A Dual Volatility Conditioning with Moderation in the Hedge Fund Return Process

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Abstract: One-month prior role of mediation by market momentum on the effect of market excess return to its contemporaneous hedge fund performance through the one-month prior level of VIX as the primary moderator is moderated if the indirect effect of the one-month prior market momentum depends on the return magnitudes of one-month prior VIX as the secondary moderator. This paper adopts quantitative approaches with behavioral implications to financial time-series applying the multiple moderator-conditional process models. Consistent with our view that hedge fund managers exhibit different skill sets in translating the implied volatilities especially in the equity market, we document some heterogeneity of processing momentum implications by dynamically adjusting risk-in/off behavior across hedge fund strategies. We subsequently show the test methodology when the indirect effect on the returns of various hedge fund strategies by market risk premium is moderated by any factor whenever two different moderators are switching their primary and the secondary roles of dependency.

Keywords: Conditional Moderated Mediation Effect, Indirect Effect, Hedge Fund Investment Style Index

1. Introduction

Investors consider many systematic risk premiums that empirically explain the cross-section of diversified portfolio returns. These factors are the unique source of returns that explain the performance of diversified portfolios. While many factors have been analyzed in academic as well as practitioners' literature, the market, the value, the size, the momentum, the volatility, the credit, and the term factors are relatively well-adopted in practice.

As the emergence of risk factors and their cross-sectional effects to stock returns were identified, the same risk factors are also applied to the literature of hedge fund performance as well as risk attributions. The literature on this topic in both equity and debt markets are voluminous. To be more specific, Fama [9] mentioned that a fund manager has stock selection and market timing skills. The former is about the ability to select the right securities on a certain risk level and the latter is about the ability to forecast the potential market movement to adjust the risk-on positions on or before the right moment. It is generally conceived that hedge funds employing dynamic trading strategies have time-varying risky asset exposures to generate option-like return profiles. Since they are lightly regulated and more flexible to engage in leverage transactions based on their short-selling capacities and the derivative exposures, hedge funds are better positioned for factor timing than the rest of traditional money managers. Unlike traditional investment portfolio diversification at the asset-class level, factor-based investing involves identifying compensated factor exposures and constructing a portfolio by these factor allocations to harvest factor premia, rather than attempting to uncorrelated pure alpha (or the active manager skills).

A better understanding exists in capital market research when we can claim how some market sentiment factors affect a specific hedge fund return. A mediation model is a causal model that any efficient econometric inference requires something more than a mere statistical linkage between and among the focal and outcome variables. Empirical work on volatility weighting was done by Hallerbach [14] for weighting a strategy by its trading volatility. Cho [4] tested various combinations of time-series convergence and divergence trades in the KOSPI 200 futures market based on the statistical strength of the publicly available implied volatility index-based ("VKOSPI") trading signals out of the reconstructed entropy algorithms. Recently, Cho [3] studied the information contents of the VIX and the VKOSPI in global hedge fund index returns by applying Fama and French [10]'s equity risk factors and its potential diversification implications to Korean equity investors.

We further distinguish between the level and the log returns in one-month prior volatility information to conditioning the dual moderation model as the primary and secondary moderators to one-month prior market momentum factor to mediate the contemporaneous excess market returns to the current excess performances of hedge fund styles. This design is practical in a way by combining both contemporary and one-month prior market risk factors in the same analysis. For instance, where the returns of hedge fund strategies and the excess market performances are contemporaneous, all other systematic risk factors adopted such as market momentum (WML_{t-1}), the level (VIX_{t-1}), and the size of return ($DVIX_{t-1}$) information of the VIX are one-month prior. At the index level as the aggregates of individual hedge funds' net-of-fee returns, the results suggest that different hedge fund investment styles show ample variation in conditioning on the information of one-month prior implied volatility returns: Investment styles such as emerging market (EM), equity long-short (LS), fund of hedge funds (FoF), and multi-strategies (MS) show relatively more significant in conditioning on the contemporaneous excess market returns measured by the indirect (or mediated) effect of one-month prior market momentums as moderated through the level and the size of one-month prior Chicago Board of Options Exchange implied volatilities versus other styles such as global macro (GM) and commodity trading advisors (CTA).

The current study is similar in spirit to the multi-factor performance attribution and conditional process analysis that we apply the basic intuition and methodology from Fama and French [10] and Hayes [15] to hedge fund style time-series. However, we differ from them in several important aspects. In this article, even though we test a moderation of moderated mediation hypothesis based on stochastic mediation and moderation literature, we maintain our attention in the connection among the known variables such as one-month prior market momentum, one-month prior level and return information of the VIX to the excess returns of contemporaneous hedge fund style indices: Whether this time-series connection can be explained by how and in what circumstances one-month prior volatility signals such as levels and returns might further capture information about one-month prior market momentum factor's conditional sensitivities to the contemporaneous, thus still unknown, hedge fund sub-index returns.

The general intuition is as follows: Out of the world of the CAPM, there are many alternative efficient portfolios and investment managers are looking to increase their return by timely tilting their portfolio towards various risk-on equivalent positions per their degree of risk aversion to garner more risk premium. This raises extra demand for risky assets and reduces demand for risk-free assets, which may explain a money manager's market-timing behavior. Certain investment styles of hedge fund managers may be better than others at identifying market concerns. These hedge fund managers are under the environment of less strictly regulated and are more flexible to engage in active trades such as short-selling and leveraging, and should be better positioned to dynamically adjust their risky asset exposures to exercise their timing ability. When the customized forecast shows a positive market sentiment, hedge fund managers of a certain investment style will be more willing and actively increase the fund's exposure to the market factor or some other specific systematic risk factors such as market momentum.

