry.	√	√	√
3. Sets up sign table of a quadratic function before drawing its graph.	√	**x**	**x**
4. Analyzes x-intercepts, vertex, being either positive definite or negative definite of a quadratic function by using only *discriminant* and coefficients of $y= ax^2+bx+c$.	√	√	√
5. Identifies the quadratic function when vertex and an arbitrary point on its graph are given, or when any three points on its graph are given.	√	**x**	√
6. Draws graphs of linear functions, and analyzes three possible positions (cutting, touching, missing) of a linear and quadratic function on coordinate plane.	√	√	√
7. Models problems that can be solved through a quadratic function, and solves them using its graph (parabola).	√	**x**	√
Total number of coverage of the learning outcomes	**7**	**4**	**6**

The results gathered from Table 6, 7 and 8 are summarized in Table 9 below.

Table 9: Results of Internal Content Analysis Conducted through Learning Outcomes

Textbook	Total number of learning outcomes addressed	Percentage of covering learning outcomes
Turkey	28	93.3%
Singapore	15	50.0%
IBDP-SL	15	50.0%

As Table 9 shows, the quadratics unit in the Turkish mathematics textbook addresses more learning outcomes compared to that of Singapore and the IBDP-SL. These results can be explained in several ways. The Singaporean mathematics textbook covers quadratics in unit 4 without using the term quadratic function. Then, while introducing the function concept in unit 9, it gives the examples of linear and quadratic functions in detail to simplify each subtopic in the functions unit, which may be considered as an indirect reference to the quadratic function concept. In addition, the IBDP-SL mathematics textbook

does not contain inequalities in the quadratics unit. Furthermore, since the Singaporean and IBDP-SL mathematics textbooks include all of the O level additional mathematics and IBDP-SL mathematics topics respectively, they allocate a narrower space for the quadratics unit compared to the Turkish mathematics textbook, which is written for students in the 10^{th} grade only.

4. Discussion

The purpose of this study was to compare the contents of the quadratics chapters in three mathematics textbooks from Turkey, Singapore and the IBDP. By using content analysis, the three quadratics units were analyzed and compared in terms of their positions and weights within the mathematics curricula they represent, and the learning outcomes they target.

4.1. External positioning of the quadratics chapters

The analysis of the three quadratics units revealed some similarities and differences in terms of the positions of chapters and weights of the units, time allocated within the programs, and at which grades the units are taught.

The positions of the quadratics units in the three textbooks gave some hints about the continuity and connections among the topics intended in the corresponding mathematics syllabi (see Tables 2-4). For instance, in the Turkish mathematics textbook, the quadratics chapter was given after polynomials, which shows that in the Turkish curricula, quadratics are considered as a special type of polynomials. In this way, students are expected to make connections with their prior knowledge about polynomials and functions. Similarly, the Singaporean mathematics textbook presents quadratics before the chapter on remainder and factor theorems that includes polynomial identities and cubic equations. Thus, in the Singaporean mathematics syllabus, polynomials are expected to serve as a conceptual umbrella for the quadratic and linear expressions as two special types of polynomials. The IBDP-SL mathematics textbook, on the other hand, makes a connection with quadratics in the topic of binomial theorem —the unit after quadratics—by expressing that the binomial expansion of $(a+b)^2$ is a quadratic expression and showing again a move from a specific to general case, even though on a different conceptual pattern. So, it became clear that in all three textbooks, the positions of the units are selected in a way that enables students to make connections among the topics and see the usefulness of each of the topics within a bigger picture.

On the other hand, regarding the time allocated to quadratics within the programs, parallel to the percent weights of the units in the three textbooks, the quadratics topic is taught in a longer period of time in Turkey and carries a relatively bigger weight.

The Turkish mathematics textbook has a smaller number of topics compared to the Singaporean and IBDP-SL mathematics textbooks, which probably results from its intention to be used by only the students in the 10^{th} grade and the other two serving for

two-year programs. Correspondingly, the percent weights of the quadratics unit over the three textbooks differ (see Table 5). It appears that the quadratics unit occupies a more dominant place in the Turkish mathematics textbook even if the percent weights of the chapters in the Singaporean and IBDP-SL mathematics textbooks are doubled—with the underlying reason that these two textbooks cover all of the topics in the two-year curricula.

4.2. Analyzing internal content

The investigation of the three chapters on quadratics in terms of their internal contents revealed that the Turkish mathematics textbook attempts to fulfill *nearly twice* of the learning outcomes covered by the Singaporean and the IBDP-SL textbooks (see Table 9). This high expectation of mathematical content stands in stark contrast with the relatively low mathematical attainment of Turkish students in international achievement measures such as PISA (Alacaci & Erbaş, 2010) and deserves further elaboration. This finding seems also to suggest that the Turkish mathematics textbook has an approach to explain every mathematical issue directly under a separate title, instead of offering students group or individual work or portfolio opportunities to investigate these topics on their own. However, this may lead to students becoming used to absorbing ready-made mathematical knowledge. This finding seems to support the presence of an implicit *encyclopedist* philosophy still in effect in Turkish education as suggested by Hesapçıoğlu (2008).

The IBDP-SL mathematics textbook, on the other hand, includes an opening problem and investigation section where students are directed to explore crucial points within the topics on their own or with their peers. It is clear that IBDP-SL mathematics textbook places students more at the center of learning process by enabling them to actively participate and make meaning from the textbook. Moreover, this feature of the textbook can give some practical ideas to teachers about how to present the mathematical knowledge. These hints may be useful for teachers since they often use textbooks to make decisions about how to teach a mathematics topic and engage students in a mathematical investigation (Russell et al., 1995).

Different from the Turkish and IBDP-SL mathematics textbooks, the Singaporean textbook presents both algebraic and geometric features of quadratics simultaneously from the start of the chapter, by exemplifying a synthetic approach in the mathematics curriculum. In addition, the textbook seems to have some organizational assumptions. Although it aims to construct background knowledge and skills about quadratics in the quadratics chapter; it uses the quadratic function examples and their graphs in the functions chapter, which is positioned five chapters later. This reflects that different curricula may have different assumptions within the inner organization of topics.

Finally, among the three subtopics—quadratic equations, inequalities, and quadratic functions—in the all-inclusive list of learning outcomes, the greatest total number of learning outcomes covered by the three textbooks belongs to quadratic functions. Even if the Singaporean mathematics textbook does not use the quadratic function terminology

within the unit, graphs of quadratic expressions are indicators of the quadratic function concept, which is presented later in the textbook. As a result, this similarity among the textbooks indicated the existence of an implicit international agreement in terms of the weight given to the *quadratic function* subtopic.

The findings of this study are delimited to the results of the analysis of one chapter from the perspective of content. However, different textbooks should be analyzed from a broader scope to discover the properties of high quality textbooks with appropriate preferences. In this way, particular features of the textbooks to meet the needs of students and teachers can be identified, and reliable criteria can be created to evaluate textbooks.

The findings of the study indicated that there is a variety of textbook approaches that can give students the opportunities to develop mathematical understanding and practice the mathematical knowledge and skills. As teachers play a significant role in the selection and use of mathematics textbooks in the classroom, they should be trained in in-service programs and during pre-service education about how to select and use mathematics textbooks effectively in teaching mathematics.

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