Exhibition Curation on the Infleuntial Usage of Plyhedra in Science, Art and Design

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Abstract: Recently, some museums have been using variety of technology to give children new cultural experiences of historical relics. Effective exhibition plan can be beneficial for overcoming time and space constraints. This article hypothesizes that polyhedron was influenced by one another in variable fields and some of them had the most innovative role in the period and later in future generation. The polyhedra theory cut across and exercise a far-reaching influence between science, art and design. The research question is how polyhedron influenced each other to the different fields. After analyzing all the connected story we plan on an exhibition telling children how these studies are intertwined, then it can provide a big picture of whole new worlds interestingly with education purposes. Method will be qualitative study including interview from experts in their fields and data collection and data analysis from literatures from ancient philosophy to contemporary art and design. Purpose of this study is for middle and high school students to have integrated thinking of science, liberal arts, and art in understanding of polyhedrons' value and meaning which was developed by particular visual language with human civilization.

Keywords: polyhedron, museum exhibition design, Platonic solid, Buckminster Fuller's structure, Leonardo da Vinci Sketch

1. Introduction

Fig. 1 Connected concepts of polyhedra

Polyhedra have diverse origins and development paths and have been used in such fields as mathematics, astrophysics, architecture, chemistry, and arts. Despite the different developments and applications, the polyhedra used in different fields share similar concepts (Figure 1). Our research question is "How have the concepts of polyhedra developed in various fields influenced one another?" With this question, we will test the hypothesis that the knowledge of polyhedra in various fields developed by making influence on one another throughout history (Figure 2). After showing the stories of the mutual influence of different concepts of polyhedra, we will suggest that they can be used in convergence education and art exhibition showing how diverse fields are intertwined.

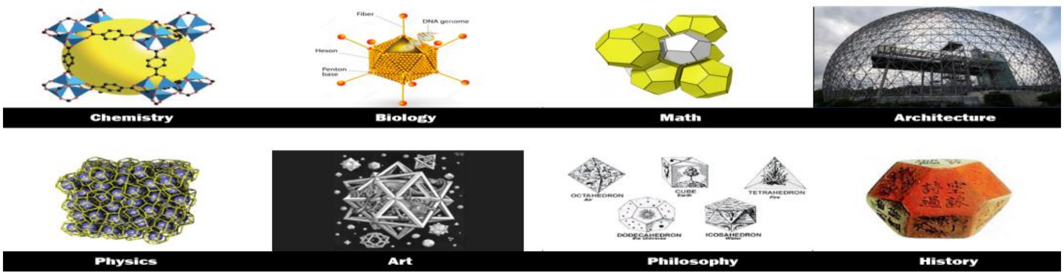


Fig. 2 The influence of polyhedral in various fields

2. Literature Review

2.1. Definition And Origin

To arrive at a rigorous definition of a polyhedron (plural "polyhedra") is a difficult task. However, broadly defined, a polyhedron is a three-dimensional geometric object constructed from flat polygonal faces, straight edges, and sharp corners or vertices. The term "polyhedron" originates from the Classical Greek *poly* (πολύεδρον), meaning many, and *hedra* (ἕδρα), meaning seat or base. The common cube, the pyramid, the soccer-ball-shaped truncated icosahedron are examples of polyhedral [1].

The origins of the polyhedron concept date back to the late Neolithic era. Hundreds of carved stone polyhedral balls dating from around 2000 BC have been found in Scotland (Figure 3). The Egyptians were also interested in polyhedra, as evidenced by their pyramids, and the Babylonians applied a similar concept to calculate the volumes of solid objects [2]. The Etruscans Ancient Italy (900–27 B.C.) were also aware of some regular polyhedra [3].

Fig. 3 Neolithic stone carvings of polyhedra

2.2. The Ancient Era: The Integration of Mathematics, Science, and the Arts

Despite the older origins of the concept, the earliest records available today are the works left by the classical Greeks. More importantly, their approach is distinct from the preceding ones insofar as they tried to prove their findings [2]. Approaching the knowledge of polyhedra mathematically, Pythagoreans and Theaetetus discovered their regularity. Pythagoreans are considered to have discovered the cube, the pyramid, and the dodecahedron, while Theaetetus is believed to have discovered the octahedron and the icosahedron. [4].

Plato (428-347 BC) described five polyhedra in detail in *Timaeus*, in which he associated each of the four classical elements comprising the world (earth, air, water, and fire) with a regular solid (Figure 4) [5]. Although he was not the first to have discovered the five regular polyhedra, he incorporated knowledge of the five regular polyhedra into his philosophical system to explain the world [6]. However, Plato neither proved the theory nor formally indicated the required properties for a regular polyhedron. It was Euclid of Alexandria (325-265 BC), the father of geometry, who proved that the five Platonic solids are the only regular polyhedra and described their construction in his mathematical and geometric treatise *Elements* [2]. Archimedes of Syracuse (287–212 BC), a Greek mathematician and engineer, further expanded Euclid's study by discovering semiregular solids called the "Archimedean solids" or "Archimedean semiregular polyhedra" (ASRP) [4].