As Frijns, Gilbert, and Zwinkels [11] discussed, the collective mutual fund managers included in the style indices may adjust their portfolio factor exposures conditioned to past factor returns rather than future returns, which is detrimental to their outperformance. If the customized a priori factor generating process shows a certain degree of (un)favorable market momentum, the hedge fund managers would tilt the fund's exposure to or from the momentum factor, under the influence of the simultaneous conditionings by prior month's level and the returns of implied volatilities, as a reliable source of general market sentiment. How these savvy and talented hedge fund managers in the heterogeneous investment style spectrum might differ in their interpretations of the moderated information contents in both one-month prior level and returns of the VIX to another one-month prior market momentum's mediation effects from contemporaneous excess market returns to contemporaneous hedge fund performance? It is generally conceived that at or near the extreme level of volatilities, money managers exhibit some usually crowded contrarian trading behavior, then it is the momentum, rather than mean-reversion trades benefits more. However, it is not clear whether the past level or the past returns of the implied volatilities better conditions the past market momentum concerning one-month posterior hedge fund return dynamics.

While we separately consider the correlation between the one-month prior market momentum, the level, and the return information of one-month prior VIX as potential explanatory variables, our focus is on incorporating these insights from interplays into a practical implication that can be adopted by hedge fund investors to recap the hedge fund managers' collective behavior on the past momentum, the level, and return information of implied equity market volatilities. Our conditional process model attempts to provide some insights into the interplays among them by offering interpretations of two- and three-way interaction terms. This study also considers whether the dual moderated mediation effect among the focal variables can add some behavioral understanding for investors in selecting hedge fund strategies.

The remainder of this article is structured as follows: The factor modeling in dual moderated mediation analysis, data analysis, and test results are presented in section 2, the statistical inferences on the indices of conditional and dual moderated mediations are discussed in section 3, and section 4 concludes.

2. Developing A Dual Moderated Mediation Factor Model

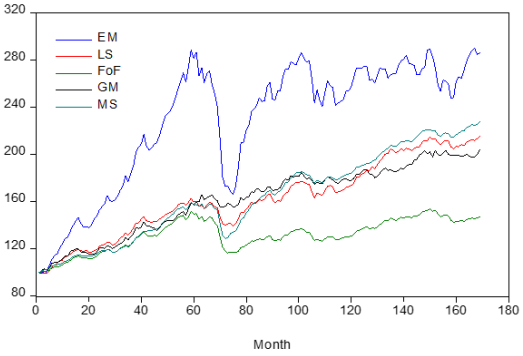
The usual factor risk models do not systematically applicable to single hedge funds. The choice of the representative average hedge fund returns in this study is the Barclays Hedge Fund Index ("BarCap") and its five main strategy classification sub-indices such as emerging market (EM), equity long-short (LS), fund of hedge funds (FoF), global macro (GM), and multi-strategies (MS). Since this study will focus on the afore-mentioned five distinct, equity-biased investment styles, the trades such as fixed income arbitrages and commodity trading advisors are excluded from the sample. As Cho [5] elaborated in his study, the BarCap indices are highly diversified, which might be helpful to reduce the potential noise of fund-specific return variation commonly observed at the levels of individual hedge funds. Because hedge funds are typically classified into diverse categories by investment style, we naturally expect that the different investment styles might show different factor moderation mechanism. By focusing on each particular style, we examine whether the investment style differences are linked to the magnitude of the factor-tilting through dual moderation dynamics by hedge fund managers.

Exhibit 1 shows the summary statistics for five BarCap sub-indices from January 2003 to December 2016. While all five styles demonstrate positive annualized excess returns, the average Sharpe ratio (excess returns divided by realized volatility) across style indices varies from 1.14 (MS) to 0.41 (FoF), due to FoF's lower

annualized returns during the sample period versus other investment styles. All return distributions are leptokurtic mostly pronounced at MS (+9.98), and have negative skewness, except for GM (+0.25). Its cumulative return is seen in the right-hand panel of Exhibit 1 that illustrates the hypothetical growth of $100 invested in February 2003 in the respective style indices.

Exhibit 1. Summary Statistics for the BarCap Sub-Indices per Investment Style in the Sample

	EM	LS	FoF	GM	MS
Total Return	186.3%	115.5%	47.4%	103.8%	127.6%
Ann. Return	7.8%	5.6%	2.8%	5.2%	6.0%
Ann. Volatility	11.5%	5.1%	5.0%	4.9%	4.7%
- Gain Stdev	6.5%	2.7%	2.2%	3.4%	2.4%
- Loss Stdev	9.7%	3.6%	4.7%	2.4%	5.4%
Sharpe Ratio	0.61	0.96	0.41	0.90	1.14
Skewness	(0.99)	(0.71)	(1.65)	0.25	(2.45)
Exc. Kurtosis	0.81	(2.06)	2.33	(2.98)	9.98
Best Month Return	11.1%	3.8%	3.0%	4.4%	3.9%
Worst Month Return	-15.6%	-4.9%	-6.8%	-3.3%	-7.6%
% Positive Months	66.1%	65.5%	64.9%	61.3%	73.8%
Avg. Draw-Down	-7.5%	-2.0%	-6.7%	-4.5%	-1.6%
Beta to S&P 500	0.62	0.29	0.25	0.17	0.22
Max Drawdown	-42.5%	-14.2%	-23.2%	-29.1%	-19.3%



