Research Article

Proportional-Integral Controller with Decouplers for an interacting TITO Process

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Abstract: Proportional-Integral (PI) controllers are currently used in several industrial processes for controlling the process parameters. In this work a method has been formulated to design the controller for a multi input and multi-output (MIMO) unstable system by considering its effective transfer function (ETF). The approach mentioned, separates the unstable poles of the system into independent loops of the MIMO process with their own ETF's. The process considered is First Order Process with Time Delay (FOPTD) and PI controllers are modelled for their diagonal components by using ETF synthesis method to counteract unstable characteristics. The design has a two-loop architecture in order to minimize the overshoot, as it is evident that the overshoot is maximum for unstable processes having a single control feedback structure. The controller design is tested for its efficacy on a Two Input Two Output (TITO) system and the results are highlighted for servo as well as regulatory response.

Keywords: TITO, MIMO, controller, process control, decouplers, proportional-integral

1. Introduction

Single-Input Single-Output (SISO) systems are relatively easier to track irrespective of their time delays. However stable systems with unstable inputs are complex and time delays in unstable systems further complicates the design process [1]. Output parameters such as settling time, overshoots as well as undershoot are challenging factors when it comes to unstable systems and literatures provide significant insights into the complications that arise during implementation. Some of the past literature provides various standard operating procedures when it comes to modelling of controllers to unstable systems [2]. A two-stage controller configuration has been proposed for an unstable system which has a single-input and single-output. The entire controller is based on proportional unit and stabilized performance is further optimized using a Integral action. It is shown that a PI controller provides better response in comparison to a basic proportional unit [3].

Another aspect about multivariable PI structures is that they are challenging to design due to the significant interactions that are present while modelling the entire system. An approach was formulated which was based on robust control configurations for MIMO systems using decentralized controllers [4]. Centralized controllers are also used in such systems to stabilize them which can be considered as a simple yet effective approach. Such an approach makes use of stable state and transition matrices which can derive the proportional and integral values to stabilize the output constraints [5]. An approach has been formulated and generalized to arrive at PI values for a multivariable system that is reliant when it comes to disturbance rejections.

In connection to the modelling of controllers to unstable MIMO systems, the available literature is scarce. An approach from the available literature proposes an optimization method but this approach cannot be directly applied to dead time processes as it doesn't provide insights for time delay systems [6]. Another approach highlights an optimization approach wherein only a single input parameter has unstable components. This method was implemented on unstable multi input-multi output system and was shown that interactions between the loops were significant. The controller design followed the tuning procedures for the decentralized PI controllers [7]. However, these controllers were unable to stabilize the control system even if a single transition matrix component comprised of unstable poles. Hence centralized controller was implemented to stabilize these uncertainties. A double PI comptroller architecture may also be used to unstable systems as mentioned above to reduce any significant interactions [8].

Concept of decouplers arise whenever there are interactions in the loops of MIMO processes [9][10]. Usually these decouplers are based on the transfer function transition matrices which lead to complicated decoupler structures. But these are capable of handling modeling errors of the processes. Here a simplified decoupler approach has been discussed that is similar to the conventional decoupler, but in this case the controller cannot be directly designed without any proper procedure to reduce the order of the system [11]. Hence the process may be prone to modelling errors during reduction [12]. Even an` optimization method has been proposed to improve the robustness and its sensitivity to noise measurements which has provided promising results when implemented on MIMO processes. Also, related work on stable systems have been carried out by adapting Equivalent transfer function (ETF/EOTF) method to formulate the models in multiloop systems.

Recent developments in the field of controller designs provide a simple approach to design the decouplers with basic approximations from ETF models. It is suggested to design controllers form the MIMO processes using such an approach. In this case ETF formulation are directly applied and decouplers are designed. After decoupling the loops, gain values are formulated for a decentralized PI controller. Compared to conventional methods of decoupling, this method provides better results when it comes to loop interactions and stability. This approach has been modeled to a unstable TITO system and the results have been highlighted in the subsequent sections.