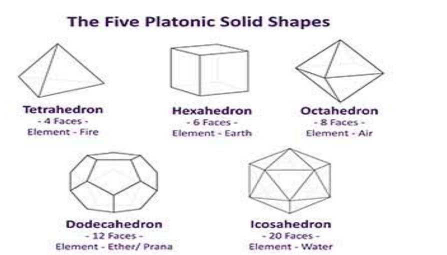


Fig. 4 The five Platonic solids

2.3. Fragmentary Traces In The Medieval Era And Preservation In The Arab World

Although the medieval era is regarded as a kind of dark age with regard to knowledge of polyhedra, some developments can be found. Qualities such as hot, dry, cold, and moist were added to the Plato's idea of the basic elements. These additional properties contributed to the development of diagnostic medicine [5]. Many cuboctahedra have been found in buildings of the medieval era, which date from the 12th to the 15th century [7].

Ancient Greek Knowledge of polyhedra was also disseminated in the Arabic world via translations made during the 8th and 9th centuries [8]. One of the crucial characteristics of Islamic artists is their genius for geometry, in which the regular polyhedra played an important role [9]. Islamic designers, who regarded repetition, variation and symmetry as important, used polyhedral geometry [10] and created new variations [11]. Polyhedra were also used in Islamic architecture. Alhambra Palace is a typical example of a building in which various kinds of polyhedra can be found. In the field of mathematics, Abu'l-Wafa used Platonic and Archimedean polyhedra and Euclidean tools to describe two- and three-dimensional constructions [12], [13]. All of this knowledge of polyhedra in Islamic civilization was handed down to Renaissance artists and modern scholars [14].

2.4. The Renaissance: An Amalgamation Of Knowledge Centered On The Arts

Italy's Renaissance

After the long dark medieval era, Renaissance artists as well as mathematicians and scientists revived the polyhedra. Their use of polyhedra was amalgamative and interdisciplinary. Paolo Uccello (1397–1475), an early Renaissance painter and mosaicist shaped the practice of using polyhedral forms in artworks by applying his knowledge of geometry to the arts [15]. Piero della Francesca (1410-1492), who was both a mathematician and an excellent painter, developed a new method of representation, perspective, by rediscovering and developing Archimedean solids. [16]. Leonardo da Vinci (1452–1519), the quintessential Renaissance man, was a passionate lover of geometry who developed a knowledge of polyhedra. His illustration of various important polyhedra can be found throughout his drawings (Figure 5) [17], [18].

Fig. 5 Leonardo da Vinci's truncated icosahedron

Luca Pacioli (1445–1517), an artist and mathematician and a friend of da Vinci's, is regarded as epitomizing the deep connection between art and mathematics (Figure 6) [19]. Fra Giovanni (1433-1515)'s *Intarsia Polyhedra* (1520) and Daniele Barbaro (1513–1570)'s *La Pratica della Perspettiva* contain illustrations of various polyhedra.

Fig. 6 Portrai of Luca Pacioli by Jacopo de Barbari

The German Renaissance

In the 16th century, the Renaissance declined in Italy and moved to northern Europe. Albrecht Durer (1471–1528), a German artist, mathematician, architect, engineer, and typographer, was an expert in printmaking. In his *Four Books on Measurement* on perspective, Durer presented one of the first examples of polyhedra nets, unfolded polyhedra that lay flat for printing. In his masterful engraving *Melancholia I*, he used various polyhedra (Figure 7). As a famous Nuremburg goldsmith, designer, and inventor of scientific instruments, Wentzel Jamnitzer (1508–1585) produced many variations of regular solids by truncating, constellating, and faceting them [20].

Fig. 7 Albrecht Durer's *Melancholia I*

2.5. The Modern Era: Developments In Mathematics And Science

In the modern era, knowledge of polyhedra began to evolve in individual fields. In science, Johannes Kepler used Platonic solids in his explanation of the orbits in which the planets move around the sun in 1596. Although we now know that Kepler's estimates of distances were wrong, his Platonic model suggested a possible mathematical synergy between microcosm and macrocosm [5]. Modern chemistry also drew upon the knowledge of the geometry of polyhedra accumulated since the time of ancient Greece [8].

In the field of mathematics, Descartes (1596–1650) discovered a formula for polyhedra and influenced Carl Friedrich Gauss (1777-1855). Gauss, in turn, affected Leonhard Euler (1707-1783), who discovered the famous Euler's polyhedron formula (V-E+F=2). By first formulating the polyhedral formula as a theorem about polyhedra, Euler made a significant contribution not only to the knowledge of polyhedra but also to the development of mathematics and other fields [1]. Augustin-Louis Cauchy (1789–1857), Adrian-Marie Legendre (1752–1833), and Ludwig Schlafli (1814-1895) further advanced the knowledge of polyhedra.

Polyhedra stars were also used in many baroque churches to represent heraldic symbols of the Pope who built the church [21]. The perspective image in Jean-Francois Niceron's *Thaumaturgus Opticus* (1638) shows a rhombicuboctahedron augmented with a pyramid on each of its square faces. The tomb in Salisbury Cathedral in England, built in 1635, also features stone polyhedra in Da Vinci's style [22].

2.6. The 20th and 21st Centuries

Knowledge of polyhedra was expanded and applied in various fields since the 20th century. In architecture, Buckminster Fuller (1895-1983), an American architect, system theorist, author, designer, mathematician, inventor, and futurist, created geodesic domes in the 1940s and 1950s. Fuller used polyhedrons to invent the hemispherical thin-shell structures (lattice-shell) that can withstand heavy loads for their size. They were used in military radar stations, civic buildings, environmental protest camps, and exhibition attractions. [23], [24].