Cho and Kim [2] test for the behavioral aspect of market timing of domestic hedge funds by newly introducing the concept of style-tilting volatility (STV) as a measure to quantify the amount of variation or dispersion of a set of individual fund's style exposures. Their method implies that market timing ability demonstrates a convex relation between fund performance and the various style as well as market factors. The analysis makes use of daily returns instead of typical hedge fund monthly returns to increase their statistical power (and the subsequent noise as well). The study evaluates the relationship between the three market-timing indicators such as STV, Volatility Timer, and Treynor-Mazuy type Market Timer, to the excess returns and Sharpe ratios of reclassified quintile hedge fund groups to identify those talented active styles and volatility timers. Some hedge funds demonstrated enhanced risk-adjusted returns through a wider range of volatility timing behavior, while their active style bets did not necessarily result in persistent outperformance compared to the peer managers.

Among Fama and French [10]'s six global equity risk factors[1], this study adopts both contemporaneous and one-month prior market excess returns, one-month prior market momentum factor together with the VIX. Global factor definitions are consistent with a global market for risk, where hedge funds operate. The analysis here intentionally excluded the usual size (SMB), value (HML), quality (RMW), and conservativeness (CMA) factors to specifically concentrate the interplay between and among contemporaneous market excess returns, one-month prior market momentum, and the one-prior implied volatility dynamics. Some risk factors didn't come up with explicitly definable returns and this rather a parsimonious approach identifies alpha as a valid signal of risk-adjusted returns. However, identifying the existence of factor-conditioned alpha based on various risk factor models per hedge fund style is not the major aim of this experiment. Goldwhite [13] suggested a hedge fund style selection framework based on the level (not the return) of the implied volatility. Our study adopts both the levels and the returns of one-month prior VIX as the representative equity market risk sentiment measures.

Based on these empirical backgrounds, we now demonstrate the performance of a simple base case model. Exhibit 2 presents the estimation results of our five-factor model estimated on all five BarCap sub-indices per investment style from January 2003 to December 2016, encompassing a couple of periods of market stress of the U.S. sub-prime mortgage and the fiscal crisis in the Eurozone, offering relatively long data of stylized hedge fund returns. The regression of excess monthly hedge fund returns against five factors performed according to Equation (1). A zero-financing premium is defined in case of WML_{t-1}. The multi-factor linear regression in Exhibit 2 is our baseline results of cross-sectional hedge fund style performance.

[Eq. 1] $$\boldsymbol{Y_t = \alpha_t + \beta_{1t} MKT_t + \beta_{2t} MKT_{t-1} + \beta_{3t} WML_{t-1} + \beta_{4t} VIX_{t-1} + \beta_{5t} DVIX_{t-1} + e_t}$$

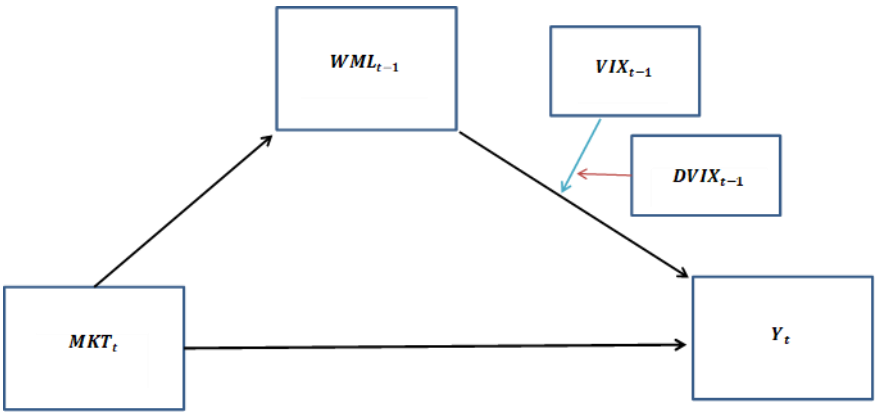
Exhibit 2. Coefficients from OLS Results

Coefficients	Emerging Markets (EM)	Equity Long-Short (LS)	Fund of Funds (FoF)	Global Macro (GM)	Multi-Strategies (MS)
Constant	0.416	0.268	0.484 **	0.342	0.081
MKT_t	0.626 ***	0.280 ***	0.247 ***	0.189 ***	0.213 ***
MKT_{t-1}	0.047	0.035 *	0.056 **	-0.031	0.085 ***
WML_{t-1}	-0.080 *	-0.020	0.001	-0.042	-0.001
VIX_{t-1}	-0.010	-0.001	-0.024 **	-0.001	0.010
$DVIX_{t-1}$	-0.004	0.000	-0.002	-0.004	0.000
R^2	0.744	0.767	0.686	0.355	0.616