2.Mathematical Modelling of the process

Amongst other, the most significant units in a chemical plant is the distillation column. This paper gives a detailed study of the mathematical model and the binary distillation column control through the ETF approach. Wood-Berry column distillation is considered in this paper which separates methanol from water. The distillation column is operated by PID controller derived through the ETF modelling. Wood and Berry experimentally modelled a column of 9 inches diameter, 8-tray binary distillation, separating methanol from water, which has been adapted to implement the controller design. The model of wood and berry can be presented as (1).

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.6e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \frac{3.8e^{-8s}}{14.9s+1} \\ \frac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} d \tag{1}$$

By having the 2nd feedback loop in automated control with $y_{r2} = 0$, the TITO system CLTF from y_1 and u_1 is given as

$$\frac{y_1}{u_1} = g_{p,11} - \frac{\left(g_{p,12} g_{p,21} g_{c,2}\right)}{\left(1 + g_{c,2} g_{p,22}\right)} \tag{2}$$

Which can be expressed in the following form,

$$\frac{y_1}{u_1} = g_{p,11} - \frac{g_{p,12} g_{p,21} \left(g_{c,2} g_{p,22}\right)}{g_{p,22}\left(1 + g_{c,2} g_{p,22}\right)} \tag{3}$$

The same may be expressed for the 2nd loop as,

$$\frac{y_2}{u_2} = g_{p,22} - \frac{g_{p,21} g_{p,12} \left(g_{c,1} g_{p,11}\right)}{g_{p,11}\left(1 + g_{c,1} g_{p,11}\right)} \tag{4}$$

Complicated equations 3 and 4 can be reduced by one of the two ways: the concept is the optimal controller approximation for another loop (the output is constant without transient) used to clarify all equations; i.e.

$$\frac{g_{c,i} g_{p,ii}}{\left(1 + \ g_{c,i} g_{p,ii}\right)} = 1 \qquad i = 1,2 \tag{5}$$

The step two is considering that ETFs will have similar open-loop model configuration. Eqs 2 and 3 can be approximated by using the best controller approximation as

$$g_{p,11}^{eff} = \frac{y_1}{u_1} = g_{p,11} - \frac{g_{p,12} g_{p,21}}{g_{p,22}} \tag{6}$$

$$g_{p,22}^{eff} = \frac{y_2}{u_2} = g_{p,22} - \frac{g_{p,12} g_{p,21}}{g_{p,11}} \tag{7}$$

Here, the successful open-loop transfer functions $g_{p,11}^{eff}$ and $g_{p,22}^{eff}$ are (EOTF)- Such EOTFs are complex models of transfer functions, so it's hard to use them explicitly for controller architecture. The corresponding EOTFs are approximated to FOPTD structures utilizing the Maclaurin sequence for controller architecture. This approach results in problems in structures with higher dimensions, in formulation and reduction of EOTFs in the process. Through utilizing the definitions of relative gain array (RGA) as well as the relative normalized gain array (RNGA), the term for ETF may also conveniently be interpreted for higher dimensional structures.

The controls are built on the ETF's diagonal elements and their closed-loop responses must be calibrated to the $g_{p,11}^{eff}$ and $g_{p,22}^{eff}$. Because the output is same, it is evident that ETF as well as the EOTF are similar. The procedure for arriving at ETF of a TITO system is given below: The standardized K_{Nij} gain, is defined as

$$K_N = K \odot T_{ar} = \begin{bmatrix} K_{N,11} & K_{N,12} \\ K_{N,21} & K_{N,22} \end{bmatrix} \tag{8}$$

Where we have K, which is the steady state process gain, Hadamard division ⊙ and T_{ar} is considered to be the mean time of residence, ie, the response speed of the y_i and u_j (controlled variable & manipulated variable). The RNGA is calculated as

$$\phi = K_N \otimes K_N^{-T} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \quad (9)$$

Where ⊗ applies to multiplication of Hadamard. RARTA, given to be the ratio of loop $y_i - u_j$ mean time of residence, on the basis that loops are open and closed to verify interactions and this shall be determined by reference to RARTA as,

$$\Gamma = \phi \odot \Lambda = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \quad (10)$$

In case of theunstable system, the Effective Transfer Function (ETF) can therefore be formulated as,

$$\hat{g}_{p,ij}(s) = \hat{k}_{p,ij} \frac{e^{-\hat{\theta}_{ij}(s)}}{\hat{\tau}_{ij}(s) - 1} \quad (11)$$

Here, $\hat{k}_{p,ij} = \frac{k_{p,ij}}{\Lambda_{i,j}}$ & $\hat{\tau}_{ij} = \gamma_{ij}\tau_{ij}$ & $\hat{\theta}_{ij} = \gamma_{ij}\theta_{ij}$

This method is only applicable when the ETF=EOTF.