In the field of mathematics, Georg Alexander Pick (1859–1942) discovered a formula for calculating the area of polygons and polyhedrons ($A = I + b/20 -1$). Pick's theorem, together with Euler's theorem, meant that knowledge of polyhedral could be more systematically employed in mathematics. The development of topology, one of the crucial branches of mathematics, was based on theories of polyhedra.

Archimedean semiregular polyhedra had a significant impact on science. The 32-faced truncated icosahedron has been particularly fascinating and useful. The "Fat Man," the first atomic bomb dropped in Nagasaki, Japan during the World War II, was developed using the 32-faced truncated icosahedron as a configuration for arranging lenses to focus the explosive shock waves of the detonators [25]. Chemists also found that the atoms of a newly discovered carbon molecule were arrayed in a structure similar to a geodesic dome and, thus, named the C60 molecule buckminsterfullerene ("buckyball").

Polyhedra can also be found in the field of arts and design. Maurits Cornelis Escher (1898–1972), a famous mathematical graphic artist, created graphic art incorporating Platonic solids as well as mathematical objects like cylinders and stellated polyhedra into his works. In his wood engraving print, *Stars*, he depicted hollowed-out compound of three interlocking regular octahedral floating in space (Figure 8). In one of his papers, Escher emphasized the importance of dimensionality [26].

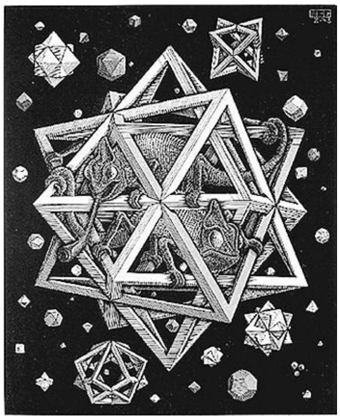
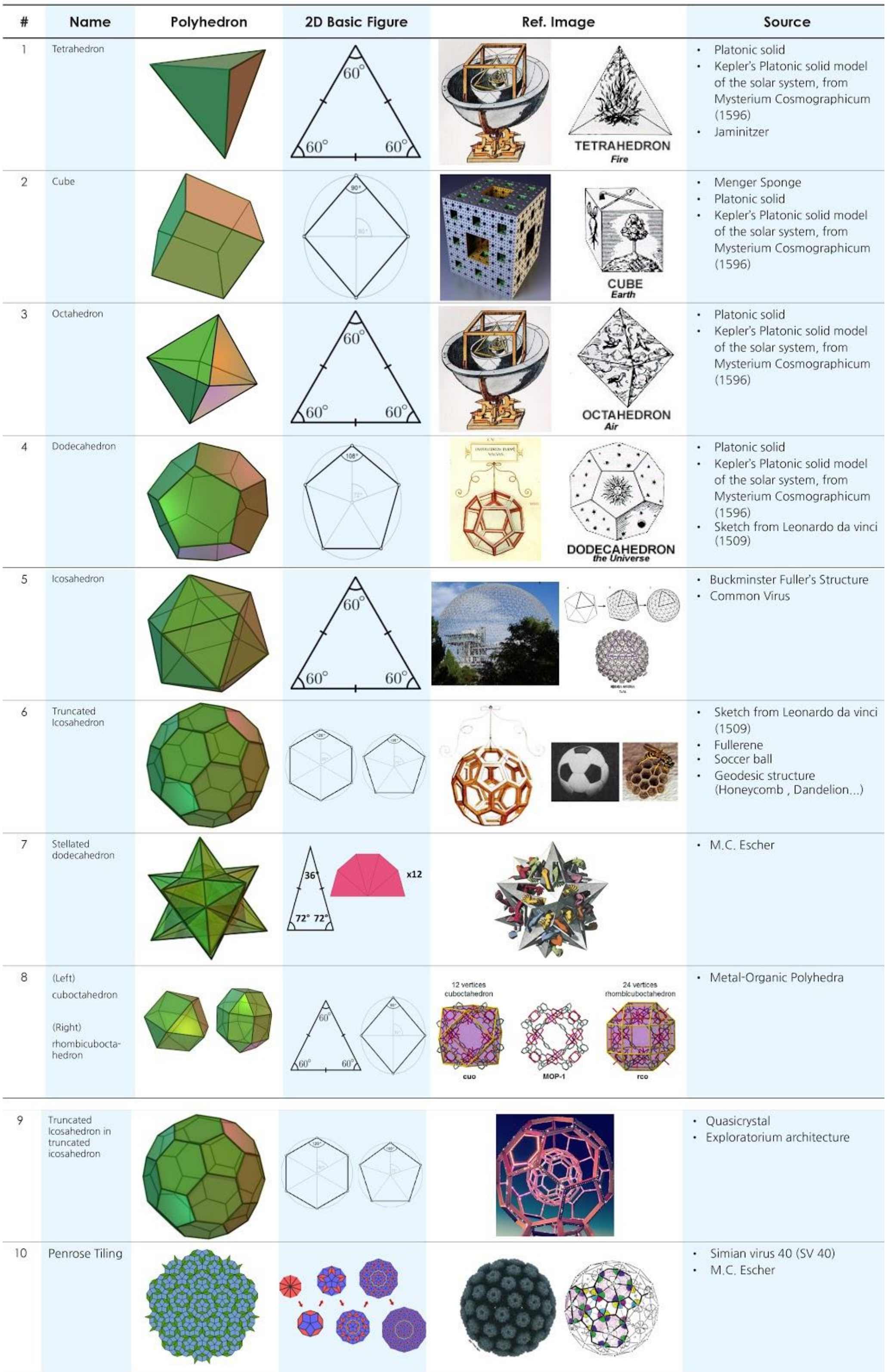


Fig. 8 Graphic artist, Escher's polyhedra artwork, *Stars*

3. Data Collection

We have found various kinds of polyhedron while historically exploring them. In order to view and compare them in a glance, we created a chart showing the details of each polyhedron. It is also useful when trying to understand the relationship among the diverse polyhedron crossing over various fields. (Figure 9).



