

A Study on Fuzzy Decision Problem of a Queueing System with Finite Capacity by LR Method

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Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021;

Published online: 20 April 2021

Abstract: In this paper, LR method has been proposed to mathematical non-linear parametric programming approach for the queueing decision problem. Using LR method, the minimal expected total cost of a crisp queueing system with finite capacity using LR – type trapezoidal fuzzy numbers is calculated. LR method is simple, short and convenient compared to alpha-cut method. Numerical example is illustrated for comparison between alpha – cut method and LR method.

Keywords: Fuzzy queue, Non – linear parametric programming, Alpha – cut method, LR – method, Trapezoidal fuzzy number

1. Introduction

Queueing decision problem plays an important role in the queueing system design that involves one (or) more decision such as the number of servers at a service facility, the efficiency of the servers. Queueing decision problem can be solved when the cost coefficient and the arrival (or) service pattern are known exactly. In Fuzzy Queueing literature, fuzzy queues are widely studied by many researchers: (H.M. Parade, 1982; C. Kao, C.C. Li, S.P. Chen, 1999; A. Nagoor Gani, 2007; J.P. Mukeba Kanyinda, 2015).

2. Fuzzy set and Fuzzy number

2.1. Fuzzy set

A fuzzy set is specified by a membership function containing the components of a domain space (or) universe 'X' in the interval [0,1], that is $\tilde{A} = \{ (Z, \mu_{\tilde{A}}(z)) ; z \in Z \}$

2.2. Membership function

$\mu_{\tilde{A}} : Z \rightarrow [0,1]$ is an interval called the degree of membership function of the fuzzy set \tilde{A} and $\mu_{\tilde{A}}(z)$ represents the membership value of $z \in Z$ in the fuzzy set \tilde{A} . The membership grade is defined by $R \rightarrow [0,1]$.

2.3. Fuzzy number

A fuzzy set \tilde{A} in the universe 'X' is a fuzzy number if and only if it satisfies the following conditions:

1. $X = \mathbb{R}$
2. \tilde{A} is normal
3. \tilde{A} is convex
4. The membership function $\mu_{\tilde{A}}$ is piecewise continuous
5. There exists one and only one $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$

3. LR – fuzzy numbers

A fuzzy number \tilde{M} is of LR – type if there exist shape functions L (for left) and R (for right) and scalars $\alpha > 0, \beta > 0$ with membership function

$$\mu_{\tilde{M}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right); & \text{if } x \leq m \\ R\left(\frac{x-m}{\beta}\right); & \text{if } x \geq m \end{cases}$$

Where the real number 'm' is called mean value of \tilde{M} , α and β are called the left and right spreads respectively.

3.1. LR – type trapezoidal fuzzy number

A fuzzy number $\tilde{M} = (m, n, \alpha, \beta)_{LR}$ is said to be LR – type trapezoidal fuzzy numbers if its membership function is given by

$$\mu_{\overline{M}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right); & \text{if } x \leq m; \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right); & \text{if } x \geq n; \beta > 0 \\ 1 & ; \text{otherwise} \end{cases}$$

3.2. Basic operations in LR – type trapezoidal fuzzy numbers

Using (J. Vahidi, 2013), If $A = (a, b, c, d)$ & $B = (e, f, g, h)$ & ‘ λ ’ is a parameter, then

$$(i) \quad A + B = (a, b, c, d) + (e, f, g, h) = (a + e, b + f, c + g, d + h)$$

$$(ii) \quad A - B = (a, b, c, d) - (e, f, g, h) = (a - f, b - e, c + h, d + g)$$

$$(iii) \quad AB = (a, b, c, d) \cdot (e, f, g, h) = (ae, bf, ag + ec, bh + fd)$$

$$(iv) \quad \frac{A}{B} = \frac{(a,b,c,d)}{(e,f,g,h)} = \left(\frac{a}{f}, \frac{b}{e}, \frac{c}{h}, \frac{d}{g}\right)$$

$$(v) \quad A^n = A \\ (a, b, c, d)^n = (a, b, c, d) \\ \text{Where } T = \{ a^n, a^{n-1}b, a^{n-2}b^2, \dots \}$$

$$T_1 = \{ d^{n-2}c^2, d^{n-1}c, d^n \}$$

$$a = \min T = k^{\text{th}} \text{ element of } T$$

$$b = \max T = l^{\text{th}} \text{ element of } T$$

$$c = \min T_1 = k^{\text{th}} \text{ element of } T_1$$

$$d = \max T_1 = l^{\text{th}} \text{ element of } T_1$$

$$\text{where } 1 \leq K \leq 4 \quad \text{and} \quad 1 \leq l \leq 4$$

$$(vi) \quad \lambda (a,b,c,d) = (\lambda a, \lambda b, \lambda c, \lambda d)$$

$$(vii) \quad 1/\lambda (a,b,c,d) = (a/\lambda, b/\lambda, c/\lambda, d/\lambda)$$