***, **, * Significance at 99.9%, 95%, 90%, respectively

In Exhibit 2, the alpha is measured to be positive and significant only FoF_t. The alpha (or active manager skill) measures the excess return, controlling for the risk premia associated with simply being exposed to these systematic risk premiums. While the exposure to the market (X_t or MKT_t) is significantly positive for all style indices, one-month prior market excess returns (X_{t-1} or MKT_{t-1}) as the factor of stale pricing measure of hedge fund returns contribute significant positive impacts to MS_t ($p < 0.01$), FoF_t ($p < 0.05$), and LS_t ($p < 0.1$) sub-indices as well. The exposure to the mediation variable (M_{t-1} or WML_{t-1}) and two moderation variables (V_{t-1} or VIX_{t-1}) and (Q_{t-1} or $DVIX_{t-1}$) are mostly negative and failed to be significant, except for FoF_t ($p < 0.05$) sub-index. The model explanatory power measured in R^2 ranges from relatively low at 0.355 (GM_t) to fairly high at 0.767 (LS_t) and 0.744 (EM_t) sub-indices. Back to our main theme of dual moderated mediation modeling, the moderation of the indirect effect can be explained by adopting some instruments of multiple interactions in either independent variable (X_t or MKT_t), mediation variable (M_{t-1} or WML_{t-1}), or two moderators such as (V_{t-1} or VIX_{t-1}) and (Q_{t-1} or $DVIX_{t-1}$) in the latter aspect of the mediation as can be seen from the conceptual diagram at Exhibit 4. Krieger and Sarge [2013] applied in their analysis of potential mediators of the link between vaccine framing and people's behaviors with this second stage dual moderated mediation model.

Exhibit 3. Conceptual Diagram of the Second Stage Dual Moderated Mediations (Equation 2 and 4)



The model in Exhibit 2 can be represented with Equation 2 and Equation 4. The effect[2] of (X_t or MKT_t) on (M_{t-1} or WML_{t-1}) is summarized in Equation 4 and depends on both (V_{t-1} or VIX_{t-1}) and (Q_{t-1} or $DVIX_{t-1}$), whereas the effect of (M_{t-1} or WML_{t-1}) on (X_t or MKT_t) is the coefficient ($\boldsymbol{a_t}$) from Equation 2. Equation 3 further includes additional predictors of (X_{t-1} or MKT_{t-1}), one-month prior excess market return, or a stale-pricing factor to account for compounding effects without necessarily denoting in our conceptual diagram in Exhibit 3.

[Eq. 2] $$\boldsymbol{M_{t-1} = i_{Mt-1} + a_t X_t + e_{Mt-1}}$$

[Eq. 3] $$\boldsymbol{Y_t = i_{Yt} + c_t' X_t + c_{t-1}' X_{t-1} + b_{it} M_{t-1}}$$

[Eq. 4]
$$\begin{aligned}\boldsymbol{Y_t} &= \boldsymbol{i_{Yt} + c_t' X_t + c_{t-1}' X_{t-1} + b_{1t} M_{t-1} + b_{2t} V_{t-1} + b_{3t} Q_{t-1} + b_{4t} M_{t-1} V_{t-1} + b_{5t} M_{t-1} Q_{t-1}} \\ &\quad \boldsymbol{+ b_{6t} V_{t-1} Q_{t-1} + b_{7t} M_{t-1} V_{t-1} Q_{t-1} + e_{Yt}} \\ &= \boldsymbol{i_{Yt} + (b_{1t} + b_{4t} V_{t-1} + b_{5t} Q_{t-1} + b_{7t} V_{t-1} Q_{t-1}) M_{t-1} + c_t' X_t + c_{t-1}' X_{t-1} + b_{2t} V_{t-1}} \\ &\quad \boldsymbol{+ b_{3t} Q_{t-1} + b_{6t} V_{t-1} Q_{t-1} + e_{Yt}} \\ &= \boldsymbol{i_{Yt} + (b_{1t} + b_{4t} VIX_{t-1} + b_{5t} DVIX_{t-1} + b_{7t} VIX_{t-1} DVIX_{t-1}) WML_{t-1} + c_t' MKT_t} \\ &\quad \boldsymbol{+ c_{t-1}' MKT_{t-1} + b_{2t} VIX_{t-1} + b_{3t} DVIX_{t-1} + b_{6t} VIX_{t-1} DVIX_{t-1} + e_{Yt}}\end{aligned}$$

Y_t is the monthly BarCap hedge fund returns over the risk-free rate ($EM_t, LS_t, FoF_t, GM_t, MS_t$), M_{t-1} is WML_{t-1}, X_t is MKT_t, V_{t-1} is VIX_{t-1}, Q_{t-1} is $DVIX_{t-1}$. One-month Treasury bills returns as the proxy of risk-free rates, MKT_t is the Fama-French excess market return, WML_{t-1} is the one-month prior return of Fama-French global market momentum factor, VIX_{t-1} is the level of one-month prior CBOE implied volatility index as a primary moderator, and $DVIX_{t-1}$ is the return of one-month prior VIX (thus, return between VIX_{t-1} and VIX_{t-2}) as a secondary moderator in this setting. The errors in Equation 2 and 3 for the estimation of WML_{t-1} and Y_t assumed to be i.i.d. with zero means.

The direct effect of MKT_t on Y_t is estimated with $\boldsymbol{c_t'}$ in Equation 3, which quantifies incremental changes in MKT_t are affecting Y_t, independent of the relevant forces from WML_{t-1} on Y_t. In Equation 3, the effect of Y_t on WML_{t-1} is unmoderated. Then, Equation 5 subsequently shows the combined effect ($\boldsymbol{\theta_{M_{t-1}\to Y_t}}$) of WML_{t-1} on Y_t.

$$[Eq.5]\boldsymbol{\theta_{M_{t-1}\to Y_t}} = \boldsymbol{b_{1t} + b_{4t}V_{t-1} + b_{5t}Q_{t-1} + b_{7t}V_{t-1}Q_{t-1}}$$
$$= \boldsymbol{b_{1t} + b_{4t}VIX_{t-1} + b_{5t}DVIX_{t-1} + b_{7t}VIX_{t-1}DVIX_{t-1}}$$

The indirect effect of MKT_t is the outcome of dual forces as in Equation 6.