#	Name	Polyhedron	2D Basic Figure	Ref. Image	Source
1	Tetrahedron		60° 60° 60°	TETRAHEDRON Fire	• Platonic solid • Kepler's Platonic solid model of the solar system, from Mysterium Cosmographicum (1596) • Jaminitzer
2	Cube			CUBE Earth	• Menger Sponge • Platonic solid • Kepler's Platonic solid model of the solar system, from Mysterium Cosmographicum (1596)
3	Octahedron		60° 60° 60°	OCTAHEDRON Air	• Platonic solid • Kepler's Platonic solid model of the solar system, from Mysterium Cosmographicum (1596)
4	Dodecahedron			DODECAHEDRON the Universe	• Platonic solid • Kepler's Platonic solid model of the solar system, from Mysterium Cosmographicum (1596) • Sketch from Leonardo da vinci (1509)
5	Icosahedron		60° 60° 60°		• Buckminster Fuller's Structure • Common Virus
6	Truncated Icosahedron				• Sketch from Leonardo da vinci (1509) • Fullerene • Soccer ball • Geodesic structure (Honeycomb , Dandelion...)
7	Stellated dodecahedron		36° 72° 72° x12		• M.C. Escher
8	(Left) cuboctahedron (Right) rhombicubocta-hedron		60° 60° 60°	12 vertices cuboctahedron, 24 vertices rhombicuboctahedron, cuo, MOP-1, rco	• Metal-Organic Polyhedra
9	Truncated Icosahedron in truncated icosahedron				• Quasicrystal • Exploratorium architecture
10	Penrose Tiling				• Simian virus 40 (SV 40) • M.C. Escher

Fig. 9 Relationships between scholars of polyhedra

4. Data Analysis

The literature review and data collection showed the complex processes that varied knowledge of polyhedra has evolved by interacting each other throughout history. This section provides a more in-depth analysis of two specific cases to show the interdisciplinary ways that knowledge has evolved.

4.1. Leonhard Euler's Influence Over The Fields, "Integration"

Being an expert in various fields, Leonhard Euler was one of the world's greatest mathematicians who made deep and groundbreaking contributions to mathematics. One of his most famous formulas (1), which has recently been voted as the second-most beautiful theorem in a survey of mathematicians, describes the relationship between the number of vertices, edges, and faces of polyhedra [1]:

$$V - E + F = 2 \qquad (1)$$

Tracing the origins of Euler's polyhedral formula, we encounter several scholars from ancient Greece. Pythagoras and Theaetetus studied convex polyhedra mathematically and discovered their regularity. Euclid's *Elements* contains the theorem that there are only five regular polyhedra. Fascinated by these five definite shapes, Plato incorporated them into an atomic theory, believing them to be the basic elements of the world. After the long medieval "dark era," there was a revival of the knowledge in the Renaissance. Polyhedra and the skeletons of polyhedra were useful subjects for artists to show their mastery of perspective. Artists such as Piero della Francesca and Albrecht Durer wrote about perspective in polyhedra. Leonardo da Vinci, Luca Pacioli, Wentzel Jamnitzer, Jacopo de Barbari, Paolo Uccello, and Fra Giovanni da Verona featured polyhedra in their artworks. In the modern era, the regular polyhedra were used by Johannes Kepler (1671-1630) to create an early model of the solar system. René Descartes (1596-1650) also used the knowledge of Ancient Greece to devise a polyhedral formula. In turn, Kepler and Descartes had significant influence on Euler [20].

Influenced by various ancient and Renaissance scholars and artists, Euler's polyhedra theorem, in turn, had a significant impact on modern mathematics and science. Many scholars worked to prove and develop Euler's polyhedral formula. Cauchy's proof, in particular, offered an approach to polyhedra using the concepts of graphs and networks. Euler's Seven Bridges of Königsberg problem and polyhedron formula, along with the works of Cauchy and Johann Benedict Listing, contributed to the birth of topology, which would later become a crucial branch of mathematics. Topology provided a whole new way of thinking about shape and space and formed the foundation for the field of cosmology.

In the 20th century, Euler's polyhedral formula came to be utilized by scholars across diverse fields beyond the world of polyhedra. In computer chips, which are integrated circuits consisting of many tiny components connected by millions of conducting tracks, finding a suitable arrangement for the components in the integrated circuits is not an easy task. Euler's polyhedral formula has been used to solve this problem from the perspective of networks. Euler's formula and characteristic have also been applied to fullerenes, carbon molecules with a polyhedral structure, of which the most famous one, Buckminsterfullerene (C60), is named after Buckminster Fuller.