4. Numerical example

We consider an (FM/M/1) : (N/FCFS) queueing model. The minimal expected total cost is derived using non-parametric programming approach. The minimal expected total cost of a crisp queueing system with finite capacity (Gross, D. 1998; Hillier, F. S., 2001) is given by

$$E(\tilde{c}) = \mu c_1 + L v + N c_3 + x P_N c_4$$

The following example is taken for comparing alpha – cut method and LR method.

A pizza unlimited restaurant has two franchises Model A has a capacity of 20 Groups of customers and Model B can seat 30 groups of customers. The monthly operating cost of Model A is a fuzzy trapezoidal number $\tilde{c}_1 = [10000 \ 11500 \ 12500 \ 14000]$ and the monthly operating cost of Model B is also a fuzzy trapezoidal number $\tilde{c}_1 = [10000 \ 15000 \ 19000 \ 20000]$. An investor wants to set up a pizza restaurant and estimates that group of customer each occupying one table arrive according to a poisson distribution at the rate of $\tilde{\lambda} = [20 \ 23 \ 27 \ 30]$ per hour. If all the tables are occupied, customers will go elsewhere. Model A will serve 26 groups per hour and Model B will serve 29 groups per hour. Because of the variation in group sizes and in types of orders, the service time is exponential. The investor estimates that the average cost of lost business per customer group per hour is Rs. 15. The cost of serving additional customer is Rs. 5. A delay in serving waiting customers is estimated as fuzzy trapezoidal number $\tilde{c}_2 = [5 \ 7 \ 12 \ 16]$ per customer group per hour. The manager of the restaurant wants to determine the optimum model so that the total expected cost per unit time is minimized.

4.1 Alpha -cut method

Model A:

$$[x_{\alpha}^L, x_{\alpha}^U] = [20 + 3\alpha; 30 - 3\alpha]$$

$$[u_{\alpha}^L, u_{\alpha}^U] = [10000 + 1500\alpha, 14000 - 1500\alpha]$$

$$[v_{\alpha}^L, v_{\alpha}^U] = [5 + 2\alpha, 16 - 4\alpha]$$

$$\text{Also } N = 20; \mu = 26 \text{ groups / hr}; c_3 = \text{Rs. } 5; c_4 = \text{Rs. } 15; \rho = \lambda / \mu$$

In $[E(c)]_{\alpha}^L$, $\rho = (20 + 3\alpha) / 26$ and in $[E(c)]_{\alpha}^U$, $\rho = (30 - 3\alpha) / 26$

In $(M/M/1) : (N/FCFS)$, $P_N = \left[\frac{(1-\rho)}{(1-\rho^{N+1})} \right] \rho^n$; for $n = 1, 2, 3, \dots, N$

$$L_s = \left[\frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{(1-\rho^{N+1})} \right] = L$$

The upper bound of $E(c)$ is given by $[E(c)]_{\alpha}^U = \mu c_1 + Lc_2 + Nc_3 + \lambda P_N c_4$

$$\begin{aligned} [E(c)]_{\alpha}^U &= 26(14000 - 1500\alpha) + (16 - 4\alpha) \left[\frac{\frac{(30-3\alpha)}{26}}{1 - \frac{(30-3\alpha)}{26}} - \frac{21 \left(\frac{30-3\alpha}{26} \right)^{21}}{1 - \left(\frac{30-3\alpha}{26} \right)^{21}} \right] \\ &\quad + 20 \times 5 + 15(30 - 3\alpha) \left[\frac{1 - \frac{(30-3\alpha)}{26}}{1 - \left(\frac{30-3\alpha}{26} \right)^{21}} \right] \left(\frac{30-3\alpha}{26} \right)^{20} \end{aligned}$$

