Applications Of Diophantine Equations In Chemical Reactions And Cryptography

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ABSTRACT: This paper is concerned with practical applications of Diophantine equations and is intended for all readers interested in applied mathematics. We briefly explain an idea of encryption in Caesar ciphering in cryptography. Linear Diophantine equations are used to solve the chemical equations.

KEYWORDS: Diophantine equations, Chemical equations, input, output, reactants, products, cryptography.

INTRODUCTION:

For thousands of years people searched for the way to send a message secretly. There is a story that, in old time, a ruler expected to send a mysterious message to his general in fight. The king took a servant, shaved his head and wrote the message on his head. He waited for the servant's hair to grow back and then sent the servant to the general. The general then shaved the servant's head and read the message. If the enemy had captured the servant, they presumably would not have known to shave his head and message would have been safe.

Cryptography is the investigation of techniques to send and get the mysterious messages. Overall we have a sender who is attempting to make an impression on recipient. We are effective if sender can impart a message to the receiver without enemy realizing what the message was.

DIOPHANTINE EQUATIONS:

Diophantine equations are named after the Greek mathematician Diophantus C.250 of Alexandria.
The simplest form of Diophantine equation is

$$ax + by = c$$

where a, b, c are given integers.

SECTION A:

SOME IMPORTANT APPLICATIONS OF DIOPHANTINE EQUATIONS IN CHEMISTRY IN DAILY LIFE:

New products are formed by matter interactions, the process is called chemical reaction.

- Digestion relies on chemical reactions between food and acids and enzymes to break down molecules into nutrients the body can absorb and use.
- Soaps and detergents act as emulsifiers to surround dirt and grime so it can be washed away from clothing, dishes, and our bodies.
- Drugs work because of chemistry. The chemical compounds may fit into the binding site for natural chemicals in our body (e.g., block pain receptors) or may attack chemicals found in pathogens, but not human cells (e.g., antibiotics).
- Cooking is a chemical change that alters food to make it more palatable, kill dangerous microorganisms, and make it more digestible. The heat of cooking may denature proteins, promote chemical reactions between ingredients, caramelize sugars, etc.

BALANCING CHEMICAL EQUATIONS:

Some application of a linear Diophantine equation in balancing of chemical equations are presented below.

Example 1:

Consider the chemical reaction

$$wBacl_2 + xAl_2(SO_4)_3 \rightarrow yAlcl_3 + zBaSO_4 \qquad (1)$$

Which leads to the equations

$w = z$ for Ba
$2w = 3y$ for Cl
$2x = y$ for Al (2)
$3x = z$ for S
$12x = 4z$ for O

The above system of equations can be easily reduces to
$w - 3x = 0,$
a linear Diophantine equation with a solution [w, x] = [3, 1]
Hence, [w, x, y, z] = [3, 1, 2, 3]
Consequently equation (1) becomes,
$3Bacl_2 + Al_2(SO_4)_3 \rightarrow 2Alcl_3 + 3BaSO_4$

Example 2:
Consider the chemical reaction
$wH_3PO_4 + xKOH \rightarrow yK_3PO_4 + zH_2O$ (3)
From the above equation, the following are received:
$3w + x = 2z$ for H
$w = y$ for P
$x = 3y$ for K (4)
$4w + x = 4y + z$ for O

The above system of equations can be easily reduced to
$4w - y - z = 0,$
a linear Diophantine equation with a solution [w, y, z] = [1, 1, 3]
Hence, [w, x, y, z] = [1, 3, 1, 3]
Consequently equation (3) becomes,
$H_3PO_4 + 3KOH \rightarrow K_3PO_4 + 3H_2O$

Example 3:
Consider the chemical reaction
$wNa_2S + xAgl \rightarrow yNal + zAg_2S$ (5)
From the above equation, the following are obtained:
$2w = y$ for Na
$w = z$ for S
$x = 2z$ for Ag (6)
$x = y$ for l

The above system of equations can be easily reduced to
$x - 2w = 0,$
a linear Diophantine equation with a solution [x, w] = [2, 1]
Hence, [w, x, y, z] = [1, 2, 2, 1]
Consequently equation (5) becomes
$Na_2S + 2Agl \rightarrow 2Nal + Ag_2S$

Example 4:
Consider the chemical reaction
$wNa + xH_2O \rightarrow yNaOH + zH_2$ (7)
From the above equation, the following are attained:
$w = y$ for Na
$2x = y + 2z$ for H (8)
$x = y$ for O

The above system of equations can be easily reduced to
$x - 2z = 0,$

a linear Diophantine equation with a solution [x, z] = [2, 1]
Hence, [w, x, y, z] = [2, 2, 2, 1]
Consequently equation (7) becomes

$$2Na + 2H_2O \rightarrow 2NaOH + H_2$$

Example 5:

Consider the chemical reaction

$$xC_6H_{12}O_6 \rightarrow yC_2H_5OH + zCO_2 \quad (9)$$

From the above equation, the following are obtained:

$$6x = 2y + z \quad \text{for C}$$
$$12x = 6y \quad \text{for H} \quad (10)$$
$$6x = y + 2z \quad \text{for O}$$

The above system of equations can be easily reduced to

$$4x - 2z = 0,$$

a linear Diophantine equation with a solution [x, z] = [1, 2]
Hence, [x, y, z] = [1, 2, 2]
Consequently equation (9) becomes

$$C_6H_{12}O_6 \rightarrow 2C_2H_5OH + 2CO_2$$

SECTION B:

CAESAR CIPHER KEY CRYPTOGRAPHY:

Cryptography is the science of devising methods that allow information to be sent in a secure form in such a way that the only person able to retrieve this information. Cryptography is considered a branch of both mathematics and computer science and is used in applications present in technologically advanced societies such as the security of ATM cards, computer passwords and electronic commerce.

A message being sent is known as plaintext. The message is then coded using cryptographic algorithm. This process is called encryption. The encrypted message is known as ciphertext and is turned back into plaintext by the process of decryption. The method of decryption is the same as that for encryption but in reverse direction.

In cryptography, cipher is an algorithm for performing encryption or decryption – a series of well defined steps that can be followed as a procedure. One of the earliest cryptographic system was used by great Roman emperor Julius Caesar around 50 (B.C.). Caesar wrote to Marcus Cicero using a rudimentary substitution cipher in which each letter of the alphabet is replaced by letter that occurs three places down the alphabet. With the last three letters cycled back to the first three letters. Under, the plain text letter the substitution alphabet for Caesar cipher is given by

A	B	C	D	E	F	G	H	I	J	K	L	M
D	E	F	G	H	I	J	K	L	M	N	O	P
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Q	R	S	T	U	V	W	X	Y	Z	A	B	C

For Example:

MATHEMATICS IS THE QUEEN OF SCIENCE AND ARITHMETIC THE QUEEN OF MATHEMATICS is transformed into

PDWKHPDWLFV LV WKH TXHHQ RI VFLHQFH DQG DULWKPHWLF WKH TXHHQ RI PDWKHPDWLFV.

Using the concept of congruence theory, Caesar cipher can be described. Plaintext is expressed numerically by transforming the character of the text into digit such as

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

Let P denotes the Plaintext and C denotes the Cipher text then $C \equiv P + 3 (mod\ 26)$

For Example:

MATHEMATICS	IS	THE	QUEEN	OF	SCIENCE
12 0 19 7 4 12 0 19 8 2 18 8 18	19 7 4	16 20 4 4 13	14 5	18 2 8 4 13 2 4	

AND ARITHMETIC THE QUEEN OF MATHEMATICS

0 13 3 0 17 8 19 7 12 4 19 7 4 16 20 4 4 13 14 5 12 0 19 7 4 12 0 19 8 2 18

Using the congruence $C \equiv P + 3(mod\ 26)$, the Caesar cipher text is presented below

15 3 22 10 7 15 3 22 11 5 21 11 21 22 10 7 19 23 7 7 16 17 8 21 5 11 7 16 5 7
PDWKHPDWLFV LV WKH TXHHQ RI VFLHQFH
3 16 6 3 20 11 22 10 15 7 22 11 5 22 10 7 19 23 7 7 16 17 8
DQG DULWKPHWLF WKH TXHHQ RI
15 3 22 10 7 15 3 22 11 5 21
PDWKHPDWLFV

To recover the plain text this procedure is reversed by using the congruence $C - 3 \equiv P(mod\ 26)$

Encrypting a message twice with some block cipher, either with the same key or by using two different keys, then the resultant encryption to be stronger in all but some exceptional circumstances.

Double encryption is done by using the congruence $C \equiv ((P + 3) + 3)(mod\ 26)$

After the double encryption, the plaintext can be presented as follows:

18 6 25 13 10 18 6 25 14 8 24 14 24 25 13 10 22 0 10 10 19 20 11
SGZNKSGZOIY OY ZNK WAKKT UL
24 8 14 10 19 8 10 6 19 9 3 20 11 22 10 15 7 22 11 5 25 13 10
YIOKTIK GTJ DULWUPHWLF ZNK
22 0 10 10 19 20 11 18 6 25 13 10 18 6 25 14 8 24
WAKKT UL SGZNKSGZOIY

To recover the plain text this procedure is reversed by using the congruence
$((C - 3) - 3) \equiv P(mod\ 26)$.

In such a way, multiple encryptions will occur in each phase and this process will be repeated number of times up to desired extent.

CONCLUSION:

In this paper, we perceive that the linear Diophantine equations plays an important role in chemical reactions and congruence are used in Caesar ciphering key in cryptography.

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