4.2. Buckminster Fuller's Influence Over The Fields, "Integration"

The design of Buckminster Fuller's geodesic dome, as explained, was based on various forms of knowledge. Fuller used the geometrical principle that triangles are twice as strong as rectangles. In order to enhance efficiency, he applied the principles of science in his design (which he called comprehensive anticipatory design science). In choosing the round shape, he considered aerodynamics and energy efficiency. Among others, the geodesic dome's design owed much to the Greek scholars including Pythagoras, Archimedes, Plato and the mathematicians of later generations [25].

The geodesic dome, in turn, has inspired many architects who have tried to build cost-effective and energy-efficient houses. One of his largest clients was the U.S. military that needed sturdy shelters that could be erected quickly for servicemen overseas. Today, there are hundreds of thousands of geodesic domes around the world, ranging from the Epcot Center at Walt Disney World to the Biosphere in Arizona to the humble solar greenhouse called the Growing Dome [27].

Fuller's inventions had a wide impact on many fields beyond architecture. The buckyball, named after him, paved the way for a new understanding of sheet materials and opened an entirely new chapter of nanoscience and nanotechnology [28]. The buckyballs, combined with graphene, known as an ideal material for next-generation electronics, have had a significant impact on smart electronics, enabling devices to run for longer and preventing them from breaking easily. This has contributed to the development of highly mobile organic devices and applications [29]. Scientists have produced inexpensive solar cells by combining buckyballs, nanotubes, and polymers [30]. Recently, polyhedra have also had a crucial impact on the field of "cluster physics" and specifically in on the development of nanoclusters. Nanoscientists have recently developed a technology to mass-produce more stable nano-silver using Platonic solids and buckyballs in diverse and creative ways [31], [32].

Applications of polyhedra can also be found in other fields. The soccer ball and the ball used in team handball, both of which use a spherical polyhedron analog to the truncated icosahedron, are good examples [33]. Polyhedra have also been used in shock absorbers [34] and interwoven Islamic geometric patterns [35]. In the field of computer science, a polyhedral model has been applied for compiler optimization [36]. Polyhedron theorems have also been used to find effective ways to arrange the wires in electric circuits. Biologists also found a way to block inflammation and prevent the spread of HIV by utilizing buckyballs.

We present our collected data in the chart below (Figure 10) which displays the relationships among diverse scholars of polyhedra. This chart shows the complex ways that philosophers, scientists, mathematicians, artists, and architects have influenced on each other in a single glance.

