Research Article

Strong Relations Of Fuzzy Automata

N. Mohanarao[1], V. Karthikeyan [2]

[1]Department of Mathematics, Government College of Engineering, Bodinayakkanur, Tamil Nadu, India
[2]Department of Mathematics, Government College of Engineering,Dharmapuri Tamil Nadu, India
[1]mohanaraonavuluri@gmail.com, [2]vkarthikau@gmail.com

Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 20 April 2021

Abstract: Strong diminution and strong dependability relation of fuzzy automata (FA) are introduced and prove that strong dependability relation is an equivalence relation and congruence relation.

Key words: Fuzzy automata (FA), Strong diminution, Strong dependability relation.

1 Introduction

Fuzzy sets was introduced by Zadeh in 1965[4] and it is used in many applications. Fuzzy automaton was introduced by Wee [3]. In this paper, strong diminution and strong dependability relation of FA are introduced and prove that strong dependability is an equivalence relation and congruence relation in FA.

2 Preliminaries

2.1 Definition [2]

A fuzzy automata is $F = (T, I, \beta)$ where,
$T -$ set of states
$I -$ set of input symbols
$\beta -$ fuzzy transition function in $T \times I \times T \to [0,1]$

2.2 Definition [2]

Let $F = (T, I, \beta)$

2.3 Definition [1]

Let $\Theta = t_1, t_2, \dots, t_r$ be a partition of T such that if $\beta(t_a, y, t_b) > 0$ for some $y \in I$ then $t_a \in t_s$ and $t_b \in t_{s+1}$. Then Θ is periodic of order $r \geq 2$. FA F is periodic of period $r \geq 2$ iff $r =$ Maxcard (Θ) and max. is taken from all partitions Θ of F otherwise F is aperiodic.

2.4 Definition [2]

Let $F = (T, I, \beta)$ be FA. A relation R on a set T is an equivalence relation if it is reflexive, symmetric and transitive.

2.5 Definition [3]

Let $F = (T, I, \beta)$ be FA. An equivalence relation R on a set T is said to be congruence relation if $\forall t_a, t_b \in T$ and $y \in I, t_a R t_b$ implies that then there exists $t_l, t_k \in T$ such that $\beta(t_a, y, t_l) > 0, \beta(t_b, y, t_k) > 0$

Note. Throughout this paper we consider deterministic, strongly connected and aperiodic FA.

3 Properties of Strong Relations of Fuzzy automata

3.1 Definition

Let $F = (T, I, \beta)$ be FA. If t_a and $t_b, t_a, t_b \in T$ are called strong diminution relation, denoted by $t_a \Upsilon t_b$ if exists a string $z \in I^*, t_k \in T, \nu \in [0,1]$ such that $\beta(t_a, z, t_k) = \nu > 0 \Leftrightarrow \beta(t_b, z, t_k) = \nu > 0$

3.2 Example

Let $F = (T, I, \beta)$ be FA, where $T = \{t_1, t_2, t_3, t_4\}, I = \{y, z\}$, and β are defined as below. $\beta(t_1, y, t_4) = 0.6$, $\beta(t_1, z, t_2) = 0.3$
$\beta(t_2, y, t_3) = 0.5$, $\beta(t_2, z, t_4) = 0.2$
$\beta(t_3, y, t_2) = 0.5$, $\beta(t_3, z, t_4) = 0.2$

$\beta(t_4, y, t_1) = 0.6$, $\beta(t_4, z, t_3) = 0.3$
The states t_2 and t_3 are strong diminution, since $\beta(t_2, yz, t_4) = 0.2 > 0 \Leftrightarrow \beta(t_3, yz, t_4) = 0.2 > 0$.

3.3 Definition

Let $F = (T, I, \beta)$ be FA. Two states t_a and t_b are said to be strong dependability related, denoted by $t_a \Omega t_b$ if for any string $z_1 \in I^*$ there exists a string $z_2 \in I^*, t_k \in T, \nu \in [0,1]$ such that

$$\beta(t_a, z_1 z_2, t_k) = \nu > 0 \Leftrightarrow \beta\big(t_j, z_1 z_2, t_k\big) = \nu > 0$$

3.4 Example

Let $F = (T, I, \beta)$ be FA, where

$T = \{t_1, t_2, t_3, t_4\}, I = \{y, z\}$, and β are defined as below.
$\beta(t_1, y, t_4) = 0.3$, $\beta(t_1, z, t_2) = 0.5$
$\beta(t_2, y, t_3) = 0.2$, $\beta(t_2, z, t_4) = 0.3$
$\beta(t_3, y, t_2) = 0.2$, $\beta(t_3, z, t_4) = 0.5$
$\beta(t_4, y, t_1) = 0.3$, $\beta(t_4, z, t_3) = 0.3$

For any string $w \in I^*$, there exists a string $yzz \in I^*$ such that

$\beta(t_1, wyzz, t_k) = \nu > 0 \Leftrightarrow \beta(t_4, wyzz, t_k) = \nu > 0$ and
$\beta(t_2, wyzz, t_l) = \nu > 0 \Leftrightarrow \beta(t_3, wyzz, t_l) = \nu > 0$

The states t_1, t_4 and t_2, t_3 are strong dependability related.