3.Controller Design and Results

The pairing of the controller should be such that it will completely eliminate any form of loop interactions. The severity of such an interaction is given by RGA model which aids in determining the effects of manipulated variable that suites the controller design. Here NI (Niederlinski index) and RGA positive values are used for a stable plant structure where;

$$NI = \frac{det[K]}{\Pi K_{ij}} \quad (12)$$

$$RGA(\Lambda) = K \otimes K^{-T} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \quad (13)$$

Unstable network pairing parameters will vary when the number of $G_p(s)$ open-loop poles which are unstable, is not same from $\hat{G}_p(s) = diag[g_{p,ii}(s)]$. Therefore, pairing is done by the one of the two methods: When (n x n) plants (1) containing an unstable poles appearing in all $G_p(s)$, the pairing, in which case, if n turns out to be odd, then NI is positive and NI is negative in cases where n is even, or with P increasingly unstable poles of $G_p(s)$ elements, that if $(n-1)P$ turns out to be even then NI is positive and NI is negative if $(n-1)P$ turns out to be odd. The TITO process flow discussed here can be seen in Figure 1.

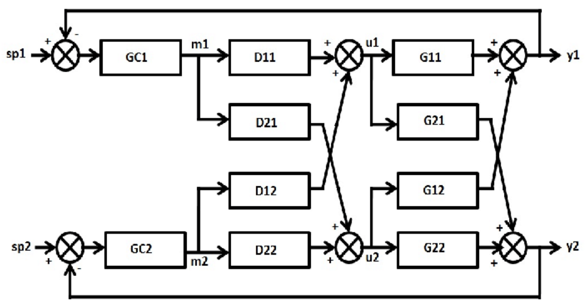

Figure 1: Decoupled TITO Process

We have, for the preceding TITO system,

$$Y(s) = G_p(s)D(s)U(s) \quad (14)$$

$$G_p(s)D(s) = \begin{bmatrix} g_{p,11} & g_{p,12} \\ g_{p,21} & g_{p,22} \end{bmatrix} \begin{bmatrix} 1 & d_{12} \\ d_{21} & 1 \end{bmatrix} = \begin{bmatrix} g_{p,11}^* & 0 \\ 0 & g_{p,22}^* \end{bmatrix} \quad (15)$$

Due to which the decoupler could be designed as,

$$d_{12}(s) = -\frac{g_{p,12}(s)}{g_{p,11}(s)}; \qquad d_{21}(s) = -\frac{g_{p,21}(s)}{g_{p,22}(s)} \tag{16}$$

Having time delay for these systems can lead to undesirable situations by equation 15. Therefore, an additional dead time (θ) must be taken into consideration into the structure matrix of the decoupler, which is incorporated to the respective ETF. The disturbance rejection capability of this closed-loop system is measured by simulation for variability in dead time, time constant, or process cost. The TITO mechanism operates like two separate loops, in the presence of the decoupler, where the controllers can be constructed separately. In the proposed work, the synthesis system derived on the unstable ETFs designs diagonal PI controllers:

$$g_{c,ii}(s) = k_{c,ii}\left(1 + \frac{1}{\tau_{I,ii}(s)}\right) \tag{17}$$

$$k_{c,ii}k_{p,ii} = 0.8668\mathcal{E}_{ii}^{-0.8228} \quad for\ 0.1 \le \mathcal{E}_{ii} \le 0.7 \tag{18}$$

$$\frac{\tau_{1,ii}}{\tau_{ii}} = 0.1523e^{7.9425\mathcal{E}_{ii}} \quad for\ 0.1 \le \mathcal{E}_{ii} \le 0.7 \tag{19}$$