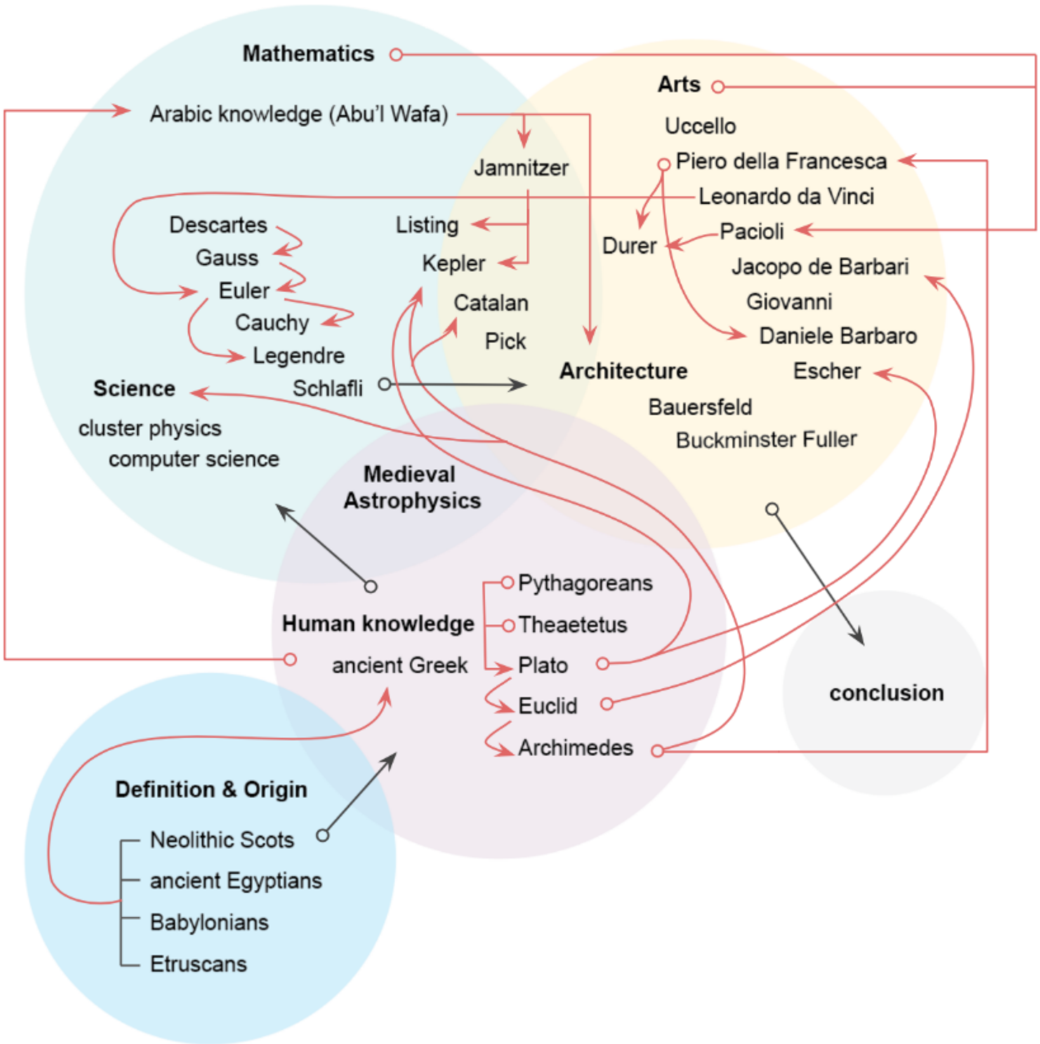


Fig. 10 How scholars of polyhedral influenced one another

5. Findings

As demonstrated in the data analysis section and displayed the above diagram, knowledge and theories of polyhedra have been developed by crossing over different fields and transcending the boundaries of space and time. Contrary to the popular belief that there is little relation between the arts and science, this paper shows the close associations between the two fields. More importantly, we have found that many great discoveries were amalgamative. Most of the key figures who have developed innovative applications of polyhedra were not confined to a narrow discipline. In the ancient, Renaissance and early modern eras, in particular, polyhedra were studied by interdisciplinary scholars whose interests lay in diverse fields. Even in the 20th and 21st centuries, when individual disciplines were firmly established, great innovations came from convergences of knowledge, as demonstrated by Euler and Fuller. These versatile innovators, in turn, had a multifaceted impact on diverse fields. The chart below (Figure 11), presenting our collected data, shows the relationships between scholars of polyhedra and how they have influenced each other.

By showing the mutual influences of diverse forms of knowledge, instead of simply surveying their development, this article provides a broad picture and a mechanism of how human knowledge has been accumulated and spread. Although we are familiar with knowledge of particular disciplines, much of our knowledge has evolved across diverse fields. In this long process, knowledge was often limited and proven wrong later. However, this knowledge carries importance for its impact on further knowledge. For example, Plato's idea of four regular polyhedra can be seen as philosophy rather than science as he did not prove it. However, his idea influenced various mathematicians, scientists, and artists. Kepler, who was one influenced by Plato's four elements theory, used the idea when he explained the orbits in which planets move around the sun. While Kepler's distance estimates proved to be wrong, his Platonic model suggested a possible mathematical synergy between microcosm and macrocosm and contributed to the development of modern science.