The lower bound of $E(c)$ is given by $[E(c)]_{\alpha}^L = \mu c_1 + Lc_2 + Nc_3 + \lambda P_N c_4$

$$\begin{aligned} [E(c)]_{\alpha}^L &= 26(10000 + 1500\alpha) + (5 + 2\alpha) \left[\frac{\frac{(20+3\alpha)}{26}}{1 - \frac{(20+3\alpha)}{26}} - \frac{21 \left(\frac{20+3\alpha}{26} \right)^{21}}{1 - \left(\frac{20+3\alpha}{26} \right)^{21}} \right] \\ &\quad + 20 \times 5 + 15(20 + 3\alpha) \left[\frac{1 - \frac{(20+3\alpha)}{26}}{1 - \left(\frac{20+3\alpha}{26} \right)^{21}} \right] \left(\frac{20+3\alpha}{26} \right)^{20} \end{aligned}$$

The values of $[E(c)]_{\alpha}^L$ lies between 2,60,120 and 2,99,150 and the values of $[E(c)]_{\alpha}^U$ lies between 3,25,260 and 3,64,400.

Model B

$$[x_{\alpha}^L, x_{\alpha}^U] = [20 + 3\alpha; 30 - 3\alpha]$$

$$[u_{\alpha}^L, u_{\alpha}^U] = [10000 + 5000\alpha, 20000 - 1000\alpha]$$

$$[v_{\alpha}^L, v_{\alpha}^U] = [5 + 2\alpha, 16 - 4\alpha]$$

Also $N = 30$; $\mu = 29$ groups / hr; $c_3 = \text{Rs. } 5$; $c_4 = \text{Rs. } 15$; $\rho = \lambda / \mu$

In $[E(c)]_{\alpha}^L$, $\rho = (20 + 3\alpha) / 29$ and in $[E(c)]_{\alpha}^U$, $\rho = (30 - 3\alpha) / 29$

The lower bound of $E(c)$ is given by $[E(c)]_{\alpha}^L = \mu c_1 + Lc_2 + Nc_3 + \lambda P_N c_4$

$$\begin{aligned} [E(c)]_{\alpha}^L &= 29(10000 + 5000\alpha) + (5 + 2\alpha) \left[\frac{\frac{(20+3\alpha)}{29}}{1 - \frac{(20+3\alpha)}{29}} - \frac{31 \left(\frac{20+3\alpha}{29} \right)^{31}}{1 - \left(\frac{20+3\alpha}{29} \right)^{31}} \right] \\ &\quad + 30 \times 5 + 15(20 + 3\alpha) \left[\frac{1 - \frac{(20+3\alpha)}{29}}{1 - \left(\frac{20+3\alpha}{29} \right)^{31}} \right] \left(\frac{20+3\alpha}{29} \right)^{30} \end{aligned}$$

The upper bound of $E(c)$ is given by $[E(c)]_{\alpha}^U = \mu c_1 + Lc_2 + Nc_3 + \lambda P_N c_4$

$$\begin{aligned} [E(c)]_{\alpha}^U &= 29(20000 - 1000\alpha) + (16 - 4\alpha) \left[\frac{\frac{(30-3\alpha)}{29}}{1 - \frac{(30-3\alpha)}{29}} - \frac{31 \left(\frac{30-3\alpha}{29} \right)^{31}}{1 - \left(\frac{30-3\alpha}{29} \right)^{31}} \right] \\ &\quad + 30 \times 5 + 15(30 - 3\alpha) \left[\frac{1 - \frac{(30-3\alpha)}{29}}{1 - \left(\frac{30-3\alpha}{29} \right)^{31}} \right] \left(\frac{30-3\alpha}{29} \right)^{30} \end{aligned}$$

The values of $[E(c)]_{\alpha}^L$ lies between 2,90,160 and 4,35,180 and the values of $[E(c)]_{\alpha}^U$ lies between 5,51,270 and 5,80,460.