$$[Eq.6]\ \boldsymbol{a_t\theta_{M_{t-1}\to Y_t}} = \boldsymbol{a_tb_{1t} + a_tb_{4t}V_{t-1} + a_tb_{5t}Q_{t-1} + a_tb_{7t}V_{t-1}Q_{t-1}}$$
$$= \boldsymbol{a_tb_{1t} + (a_tb_{4t} + a_tb_{7t}Q_{t-1})V_{t-1} + a_tb_{5t}Q_{t-1}}$$
$$= \boldsymbol{a_tb_{1t} + (a_tb_{4t} + a_tb_{7t}DVIX_{t-1})VIX_{t-1} + a_tb_{5t}DVIX_{t-1}}$$

Finally, $\boldsymbol{a_tb_{7t}}$ quantifies the sensitivities of the indirect effect in MKT_{t-1} since both VIX_{t-1} and $DVIX_{t-1}$ are changing. According to Hayes [2018], both $\boldsymbol{a_tb_{4t}}VIX_{t-1} + \boldsymbol{a_tb_{7t}}VIX_{t-1}DVIX_{t-1}$ and $\boldsymbol{a_tb_{5t}}DVIX_{t-1} + \boldsymbol{a_tb_{7t}}VIX_{t-1}DVIX_{t-1}$ terms are the "indices of partial moderated mediation" of MKT_t's indirect effect by VIX_{t-1} and $DVIX_{t-1}$, which "quantify the relationship between one moderator and the size of MKT_t's indirect effect on Y_t through WML_{t-1} when the second moderator is held constant."

If we draw the relationship between $DVIX_{t-1}$ and an indirect effect of MKT_t, with VIX_{t-1} on the horizontal axis, $(\boldsymbol{a_tb_{4t}} + \boldsymbol{a_tb_{7t}}DVIX_{t-1})$ in Equation 6 summarizes the sensitivities of the indirect effect of MKT_t on Y_t when VIX_{t-1} varies, concerning fixed $DVIX_{t-1}$. $\boldsymbol{a_tb_{4t}}$ estimates how the indirect force of MKT_t changes with VIX_{t-1} when $DVIX_{t-1}$ equals to zero and $\boldsymbol{a_tb_{7t}}$ quantifies how that moderation of the indirect effect of MKT_t by VIX_{t-1} as $DVIX_{t-1}$ changes. With a little algebra, Equation 6 can be modified as in Equation 7, concerning the linear association between WML_{t-1} and MKT_t is a function of $DVIX_{t-1}$: $(\boldsymbol{a_tb_{5t}} + \boldsymbol{a_tb_{7t}}VIX_{t-1})$.

$$[Eq.7]\ \boldsymbol{a_t\theta_{M_{t-1}\to Y_t}} = \boldsymbol{a_tb_{1t} + a_tb_{4t}V_{t-1} + a_tb_{5t}Q_{t-1} + a_tb_{7t}V_{t-1}Q_{t-1}}$$
$$= \boldsymbol{a_tb_{1t} + (a_tb_{5t} + a_tb_{7t}V_{t-1})Q_{t-1} + a_tb_{4t}V_{t-1}}$$
$$= \boldsymbol{a_tb_{1t} + (a_tb_{5t} + a_tb_{7t}VIX_{t-1})DVIX_{t-1} + a_tb_{4t}VIX_{t-1}}$$

A logic of symmetry exists to this dual moderated mediation model because Equation 6 can be written in an alternative form as in Equation 7, where $DVIX_{t-1}$ is the first moderator, VIX_{t-1} is the second moderator, and $(\boldsymbol{a_tb_{5t}} + \boldsymbol{a_tb_{7t}}VIX_{t-1})$ is "the index of conditional moderated mediation" by $DVIX_{t-1}$. It demonstrates the conditioned indirect effect of MKT_t by $DVIX_{t-1}$ moderated on the value of VIX_{t-1}. Therefore $\boldsymbol{a_tb_{7t}}$ is symmetrical to either VIX_{t-1} or $DVIX_{t-1}$ is the first or second moderator. If $\boldsymbol{a_tb_{7t}}$ is not zero, VIX_{t-1} moderates the moderation of the indirect effect of MKT_t by $DVIX_{t-1}$ or $DVIX_{t-1}$ moderates the indirect effect of MKT_t by VIX_{t-1}.

3. Statistical Inference

In Exhibit 4, there is a three-way interaction term (WML_{t-1}*VIX_{t-1}*$DVIX_{t-1}$). Probing this interaction reveals how one-month prior VIX level moderates the effect of the contemporaneous excess market returns on one-month prior market momentum per size of one-month before VIX returns, which is the inference methodology adopted in this article.