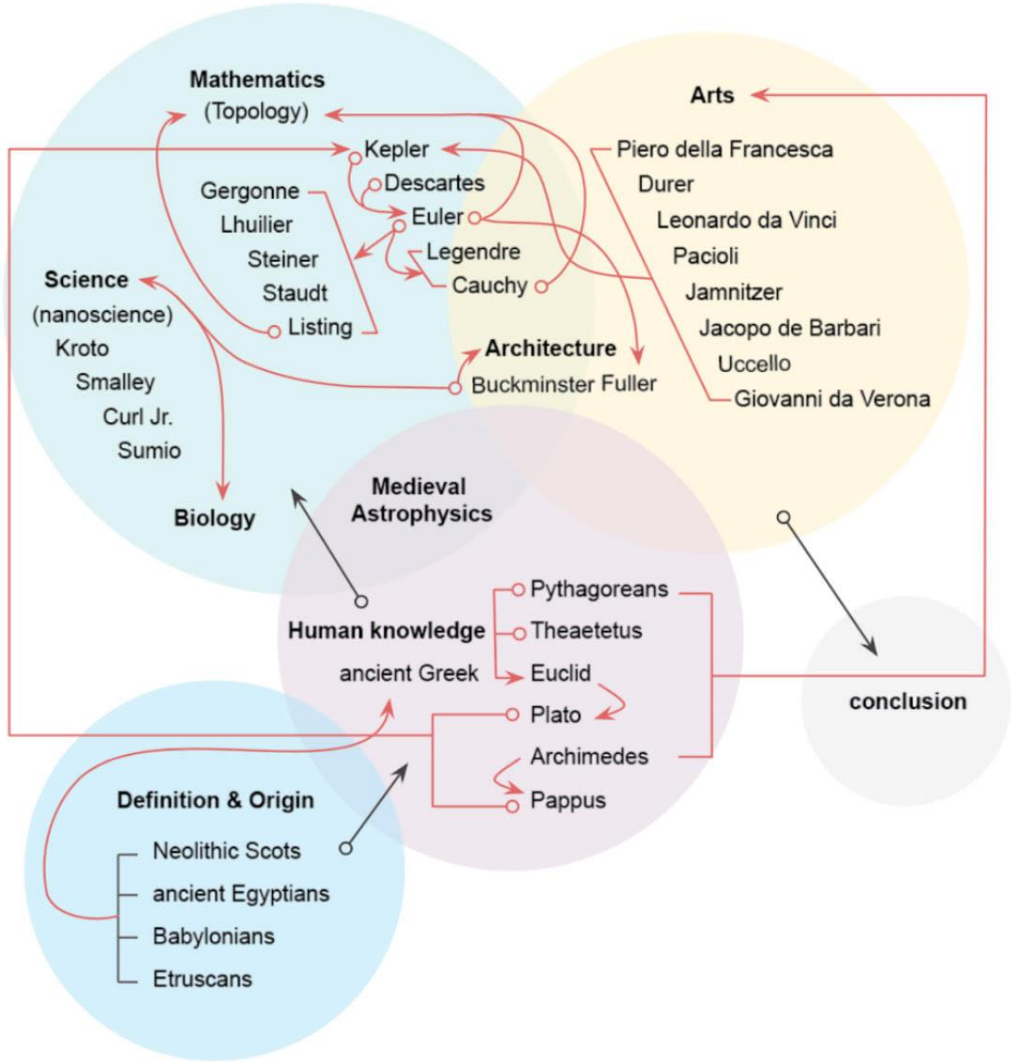


Fig. 11 The flow of polyhedral exhibition

6. Conclusion

Since the ancient era, polyhedra have attracted many thinkers who have tried to understand and explain the world in different ways. Some developed mathematical theories of polyhedral, while others made efforts to understand the cosmos using them. Many artists have also used polyhedra in their works. Scientists have been inspired by polyhedra and architects have created efficient structures using them. Recently, the knowledge has been actively utilized in diverse fields including nanoscience and computer science.

This history of knowledge of polyhedra can be an effective tool for teaching theories and concepts in the fields of mathematics, science, and arts. Especially, our findings could be used to help students more easily access to mathematical theorems and scientific theories. For example, a math activity entitled Geodesic Dome Challenge was designed and run as a countywide interschool challenge during the 2019–20 academic year with the goal of "providing context and fun to Math curriculum" for KS 2, 3, and 4 students [37].

The intriguing stories of polyhedra could inspire curators to design exhibitions showing how knowledge has been formed [38]. Representing the intriguing ways in which diverse figures in various fields influenced each other in the development of polyhedra can help understand how knowledge of science, art, mathematics, philosophy, and architecture has been crossed over throughout history [39]. The data collected in this paper would also provide useful content for such an exhibition. (Figures 12 and 13) Furthermore, this article will contribute to museum education by providing useful ideas and contents for curating an exhibition about the integration of art and science [40]. By demonstrating the specific ways in which scholars and artists derived inspiration and ideas from polyhedra, the exhibition could enhance its educational role. It is hoped that this article will offer ideas and insights to curators and educators.

Fig. 12 An example of a polyhedron design exhibition

Fig. 13 An example of a polyhedron design exhibition

References

1. D. S. Richeson, “Euler’s Gem“, Princeton University Press (2008)
2. G. Kiper, “Polyhedra: A historical review. Ankara – February 2007”, (2007)
    A. C. Sparavigna, "An Etruscan Dodecahedron" *arXiv preprint arXiv:1205.0706,* (2012) Retrieved from https://arxiv.org/abs/1205.0706
3. M. Senechal, ed, “Shaping space: exploring polyhedra in nature, art, and the geometrical imagination”, Springer Science & Business Media, (2013)
4. M. Beech, “The Large Hadron Collider: Unraveling the Mysteries of the Universe”, Springer Science & Business Media, (2010)
5. B. Artmann. In J. Milo (Ed.) *Eves' circles.* Washington, D.C.: Mathematical Association of America, (19 94)
6. H. Hisarligil, and B. B. Hisarligil, "The Geometry of Cuboctahedra in Medieval Art in Anatolia", *Nexus Network Journal* 20, no. 1, pp. 125-152, (2018)