4.2 L. R. Method

Model A

The minimal expected total cost of a crisp queueing system with finite Capacity is given by

$E(c) = \mu c_1 + Lc_2 + Nc_3 + \lambda P_N c_4$, where

$$L_s = \left[\frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{(1-\rho^{N+1})} \right] = L$$

$$L = \left[\frac{(\lambda/\mu)}{1-(\lambda/\mu)} - \frac{(21)(\lambda/\mu)^{21}}{1-(\lambda/\mu)^{21}} \right]$$

$$L = \left[\frac{<20,23,27,30>/26}{1-(<20,23,27,30>/26)} - \frac{(21)(<20,23,27,30>/26)^{21}}{1-(<20,23,27,30>/26)^{21}} \right]$$

$$L = \left[\frac{<20/26,23/26,27/26,30/26>}{1-<20/26,23/26,27/26,30/26>} - \frac{(21)(<20/26,23/26,27/26,30/26>)^{21}}{1-(<20/26,23/26,27/26,30/26>)^{21}} \right], \quad \text{using 3.2(vii)}$$

$$L = \left[\frac{<0.7692,0.8846,1.0384,1.1538>}{1-<0.7692,0.8846,1.0384,1.1538>} - \frac{(21)<0.7692,0.8846,1.0384,1.1538>^{21}}{1-<0.7692,0.8846,1.0384,1.1538>^{21}} \right]$$

To find the value of $<0.7692, 0.8846, 1.0384, 1.1538>^{21}$:

Using 3.2(v)

$$T = \{ (0.7692)^{21}, (0.7692)^{20}(0.8846), (0.7692)^{20}(0.8846), (0.7692)^{19}(0.8846)^2 \}$$

$$T = \{ 0.0040, (0.0052)(0.8846), (0.0052)(0.8846), (0.0068)(0.7825) \}$$

$$T = \{ 0.0040, 0.0045, 0.0045, 0.0053 \}$$

$$T_1 = \{ (1.1538)^{19}(1.0384)^2, (1.1538)^{20}(1.0384), (1.1538)^{20}(1.0384), (1.1538)^{21} \}$$

$$T_1 = \{ (15.1523)(1.0782), (17.4827)(1.0384), (17.4827)(1.0384), (20.1716) \}$$

$$T_1 = \{ 16.3372, 18.1540, 18.1540, 20.1716 \}$$

$$a = \min \{T\} = 0.0040; b = \max \{T\} = 0.0053, c = \min \{T_1\} = 16.3372; d = \max \{T_1\} = 20.1716$$

$$<0.7692, 0.8846, 1.0384, 1.1538>^{21} = <0.0040, 0.0053, 16.3372, 20.1716>$$

$$L = \left[\frac{<0.7692,0.8846,1.0384,1.1538>}{1-<0.7692,0.8846,1.0384,1.1538>} - \frac{(21)<0.0040,0.0053,16.3372,20.1716>}{1-<0.0040,0.0053,16.3372,20.1716>} \right]$$

$$L = \left[\frac{<0.7692,0.8846,1.0384,1.1538>}{1-<0.7692,0.8846,1.0384,1.1538>} - \frac{<0.084,0.1113,343.0812,423.6036>}{1-<0.0040,0.0053,16.3372,20.1716>} \right]$$

With reference from [8] & [9], taking ‘1’ as trapezoidal fuzzy number we have $1 = <0.8788, 1.1422, 1.2822, 1.4922>$ with 0.2634, 0.14 & 0.21 taken as the difference.

$$L = \left[\frac{<0.7692,0.8846,1.0384,1.1538>}{<0.8788,1.1422,1.2822,1.4922>-<0.7692,0.8846,1.0384,1.1538>} - \frac{<0.084,0.1113,343.0812,423.6036>}{<0.8788,1.1422,1.2822,1.4922>-<0.0040,0.0053,16.3372,20.1716>} \right]$$

$$L = \left[\frac{<0.7692,0.8846,1.0384,1.1538>}{<-0.0058,0.3730,2.436,2.5306>} - \frac{<0.084,0.1113,343.0812,423.6036>}{<0.8735,1.1382,21.4538,17.8294>} \right], \quad \text{using 3.2(ii)}$$

$$L = <2.0621, -152.5172, 0.4103, 0.4736> - <0.0738, 0.1274, 19.2424, 19.7449>, \quad \text{using 3.2(iv)} \quad L = <1.9347, -152.591, 20.1552, 19.716>$$