3.5 Remark

(i) Strong diminution relation is not an equivalence relation in FA. Since transitive relation does not exists.

3.6 Example

Let $F = (T, I, \beta)$ be FA, where

$T = \{t_1, t_2, t_3, t_4\}, I = \{y, z\}$ and β are defined as below.
$\beta(t_1, y, t_3) = 0.3$, $\beta(t_1, z, t_1) = 0.5$
$\beta(t_2, y, t_1) = 0.7$, $\beta(t_2, z, t_1) = 0.5$
$\beta(t_3, y, t_4) = 0.3$, $\beta(t_3, z, t_4) = 0.3$
$\beta(t_4, y, t_2) = 0.6$, $\beta(t_4, z, t_4) = 0.7$

In the above FA F $\exists$ a string $zz \in I^*$ such that $\beta(t_1, zz, t_1) = 0.5 > 0 \Leftrightarrow \beta(t_2, zz, t_1) = 0.5 > 0$

Therefore the states t_1 and t_2 are strong diminution.

Also there exists a string $yz \in I^*$ such that $\beta(t_2, yz, t_1) = 0.5 > 0 \Leftrightarrow \beta(t_4, yz, t_1) = 0.5 > 0$

Therefore the states t_2 and t_4 are strong diminution but t_1 and t_4 are not strong diminution for any string $w \in I^*$

Hence strong diminution relation is not transitive.

Theorem 3.1 Let $F = (T, I, \beta)$ be an FA. Strong dependability relation on FA is an equivalence relation.

Proof.

Let $F = (T, I, \beta)$ be an FA. Clearly strong dependability relation on FA F is reflexive and symmetric. Now to prove strong dependability relation is an equivalence relation it is enough to prove that it is transitive. Let $t_a \Omega t_b$ and $t_b \Omega t_k$

To prove $t_a \Omega t_k$, we need to prove for any string $v_1 \in I^*$, there exists a string
$v \in I^*, t_n \in T$ such that

$$\beta(t_a, w_1 w_2, t_n) = \nu > 0 \Leftrightarrow \beta(t_k, v_1 v, t_n) = \nu > 0$$

Since $t_a \Omega t_b$ for any string $v_1 \in I^*$ there exists a string $v_2 \in I^*$ and $t_m \in T$ such that

$$\beta(t_a, v_1 v_2, t_m) = \nu > 0 \Leftrightarrow \beta_{N^*}(t_b, v_1 v_2, t_m) = \nu > 0$$

Since $t_b \Omega t_k$, for any string $v_1 v_2 \in I^*$ there exists a string $v_3 \in I^*, t_n \in T$ such that
$\beta(t_b, v_1 v_2 v_3, t_n) = \nu > 0 \Leftrightarrow \beta(t_k, v_1 v_2 v_3, t_n) = \nu > 0$
$\beta(t_b, v_1 v_2 v_3, t_n) = \nu > 0 \Leftrightarrow \beta(t_a, v_1 v_2 v_3, t_n) = \nu > 0$
$\beta(t_a, v_1 v_2 v_3, t_n) = \nu > 0 \Leftrightarrow \beta(t_k, v_1 v_2 v_3, t_n) = \nu > 0$
Now, choose $v_2 v_3 = v$.

For any string $v_1 \in I^*$ there exists string $v \in I^*$ and $t_n \in T$ such that $\beta(t_a, v_1 v, t_n) = \nu > 0 \Leftrightarrow \beta(t_k, v_1 v, t_n) = \nu > 0$.

Hence, $t_i \Omega t_k$

Theorem 3.2

Let $F = (T, I, \beta)$ be an FA. Strong dependability relation on FA is a congruence relation.

Proof.

Let $F = (T, I, \beta)$ be an FA. Strong dependability relation is an equivalence relation. Consider the equivalence classes using strong dependability relation. Let $t_a \Omega t_b, t_a, t_b \in [T_i], i \in I$. Since, F is deterministic and strongly connected, $\exists y \in I, t_l, t_k \in T$ such that $\beta(t_a, y, t_l) = \nu > 0, \beta(t_b, y, t_k) = \nu > 0, t_l \Omega t_k,\ \ t_l, t_k \in [T_j], j \in I$. Hence, Strong dependability relation is congruence relation.

4 Conclusion

We introduce strong diminution and strong dependability relation in FA. We have shown by example that strong diminution is not an equivalence relation. Finally prove that strong dependability relation is an equivalence relation.

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