Exhibit 4. Dual Moderated Mediation Model Estimates

Coefficients	Emerging Markets (EM)	Equity Long-Short (LS)	Fund of Funds (FoF)	Global Macro (GM)	Multi-Strategies (MS)
Constant	0.2009	0.2087 **	0.0288	0.2445 **	0.3367 ***
MKT_t	0.622 **	0.2866 ***	0.2475 ***	0.19863 ***	0.2049 ***
MKT_{t-1}	0.0195	0.0529 **	0.0582 **	-0.0104	0.0661 **
Direct M (C'_{1t}+C'_{t-1})	-0.3553 ***				
M (WML_{t-1})	-0.0427	-0.0341 *	-0.0103	-0.0419	-0.0025
V (VIX_{t-1})	-0.0200	-0.0044	-0.02634 *	-0.0057	0.0098
Q ($DVIX_{t-1}$)	-0.0066	0.0010	-0.0011	-0.0035	-0.0003
WML_{t-1}*VIX_{t-1}	-0.005828 **	0.0014	0.0006	-0.0001	-0.0008
WML_{t-1}*$DVIX_{t-1}$	-0.0031	-0.00158 *	-0.00256 ***	-0.0004	-0.00286 ***
VIX_{t-1}*$DVIX_{t-1}$	-0.0004	0.0004	-0.0002	0.0009	-0.0009
WML_{t-1}*VIX_{t-1}*$DVIX_{t-1}$	0.00033 ***	0.00011 **	0.000164 ***	0.0000	0.000181 ***
R^2	0.7555	0.7778	0.6975	0.3709	0.6434
R^2 Increment	0.0073 ***	0.004 *	0.0096 ***	0.0008	0.01359 ***
F(1, 157, HCO) Stat.	9.1630	3.6880	7.9350	0.2386	7.8930

***, **, * Significance at 99.9%, 95%, 90%, respectively with Huber-White Heteroskedasticity consistent standard errors

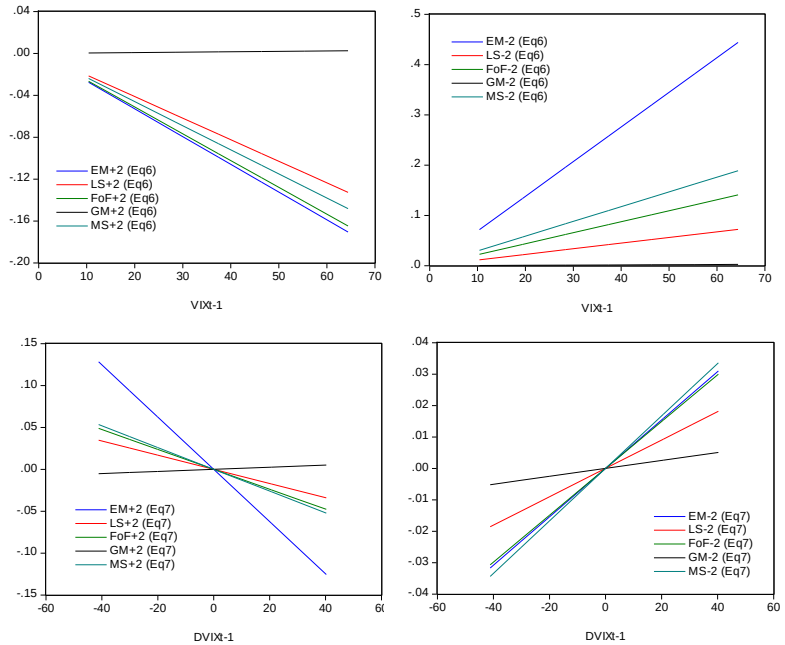
The constant reported in Exhibit 4 indicates the excess returns (or alphas) after controlling for the three-way interaction term or dual moderated mediation effects. While the only alpha of FoF_t index before the controlling of the dual moderated mediation effect in Exhibit 2 was significantly positive, the alphas of LS_t and MS_t sub-indices were become significantly positive at a 5% level after controlling for the dual moderated mediation effects. On the

other hand, the alpha of FoF_t index after the controlling of the dual moderated mediation effect becomes insignificant. The dual moderated mediation effect nevertheless drives the FoF_t alpha to be insignificant. This is another expression that the dual moderated mediation effect can explain the FoF_t index and an illustration of the importance of double-layer fees in the space.

In terms of improvement in model fit, the BarCap EM_t, LS_t, FoF_t, and MS_t sub-indices are statistically significant after the three-way interaction term is included. The incremental increase in R^2 due to allowing the moderation of X_t's effect by ($WML_{t-1} * VIX_{t-1} * DVIX_{t-1}$) is ΔR^2 = 0.0073, F(1, 157) = 9.163, p = 0.0029 for the BarCap EM_t sub-index, ΔR^2= 0.0040, F(1, 157) = 3.688, p = 0.0566 for LS_t index, ΔR^2= 0.0096, F(1, 157) = 7.935, p = 0.0055 for FoF_t index, ΔR^2= 0.0008, F(1, 157) = 0.238, p = 0.625 for GM_t index, and ΔR^2= 0.0136, F(1, 157) = 7.89, p = 0.0056 for MS_t index. This is the same as the inference when framed instead as a test of (i) the improvement in model fit due to the three-way interaction terms when the sign of the log-returns of one-month prior VIX differences in the indirect effect of the one-month prior market momentum factor appears to vary with the level of one-month prior VIX or (ii) the test of statistical significance as the regression coefficient of $\boldsymbol{a_t b_{7t}} VIX_{t-1} DVIX_{t-1}$. A hypothesis test for $\boldsymbol{a_t b_{7t}} VIX_{t-1} DVIX_{t-1}$ in the regression analysis is mathematically equivalent to this hypothesis test for ΔR^2.

Using Equation 6, if VIX_{t-1} is on the horizontal axis as depicted in two upper panels in Exhibit 5, where the indirect effect is on the other, and evolving estimates are used for changes in $DVIX_{t-1}$ and the set of lines as the outcome linking VIX_{t-1} to the causal effect. The slope is $\boldsymbol{a_t b_{4t}} VIX_{t-1} + \boldsymbol{a_t b_{7t}} DVIX_{t-1}$, which is "the index of partial moderated mediation" by VIX_{t-1}. The two lower panels in Exhibit 6 show the swapping roles of VIX_{t-1} and $DVIX_{t-1}$ by replacing $DVIX_{t-1}$ on the other and using evolving values of VIX_{t-1}, with the slope $\boldsymbol{a_t b_{5t}} + \boldsymbol{a_t b_{7t}} V_{IXt-1}$, "the index of partial moderated mediation" by $DVIX_{t-1}$. The two upper panels of Exhibit 5 show the indirect effect to VIX_{t-1} levels differed the size of $DVIX_{t-1}$ in ±2σ terms.