7. S. Alvarez, "Polyhedra in (inorganic) chemistry", *Dalton Transactions* 13, pp. 2209-2233, (2005)
8. V. J. Cornell, "*Voices of Islam: Voices of art, beauty, and science*", Vol. 4. Greenwood Publishing Group, (2007)
9. B. G. Thomas, "*Form, Shape and Space: An Exhibition of Tilings and Polyhedra*", The University of Leeds International Textiles Archive (ULITA), (2007) Retrieved from https://www.ncetm.org.uk/public/files/217083/Form+Shape+and+Space+Exhibition.pdf
10. J. Bonner, "*Islamic geometric patterns: their historical development and traditional methods of constru ction*", Springer, (2017)
11. R. Sarhangi, "Illustrating Abu al-Wafā' Būzjānī: Flat Images, Spherical Constructions", *Iranian Studies* 4 1, no. 4, pp. 511-523, (2008)
12. R. Herz-Fischler, "*A mathematical history of the golden number*", Dover Publications, (1998)
13. P. R. Cromwell, "*Polyhedra",* Cambridge University Press, (1999)
14. G. W. Hart, "Paolo Uccello's Polyhedra", (1998a) Retrieved from https://www.georgehart.com/virtual-polyhedra/uccello.html
15. G. W. Hart, "Piero della Francesca's Polyhedra" (1998b) Retrieved from https://www.georgehart.com/virtual-polyhedra/piero.html
16. F. J. Swetz, "Leonardo da Vinci's Geometric Sketches", *Convergence,* (2010) DOI:10.4169/loci002559
17. G. W. Hart, "Leonardo da Vinci's Polyhedra", (1998c) Retrieved from http://www.georgehart.com/virtual-polyhedra/leonardo.html
18. J. J. O'Connor , and E. F. Robertson, "Luca Pacioli. *Mathematics History*", University of St Andrews, Scotland, (1999)
19. F. J. Swetz, "Mathematical treasure: Wenzel Jamnitzer's Platonic solids", *Convergence*, (2013) Retrieved from https://www.maa.org/press/periodicals/convergence/leonardo-da-vincis-geometric-sketches-introduction
20. T. Tarnai, J. Krahling, and S. Kabai, "Star polyhedra: From St Mark's Basilica in Venice to Hungarian Protestant Churches", Proceedings from *The 13th International Conference on Geometry and Graphics*, Dresden, Germany: Technische Universität, (2008)
21. W. G. Hart, "Miscellaneous Polyhedral Art" (1998d) Retrieved from https://www.georgehart.com/virtual-polyhedra/art-misc.html
22. M. DiChristina, "Elliptical dome", *Popular Science, (1990)*
23. T. Horton, "The dome", *Mother Earth News,* (1999, June/July)
24. H. K. Lee, "Integrative ways that the knowledge of polyhedral have developed in history: Implications for art and design education", *International Journal of Education and Learning*, 9, no. 2, pp. 27-34, (2020)
25. M. Emmer, and D. Schattschneider, ed, "*M.C. Escher's Legacy: A Centennial Celebration*", Springer, (2005)
26. N. Fischer, "The Energy Efficient Geodesic Dome Design", (2012) Retrieved from https://www.builddirect.com/blog/the-energy-efficient-geodesic-dome-design/
27. D. R. M. Walton, and H. W. Kroto, "Fullerene: CHEMICAL COMPOUND", (1998), *Encyclopedia Britannica*, Updated (2004) Retrieved from https://www.britannica.com/science/fullerene
    A. Zettl, and C. Ojeda-Aristizabal, "Buckyball Marries Graphene", *Office of Science Website*, (2018) Retrieved from https://www.energy.gov/science/bes/articles/buckyball-marries-graphene
28. Understanding Nanotechnology (n. d.), "Buckyballs: Uses and Applications." Retrieved from https://www.understandingnano.com/buckyballs-fullerenes.html
29. H. Yang, Y. Wang, H. Huang, et al., "All-thiol-stabilized Ag44 and Au12Ag32 nanoparticles with single-crystal structures", *Natature Communications* 4, no. 1, pp. 1-8, (2013) https://doi.org/10.1038/ncomms3422
30. Z. Wang, et al. (2017). , "Assembly of silver Trigons into a buckyball-like Ag180 nanocage", *Proceedings of the National Academy of Sciences* 114, no. 46, pp. 12132-12137, (2017) DOI:10.1073/pnas.1711972114
31. E. Gibney, "First 'Buckyball' Molecules Created from Boron", (2014) Retrieved from https://www.scientificamerican.com/article/first-buckyball-molecules-created-from-boron/
32. U.S. Patent and Trademark Office, *Official Gazette* vol. 1007. No. 1-2, (1981)
33. C. S. Kaplan, (n. d.) "Interwoven Islamic geometric patterns." Retrieved from http://isohedral.ca/interwoven-islamic-geometric-patterns/
34. U. K. Bondhugula, "Effective Automatic Parallelization and Locality Optimization Using The Polyhedral Model", *Ph.D. Dissertation*, Computer Science and Engineering, Ohio State University, Springer, pp. 182–183, (2008), ISBN 978-3-540-28849-7.
35. Medway Education Business Partnership, "Geodesic Dome Challenge.", (2019) Retrieved from http://www.mebp.org/infogeodome.php
36. Younwoo Lee, Sang-Hee Lee.A Study on Teachers' Perceptions of the Developmental Characteristics and

Educational Supports for At-risk Young Children from Multicultural Families in South Korea. *International Journal of Child Warfare Promotion and Management*. Vol. 2. No. 2. Oct. 2018.GVPress. pp:1-6.http://dx.doi.org/10.21742/IJCWPM.2018.2.2.01

37. H. K. Lee, and M. K. Kim, "Importance of Design Museum from the Perspective of Millennium Generation", *International Journal of Art and Culture Technology*, 4, no.2, (2020)
38. K. Koo, Y. Heo, H. Lee, "A Basic Study on the Spatial Cognition Analysis for the Prevention of Museum Robbery", International Journal of Smart Home, 14, no.2, (2020)
39. KHATOON, KAREEMA. "DESIGN AND PERFORMANCE EVALUATION OF POLYHEDRAL SWEPT BACK HIGH WING CHUCK GLIDER FOR BETTER L/D RATIO." International Journal of Mechanical and Production Engineering Research and Development (IJMPERD) 8.3 (2018) 1-12
40. Menon, V. A. I. S. H. A. L. I., and S. KAUVERY Bai. "Exploring the Influence of Cubism Art Movement on Fashion." International Journal of Textile and Fashion Technology (IJTFT) 8.1 (2018): 17-26.
41. Singh, Mamta, and Rajendra Singh. "Art And Architectureof Jain Temples In Uttar Pradesh, India." International Journal of Humanities and Social Sciences (IJHSS) 4.4 (2015): 71-82
42. MISHRA, AKHAYA KUMAR. "SCULPTURAL ART OF JAINS IN ODISHA: A STUDY." International Journal of Humanities And Social Sciences (IJHSS) 6.4 (2017) 115-126
43. Arora, Harcharan Singh. "THE CHARISMATIC ART OF ARTICULACY AND ORATORY IN SHAKESPEARE'S SELECT PLAYS-AN OVERVIEW." International Journal of Linguistics and Literature (IJLL) 7.4 (2018) 25-32
44. THONGNOPPAKUN, WARANGKANA, and CHOKCHAI YUENYONG. "DEVELOPING PRESERVICE SCIENCE TEACHERS'PEDAGOGICAL CONTENT KNOWLEDGE USING CORES-BASED ACTIVITY." International Journal of Educational Scien