To find the value of P_N :

$$P_N = \left[\frac{(1-\rho)}{(1-\rho^{N+1})} \right] \rho^n; \text{ for } n = 1, 2, 3, \dots, N$$

$$P_N = \left[\frac{(1-(\lambda/\mu))}{(1-(\lambda/\mu)^{21})} \right] (\lambda/\mu)^{20}$$

$$P_N = \left[\frac{1-<\frac{20,23,27,30}{26}>}{1-<\frac{20,23,27,30}{26}>^{21}} \right] <\frac{20,23,27,30}{26}>^{20}$$

$$P_N = \left[\frac{1-(\frac{20,23,27,30}{26,26,26,26})}{1-(\frac{20,23,27,30}{26,26,26,26})^{21}} \right] <\frac{20}{26}, \frac{23}{26}, \frac{27}{26}, \frac{30}{26}>^{20}, \quad \text{using 3.2(vii)}$$

$$P_N = \left[\frac{1-(0.7692,0.8846,1.0384,1.1538)}{1-(0.7692,0.8846,1.0384,1.1538)^{21}} \right] (0.7692, 0.8846, 1.0384, 1.1538)^{20}$$

$$P_N = \left[\frac{1-(0.7692,0.8846,1.0384,1.1538)}{1-(0.0040,0.0053,16.3372,20.1716)} \right] (0.7692, 0.8846, 1.0384, 1.1538)^{20}$$

To find the value of $< 0.7692, 0.8846, 1.0384, 1.1538 >^{20}$:

Using 3.2(v),

$$T = \{(0.7692)^{20}, (0.7692)^{19} (0.8846), (0.7692)^{18} (0.8846)^2\}$$

$$T = \{0.0052, (0.0068)(0.8846), (0.0068)(0.8846), (0.0088)(0.7825)\}$$

$$T = \{0.0052, 0.0060, 0.0060, 0.0068\}$$

$$T_1 = \{(1.1538)^{18}(1.0384)^2, (1.1538)^{19}(1.0384), (1.1538)^{19}(1.0384), (1.1538)^{20}\}$$

$$T_1 = \{(13.1325)(1.0782), (15.1523)(1.0384), (15.1523)(1.0384), (17.4827)\}$$

$$T_1 = \{14.1594, 15.7341, 15.7341, 17.4827\}$$

$$a = \min \{T\} = 0.0052 ; b = \max \{T\} = 0.0068 ; c = \min \{T_1\} = 14.1594, d = \max \{T_1\} = 17.4827$$

$$< 0.7692, 0.8846, 1.0384, 1.1538 >^{20} = < 0.0052, 0.0068, 14.1594, 17.4827 >$$

$$P_N = \left[\frac{1-(0.7692,0.8846,1.0384,1.1538)}{1-(0.0040,0.0053,16.3372,20.1716)} \right] (0.0052, 0.0068, 14.1594, 17.4827)$$

Taking '1' as trapezoidal fuzzy number, $1 = <0.8788, 1.1422, 1.2822, 1.4922>$

$$P_N = \left[\frac{(0.8788,1.1422,1.2822,1.4922)-(0.7692,0.8846,1.0384,1.1538)}{(0.8788,1.1422,1.2822,1.4922)-(0.0040,0.0053,16.3372,20.1716)} \right]$$

$$(0.0052, 0.0068, 14.1594, 17.4827)$$

$$P_N = \left[\frac{<-0.0058,0.3730,2.436,2.5306>}{<0.8735,1.1382,21.4538,17.4827>} (0.0052, 0.0068, 14.1594, 17.4827), \right. \quad \text{using 3.2(ii)}$$

$$P_N = <-0.0050, 0.4270, 0.1366, 0.1179> (0.0052, 0.0068, 14.1594, 17.4827)$$

$$P_N = <-0.000026, 0.00290, (-0.0707+0.0007), (7.4651+0.00080) >, \quad \text{using 3.2(iii)}$$

$$P_N = <-0.000026, 0.00290, 0.0714, 7.4659 >$$

Now the expected cost ,

$$E(c) = \mu c_1 + Lc_2 + Nc_3 + \lambda P_N c_4$$

$$E(c) = 26 < 10000, 11500, 12500, 14000 > + < 1.9347, -152.59, 20.1552, 19.716 >$$