Where, $\mathcal{E}_{ii} = \frac{\theta_{ii}}{\tau_{ii}}$

Now, the controller design is carried out for the Wood and Berry described in Equation 1. As the count of unstable open loop poles are equal for $G_p(s)$ and $\hat{G}_p(s)=diag[g_{p,ii}(s)]$, it is evident that the pairing considerations for the system would be equal to that for a stable structure. To decide the pairing of this system, it calculates RGA & K;

$$K = \begin{bmatrix} 12.8 & -18.6 \\ 6.6 & -19.4 \end{bmatrix} \qquad RGA(\Lambda) = \begin{bmatrix} 1.977 & -0.977 \\ -0.977 & 1.977 \end{bmatrix} \tag{20}$$

Since NI = 19.02 is positive as eq 12 which determines, the pairing can be untouched as of now. The procedure fot calculating dynamic elements like the normalized matrix gain (K_N), the mean/average residence time (T_{ar}), as well as the RARTA matrix (Γ), RNGA(ϕ) are determined using equations 8, 9, 10 & 11:

$$T = \begin{bmatrix} 17.7 & 24 \\ 17.9 & 17.4 \end{bmatrix} \quad K_N = \begin{bmatrix} 0.72 & -0.77 \\ 0.36 & -1.11 \end{bmatrix};$$
$$\phi = \begin{bmatrix} 1.54 & 0.54 \\ -0.54 & 1.54 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} 0.78 & 0.56 \\ 0.56 & 0.78 \end{bmatrix} \tag{21}$$

Hence, using the previous definitions, we can arrive at the ETF matrix as

$$\hat{G}_p = \begin{bmatrix} \frac{6.42e^{-0.783s}}{13.08s+1} & \frac{19.02e^{-1.68s}}{11.79s+1} \\ \frac{-6.751e^{-3.93s}}{6.12s+1} & \frac{-9.809e^{-2.35s}}{11.28s+1} \end{bmatrix} \tag{22}$$

By using equation 16, we can arrive at the simplified decouplers as given by,

$$D(s) = \begin{bmatrix} 1 & \frac{(310.6s+18.6)e^{-2s}}{268.8s+12.8} \\ \frac{(211.5s+19.4)e^{-4s}}{95.04s+6.6} & 1 \end{bmatrix} \tag{23}$$

We arrive at the Proportional-Integral controllers for the diagonal elements of effective transfer function as shown below.

$$G_c(s) = \begin{bmatrix} -1.456(1+\frac{1}{8.79s}) & 0 \\ 0 & -1.34(1+\frac{1}{17.32s}) \end{bmatrix}$$

The PI controllers are modeled from ETF, as indicated earlier. Simulation obtains the related closed-loop answers. On EOTF, the same controller parameters are implemented, and closed-loop answers are considered to be similar.

Hence the EOTF is represented in the reduced form (ETF). Figure 2 displays separately the output of the controlled system in fixed point y1 and y2 for a change of step. The overshoot is observed as quite high which can be considered as the standard delay value of usual unstable systems. It is noteworthy that there is no interaction amongst the loops.