Exhibit 5. Indirect Effect of Contemporaneous Excess Market Returns and One-Month Prior VIX Levels as the Function of the Log Returns of one-Month Prior VIX.



In the upper left-hand panel of five BarCap sub-indices in Exhibit 5, the indirect effect of contemporaneous excess market returns is negative, while the magnitude is declining function to the level of one-month prior VIX for a large jump (+2σ) log-returns of one-month prior VIX (= a jump in $DVIX_{t-1}$) within the historically realized range of one-month prior VIX levels (VIX_{t-1}±2σ). A negative indirect effect is strongest among EM_t and FoF_t style hedge fund managers compared to LS_t and MS_t style mangers. On the other hand, in the upper right-hand panel in Exhibit 6, the indirect effect of contemporaneous market excess returns is positive, while the magnitude is increasing function to the level of one-month prior VIX for a large negative (-2σ) log-returns of one-month prior VIX (= a dump in $DVIX_{t-1}$) within the historically realized range of VIX_{t-1} (±2σ). A positive indirect effect is strongest among EM_t and MS_t style hedge fund managers compared to LS_t and FoF_t style mangers. Therefore, the famous asymmetric indirect effect between the historically realized ranges of one-month prior VIX returns ($DVIX_{t-1}$+2σ) and ($DVIX_{t-1}$-2σ) is more pronounced at MS_t and FoF_t style hedge fund managers compared to EM_t and LS_t style mangers.

In the lower left-hand panel in Exhibit 5, the indirect effect of contemporaneous excess market returns is negative, while the magnitude is declining function to the return of one-month prior VIX for a high level (+2σ) of

one-month prior VIX within the historically realized range of one-month prior VIX returns ($DVIX_{t-1}$±2σ). A negative indirect effect is strongest among EM_t hedge fund managers. For the most style managers, the indirect effect is a positive function when one-month prior VIX moves downward (thus, negative $DVIX_{t-1}$) when the VIX level is already high. In the lower right-hand panel in Exhibit 6, the indirect effect of contemporaneous market excess returns is a positive function when one-month prior VIX moves upward (thus, positive $DVIX_{t-1}$) when the VIX level is already low. A positive indirect effect is strongest among EM_t, MS_t, and FoF_t hedge fund managers. To quantify the indirect effect of contemporaneous excess market returns or how the indirect effect is moderated by one-month prior levels and returns of VIX, the indirect effect of contemporaneous excess market returns on the BarCap sub-index returns are summarized in Exhibit 6.

Exhibit 6. Comparing Indirect, Partial Moderated Mediation Effects in Equation 6 and 7

Direct Effect of X on Y	0.6226			
Indirect Effects	ab_1	ab_4VIX_{t-1}	ab_5DVIX_{t-1}	$ab_7VIX_{t-1}DVIX_{t-1}$
EM	(0.2210)	0.0021	0.0011	(0.0001)
LS	(0.1018)	(0.0005)	0.0006	(0.0000)
FoF	(0.0879)	(0.0002)	0.0009	(0.0001)
GM	(0.0706)	0.0000	0.0001	0.0000
MS	(0.0728)	0.0003	0.0010	(0.0001)
ab_4VIX_{t-1}+$ab_7VIX_{t-1}DVIX_{t-1}$	Case 1: +2σV +2σQ	Case 2: +2σV -2σQ	Case 3: -2σV +2σQ	Case 4: -2σV -2σQ
EM	(0.1705)	0.4440	(0.0276)	0.0718
LS	(0.1326)	0.0722	(0.0214)	0.0117
FoF	(0.1647)	0.1407	(0.0266)	0.0227
MS	(0.1481)	0.1889	(0.0239)	0.0305
ab_5VIX_{t-1}+$ab_7VIX_{t-1}DVIX_{t-1}$	Case 1: +2σV +2σQ	Case 2: +2σV -2σQ	Case 3: -2σV +2σQ	Case 4: -2σV -2σQ
EM	0.1283	(0.0317)	(0.1255)	0.0310
LS	0.0348	(0.0186)	(0.0340)	0.0182
FoF	0.0489	(0.0307)	(0.0478)	0.0300
MS	0.0534	(0.0344)	(0.0522)	0.0336