$$< 5.7, 12, 16 > + 100 + < 20, 23, 27, 30 > < -0.000026, 0.00290, 0.0714,$$

$$7.4659 > (15)$$

Taking '100' as trapezoidal fuzzy number, $100 = < 110, 115, 120, 126 >$

$$E(c) = < 260000, 299000, 325000, 364000 > + < 9.6735, -1068.137, (23.2164 + 100.776),$$

$$(-2441.456 + 138.012) > + < 110, 115, 120, 126 > +$$

$$< -0.000026, 0.00290, 0.0714, 7.4659 > < 300, 345, 405, 450 >$$

$$E(c) = < 260000, 299000, 325000, 364000 > + < 9.6735, -1068.137, 123.9924, -2303.444 > +$$

$$< 110, 115, 120, 126 > + < -0.0078, 1.0005, (-0.01053 + 21.42), (1.305 + 2575.7355) >$$

$$E(c) = < 260000, 299000, 325000, 364000 > + < 9.6735, -1068.137, 123.9924, -2303.444 > +$$

$$< 110, 115, 120, 126 > + < -0.0078, 1.0005, 21.40947, 2577.0405 >$$

$$E(c) = < 260120, 298048, 325265, 364400 >$$

Model B :

The minimal expected total cost of a crisp queueing system with finite capacity is given by

$$E(c) = \mu c_1 + Lc_2 + Nc_3 + \lambda P_N c_4$$

$$L_s = \left[\frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{(1-\rho^{N+1})} \right] = L$$

$$L = \left[\frac{(\lambda/\mu)}{1-(\lambda/\mu)} - \frac{(31)(\lambda/\mu)^{31}}{1-(\lambda/\mu)^{31}} \right]$$

$$L = \left[\frac{\langle 20,23,27,30 \rangle / 29}{1-(\langle 20,23,27,30 \rangle / 29)} - \frac{(31)(\langle 20,23,27,30 \rangle / 29)^{31}}{1-(\langle 20,23,27,30 \rangle / 29)^{31}} \right]$$

$$L = \left[\frac{\langle 20/29,23/29,27/29,30/29 \rangle}{1-\langle 20/29,23/29,27/29,30/29 \rangle} - \frac{(31)(\langle 20/29,23/29,27/29,30/29 \rangle)^{31}}{1-(\langle 20/29,23/29,27/29,30/29 \rangle)^{31}} \right], \quad \text{using 3.2(vii)}$$

$$L = \left[\frac{\langle 0.6896,0.7931,0.9310,1.0344 \rangle}{1-\langle 0.6896,0.7931,0.9310,1.0344 \rangle} - \frac{(31)\langle 0.6896,0.7931,0.9310,1.0344 \rangle^{31}}{1-(\langle 0.6896,0.7931,0.9310,1.0344 \rangle)^{31}} \right]$$

To find the value of $\langle 0.6896,0.7931,0.9310,1.0344 \rangle^{31}$:

Using 3.2(v)

$$T = \{ (0.6896)^{31}, (0.6896)^{30}(0.7931), (0.6896)^{30}(0.7931), (0.6896)^{29}(0.7931)^2 \}$$

$$T = \{ 0.00000992, (0.0000144)(0.7931), (0.0000144)(0.7931), (0.00002)(0.6290) \}$$

$$T = \{ 0.00000992, 0.0000113, 0.0000113, 0.0000131 \}$$

$$T_1 = \{ (1.0344)^{29}(0.9310)^2, (1.0344)^{30}(0.9310), (1.0344)^{30}(0.9310), (1.0344)^{31} \}$$

$$T_1 = \{ (2.6728)(0.8668), (2.7583)(0.9310), (2.7583)(0.9310), (2.86036) \}$$

$$T_1 = \{ 2.31689, 2.5679, 2.5679, 2.86036 \}$$

$$a = \min \{T\} = 0.00000992$$

$$b = \max \{T\} = 0.0000131$$

$$c = \min \{T_1\} = 2.31689$$

$$d = \max \{T_1\} = 2.86036$$

$$\langle 0.6896,0.7931,0.9310,1.0344 \rangle^{31} = \langle 0.00000992, 0.0000131, 2.31689, 2.86036 \rangle$$

$$L = \left[\frac{\langle 0.6896,0.7931,0.9310,1.0344 \rangle}{1-\langle 0.6896,0.7931,0.9310,1.0344 \rangle} - \frac{(31)\langle 0.00000992,0.0000131,2.31689,2.86036 \rangle}{1-\langle 0.00000992,0.0000131,2.31689,2.86036 \rangle} \right]$$

$$L = \left[\frac{\langle 0.6896,0.7931,0.9310,1.0344 \rangle}{1-\langle 0.6896,0.7931,0.9310,1.0344 \rangle} - \frac{\langle 0.00030,0.00040,71.8236,88.6712 \rangle}{1-\langle 0.00000992,0.0000131,2.31689,2.86036 \rangle} \right]$$