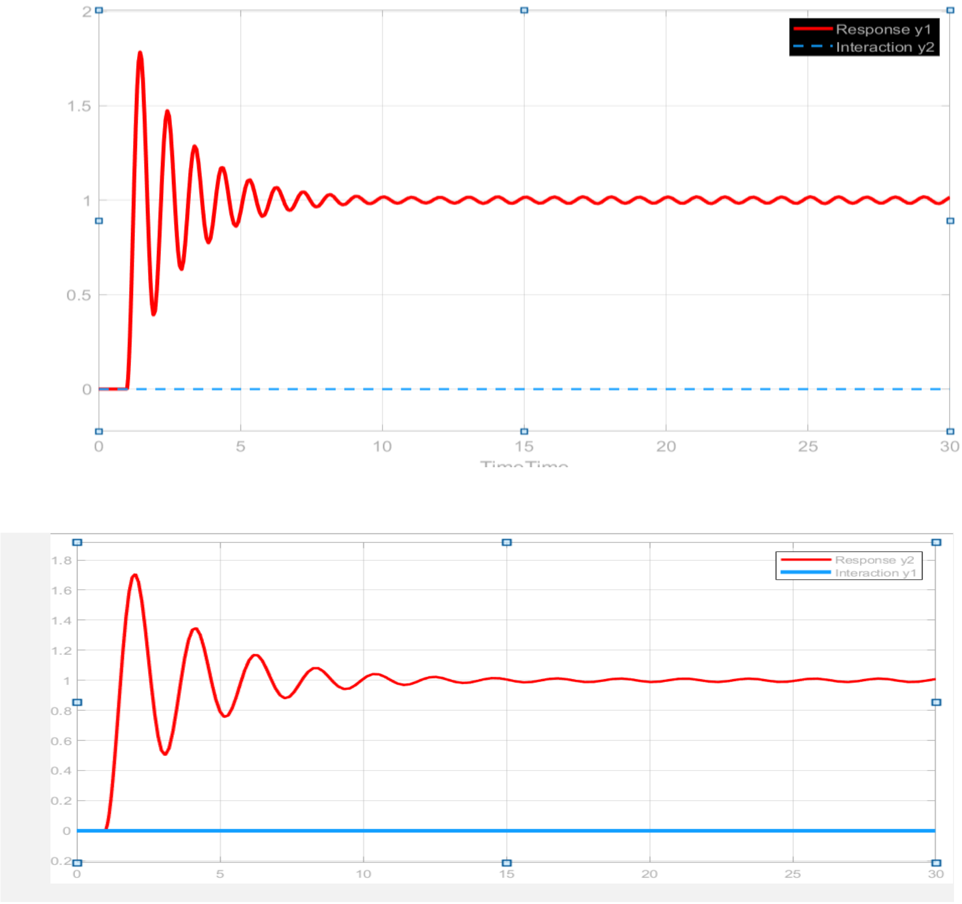

Figure 2: PI Controller output with different decouplers for a phase input in y1 and y2.

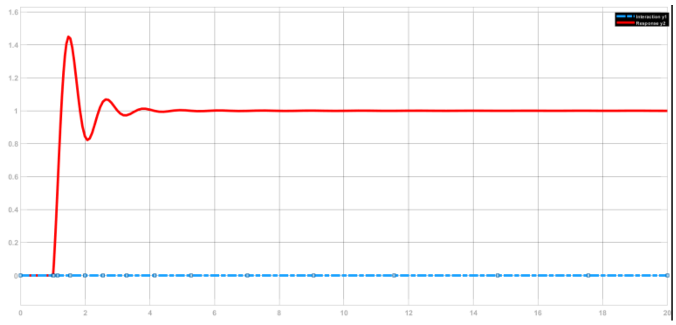

Figure 3 shows the dual-loop controller 's responses are far superior to a one-loop PI control system. Here the loop-interactions are considered to be marginal and the second control is tuned for setpoint-tracking.

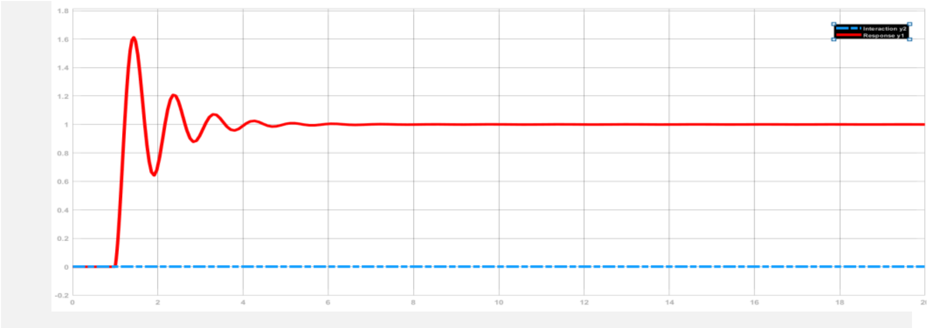

Figure 3: The dual-loop controller's responses for a phase input in y1 and y2.

4.Conclusion

Multivariable PI controllers are discussed in detail and same are modelled for unstable MIMO systems with dead time, derived from the equivalent transfer function (ETF) model. The method makes use of simplified decouplers which segregate the entire complex and unstable MIMO systems into individual loops. Here the ETFs comprising of unstable poles is considered for a decoupled process model. Synthesis method is implemented for modelling the PI controllers on the diagonal components. Since the overshoot is found to be greater, it is suggested to minimize the overshoot by a two-loop control system. An example of TITO unstable control structure with minimum interactions is given to demonstrate the efficacy of the above-mentioned method

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