Case 1 in Exhibit 6 represents +2σVIX_{t-1} +2σ$DVIX_{t-1}$, which is the case of a jump in $DVIX_{t-1}$ at a high level of VIX_{t-1}. The partial moderated mediation effect in the second panel shows all negative in EM_t, LS_t, FoF_t, GM_t, and MS_t styles. Case 2 represents +2σVIX_{t-1}-2σ$DVIX_{t-1}$, which is the case of a dump in $DVIX_{t-1}$ at a high level of VIX_{t-1}. The partial moderated mediation effect is mostly positive in all four styles, while the effect in LS_t is less than a half of the effects in all other indices, which implies its partial moderated mediation effect is smallest among other styles. Another observation is that there is a piece of evidence of asymmetry of the partial moderated mediation effects in these jump and dump cases at a high level of VIX_{t-1} (Eq. 6), which are most pronounced in EM_t and LS_t sub-indices. It is interesting to note from the 3rd panel that this asymmetry of the partial moderated mediation effects in EM_t both cases at the large $DVIX_{t-1}$ continues to be observed even when VIX_{t-1} is the secondary moderator (Eq. 7). Case 3 represents -2σVIX_{t-1} +2σ$DVIX_{t-1}$, which is the case of a jump in $DVIX_{t-1}$ at a low level of VIX_{t-1} and Case 4 represents -2σVIX_{t-1}-2σ$DVIX_{t-1}$, which is the case of a dump in $DVIX_{t-1}$ at a low level of VIX_{t-1}. The partial moderated mediation effect remained to be small negative in Case 3, while the effect is small positive in Case 4 in both the second and the third panel. The absolute magnitude of asymmetry of the partial moderated mediation effects in these jump and dump cases at the low levels of VIX_{t-1} cannot be observed in most sub-indices, except for EM_t. Interestingly, the signs of these asymmetries of the partial moderated mediation effects in all sub-indices at both cases at the large size of $DVIX_{t-1}$ is consistent when VIX_{t-1} is the secondary moderator (Eq. 7) versus the case when $DVIX_{t-1}$ is the secondary moderator (Eq. 6).

Since the covariance between one-month prior market momentum returns and one-month prior realized shocks measured by VIX as a reliable source of general market sentiment might be helpful to understand the contemporaneous hedge fund return profiles, it is intuitive whether the differences in the magnitude of indirect and partial effects might be able to explain more responsive managers within certain investment styles by tilting their risky asset exposures more dynamically to timely adjust their overall passive market exposures accordingly. Therefore, if any in-house *a priori* factor generating process shows a certain degree of unfavorable market momentum environment (or when $DVIX_{t-1}$ moves in a jump of +2σ) at a high level of VIX_{t-1}, FoF_t, MS_t, and GM_t investment style managers would tilt the fund's exposure to or from the momentum factor, under the influence of the simultaneous considerations of prior month's level and the returns of implied volatilities, which would reduce overall passive market exposures of their strategy returns, when we choose an appropriate market benchmark such as the S&P 500 as the beta for equity-based hedge fund styles.

4. Conclusion

For a money manager who believes the bullish market, it makes sense to tactically allocate to risky assets and particularly building in high-beta trading positions, as they are likely to benefit most out of the rising market. It should be the other way round when it comes to a bearish view. According to this reasoning, an overconfident market timer might actively exploit the general market sentiment of risk preference as represented by the implied equity market volatilities

This paper applied the concept of conditional and the moderation of moderated mediation to hedge fund return time-series. VIX_{t-1} substantially moderates the effect of mediation of MKT_t's on Y_t through WML_{t-1} if the magnitude of the indirect effect of MKT_t is linked to VIX_{t-1} condition on a specific value of a second moderator

$DVIX_{t-1}$. The moderation of MKT_t's indirect effect by VIX_{t-1} is moderated by $DVIX_{t-1}$ if DVIX_{t-1} is linked to the sensitivities of the indirect effect of MKT_t as VIX_{t-1} moves.

From Exhibit 3, the regression coefficient of the three-way interaction of the BarCap GM_t index is statistically insignificant and as can be seen in Exhibit 6, the coefficient of $\boldsymbol{a_t b_{7t}} VIX_{t-1} DVIX_{t-1}$ is nil that it is less meaningful to interpret the GM_t's indirect effect concerning dual moderators in this analysis. In cases of the BarCap EM_t, LS_t, FoF_t, and MS_t sub-indices, the coefficients of $\boldsymbol{a_t b_{7t}} VIX_{t-1} DVIX_{t-1}$ are small negatives but all statistically different from zero. Again, for the BarCap GM_t sub-index, the moderation of the indirect effect of contemporaneous excess market returns by one-month prior VIX level does not differ between the size of the one-month prior VIX returns.

Both EM_t and LS_t style hedge fund managers demonstrated a more consistent pattern of the interplay among MKT_t, WML_{t-1}, and VIX_{t-1} when $DVIX_{t-1}$ moves either in extreme +2σ or -2σ range. However, MS_t and FoF_t style managers demonstrated a more active, thus asymmetric pattern of the interplay among MKT_t, WML_{t-1}, and VIX_{t-1} when $DVIX_{t-1}$ moves in a jump of +2σ versus in a dump of -2σ situations. This can be confirmed from the fact that, for EM_t and LS_t style indices, where the passive market exposure (beta) to the benchmark such as the S&P 500 index is the highest among others on average, by recording 0.62 for EM_t and 0.29 for LS_t, while same betas recorded as 0.25 for FoF_t, 0.22 for MS_t, and lowest at 0.17 for GM_t, where the indirect effect is nil. Lastly, the dual moderation effect remains to be negative when the passive market benchmark to the investment style return is in the highly sensitive range (when $\boldsymbol{\beta}_{S\&P500}$ & $\boldsymbol{\beta}_{MXKR} > 1$), which might be the regime of actively tilting risky-asset exposures through the volatility moderation feedbacks by the hedge fund style managers. However, both the size and the signs of the dual moderation effect are varying when the betas are at low positive or even in the negative range.

While we have provided a framework for understanding the hedge fund manager's volatility moderation feedback process per investment style categories, further study is needed to determine their impact on risky asset exposure adjustment decisions. We have made some simplifications by intentionally dropping other systematic risk factors such as size and value to concentrate our main point of one-month prior momentum to the past VIX dynamics. The same analysis can be extended to include situations where the more sophisticated cases by containing factors that are missing from this conditional process model.

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