With reference from [8] & [9], taking '1' as trapezoidal fuzzy number we have $1 = \langle 0.7412, 1.0046, 1.1446, 1.3546 \rangle$ with 0.2634, 0.14 & 0.21 taken as the difference.

$$L = \left[\frac{\langle 0.6896,0.7931,0.9310,1.0344 \rangle}{\langle 0.7412,1.0046,1.1446,1.3546 \rangle - \langle 0.6896,0.7931,0.9310,1.0344 \rangle} - \frac{\langle 0.00030,0.00040,71.8236,88.6712 \rangle}{\langle 0.7412,1.0046,1.1446,1.3546 \rangle - \langle 0.00000992,0.0000131,2.31689,2.86036 \rangle} \right]$$

$$L = \left[\frac{\langle 0.6896,0.7931,0.9310,1.0344 \rangle}{\langle -0.0519,0.315,2.179,2.2856 \rangle} - \frac{\langle 0.00030,0.00040,71.8236,88.6712 \rangle}{\langle 0.7411,1.0045,4.00496,3.67149 \rangle} \right], \quad \text{using 3.2(ii)}$$

$$L = \langle 2.1892, -15.2813, 0.4073, 0.4747 \rangle - \langle 0.00029, 0.00053, 19.5625, 22.1403 \rangle, \quad \text{using 3.2(iv)}$$

$$L = \langle 2.18867, -15.28159, 22.5476, 20.0377 \rangle$$

To find the value of P_N :

$$P_N = \left[\frac{(1-(\lambda/\mu))}{(1-(\lambda/\mu)^{31})} \right] (\lambda/\mu)^{30}$$

$$P_N = \left[\frac{1-\langle \frac{20,23,27,30}{29} \rangle}{1-\langle \frac{20,23,27,30}{29} \rangle^{31}} \right] \langle \frac{20,23,27,30}{29} \rangle^{30}$$

$$P_N = \left[\frac{1 - \left(\frac{20 \ 23 \ 27 \ 30}{29 \ 29 \ 29 \ 29} \right)}{1 - \left(\frac{20 \ 23 \ 27 \ 30}{29 \ 29 \ 29 \ 29} \right)^{31}} \right] < \frac{20}{29}, \frac{23}{29}, \frac{27}{29}, \frac{30}{29} >^{30}, \quad \text{using 3.2(vii)}$$

$$P_N = \left[\frac{1 - (0.6896, 0.7931, 0.9310, 1.0344)}{1 - (0.6896, 0.7931, 0.9310, 1.0344)^{31}} \right] (0.6896, 0.7931, 0.9310, 1.0344)^{30}$$

$$P_N = \left[\frac{1 - (0.7692, 0.8846, 1.0384, 1.1538)}{1 - (0.00000992, 0.0000131, 2.31689, 2.86036)} \right] (0.6896, 0.7931, 0.9310, 1.0344)^{30}$$

To find the value of $< 0.6896, 0.7931, 0.9310, 1.0344 >^{30}$:

Using 3.2(v),

$$T = \{(0.6896)^{30}, (0.6896)^{29} (0.7931), (0.6896)^{29} (0.7931), (0.6896)^{28} (0.7931)^2\}$$

$$T = \{(0.00001438), (0.0002086)(0.7931), (0.0002086)(0.7931), (0.00003024)(0.62900761)\}$$

$$T = \{0.00001438, 0.00001658, 0.00001658, 0.00001902\}$$

$$T_1 = \{(1.0344)^{28}(0.9310)^2, (1.0344)^{29}(0.9310), (1.0344)^{29}(0.9310), (1.0344)^{30}\}$$

$$T_1 = \{(2.57797)(0.866761), (2.666655)(0.9310), (2.666655)(0.9310), (2.758388)\}$$

$$T_1 = \{2.234483, 2.48265, 2.48265, 2.758388\}$$

$$a = \min \{T\} = 0.00001438$$

$$b = \max \{T\} = 0.00001902$$

$$c = \min \{T_1\} = 2.234483$$

$$d = \max \{T_1\} = 2.758388$$

$$<0.6896, 0.7931, 0.9310, 1.0344>^{30} = <0.00001438, 0.00001902, 2.2344838, 2.758388>$$

$$P_N = \left[\frac{1 - (0.6896, 0.7931, 0.9310, 1.0344)}{1 - (0.00000992, 0.0000131, 2.31689, 2.86036)} \right] < 0.00001438, 0.00001902, 2.2344838, 2.758388>$$

Taking '1' as trapezoidal fuzzy number, $1 = <0.7412, 1.0046, 1.1446, 1.3546>$

$$P_N = \left[\frac{(0.7412, 1.0046, 1.1446, 1.3546) - (0.6896, 0.7931, 0.9310, 1.0344)}{(0.7412, 1.0046, 1.1446, 1.3546) - (0.00000992, 0.0000131, 2.31689, 2.86036)} \right]$$

$$<0.00001438, 0.00001902, 2.2344838, 2.758388>$$

$$P_N = \left[\frac{<-0.0519, 0.315, 2.179, 2.2856>}{<0.7411, 1.0045, 4.00496, 3.67149>} \right] <0.00001438, 0.00001902, 2.2344838, 2.758388>$$

$$P_N = <-0.0516, 0.4250, 0.5935, 0.5707> <0.00001438, 0.00001902, 2.2344838, 2.758388>$$

$$P_N = <-0.0000007, 0.00000808, (-0.115299+0.00000853), (1.1722+0.0000108)>, \quad \text{using 3.2(iii)}$$

$$P_N = <-0.0000007, 0.00000808, -0.11529047, 1.1722108>$$

Now the expected cost ,

$$E(c) = \mu c_1 + Lc_2 + Nc_3 + \lambda P_N c_4$$

$$E(c) = 29 < 10000, 15000, 19000, 20000> + < 2.18869, -15.284286, 22.5476, 20.0377>$$

$$< 5, 7, 12, 16> + (30)(5) + (15) < 20, 23, 27, 30>$$

$$< -0.0000007, 0.00000808, -0.11529047, 1.1722108>$$

$$= < 290000, 435000, 551000, 580000> + < 10.94345, -106.9894,$$

$$(26.26428+112.738), (-244.548576+140.2639)> + 150 + < -0.0000007,$$

$$0.00000808, -0.11529047, 1.1722108> < 300, 345, 405, 450>$$

Taking '150' as trapezoidal fuzzy number, $150 = \langle 149, 154, 158, 160 \rangle$

$$E(c) = \langle 290000, 435000, 551000, 580000 \rangle + \langle 10.94345, -106.9894, 139.00228, -104.2846 \rangle \\ + \langle 149, 154, 158, 160 \rangle + \langle -0.00021, 0.00278, (-34.5871 - 0.000283), \\ (404.4127 + 0.003636) \rangle$$

$$E(c) = \langle 290000, 435000, 551000, 580000 \rangle + \langle 10.94345, -106.9894, 139.00228, -104.2846 \rangle \\ + \langle 149, 154, 158, 160 \rangle + \langle -0.00021, 0.00278, -34.5873, 404.41636 \rangle$$

$$E(c) = \langle 290160, 435047, 551262, 580460 \rangle$$

5. Conclusion

In this paper, the minimal expected total cost of a crisp queueing system in (FM/M/1) : (N/FCFS) queueing model using non – linear parametric programming approach has been developed by LR method using LR – type trapezoidal fuzzy numbers. Comparison has been given between alpha – cut method and LR method.

In alpha – cut method, the minimal total cost lie between 260120 and 364400 for Model A and the minimal total cost lie between 290160 and 580460 for Model B. In LR method, the left spread and the right spread are calculated as 260120 and 364400 which give the lower bound and upper bound of the fuzzy measure in Model A. Similarly the left and right spreads are calculated as 290160 and 580460 which give the lower bound and upper bound of the fuzzy measure in model B. Therefore LR method is simple and convenient for decision making problems.

Therefore the manager decides to prefer Model A, since the minimal expected total cost is obtained from the specified model in which the minimal total cost of Model A is 260120.

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