Euler's Equations Of Motion By Using Interval Function

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Abstract: Euler's Equations of Motion play a role in deciding the approximated solution to that same interval valued problem. The interval valued Euler equations are added and extracted. Euler's Equations are used to determine the shortest path a particle may follow from one location to another location within single-variable functions. When discussing the motion of the particle and its direction, the relationship between the points on the surface and the coordinates of the particle may be investigated.

Key Words: Euler's Equation, Interval methods, optimal Solution

1. Introduction:

The calculus of variations itself and extensions was used to find the optimum model that provides the maximum the economic model's benefit while also satisfying the system's constraints. Optimal control, transport phenomenon, optics, elasticity, vibrations, solid body statics and dynamics, and navigation are really only a few of engineering as well as physics problems that call for the use of an optimum mechanism rather than an optimal point.

This essay would include a concise overview of other more critical topics in the field. In the case with multiple functions and independent variables with or without limits, Euler's equation and Euler's interval equations are all included. It starts with a derivation for Euler's Equation eventually going on to Euler's motion equations with interval values. The purpose of this paper is to help readers understand some steps involved in determining the optimal feature variation problem.

The optimum values of a function was determined to be an ideal function throughout the calculus of variations. A functional is really a function that is constructed on the complete paths of one or even more functions, rather than a collection of independent variables. The integral is a function that occurs throughout the integrand including its integral also in calculus of variations, as well as the function which appears in the integrand of both the integral should be selected to minimize or maximize the value of its integral.

2. Preliminaries: Interval arithmetic [1-8],[21]

Let, $\tilde{k} = [k_1, k_2]$, $\tilde{l} = [l_1, l_2]$

(I). Addition

$$\tilde{k} + \tilde{l} = [k_1 + l_1, k_2 + l_2]$$

(II). Subtraction

$$\tilde{k} - \tilde{l} = [k_1 - l_2, k_2 - l_1]$$

(III). Multiplication

$\tilde{k}.\tilde{l} = [\min(k_1 l_1, k_1 l_2, k_2 l_1, k_2 l_2), \max(k_1 l_1, k_1 l_2, k_2 l_1, k_2 l_2)]$

(IV). Division

$\frac{[p,q]}{[r,s]} = [p,q].[\frac{1}{r},\frac{1}{s}]$ If $0 \notin [r,s]$

(V). $\mu\tilde{k} = [\mu k_1, \mu k_2]\ for \mu \geq 0$

$[\mu k_2, \mu k_1]\ for \mu < 0$

(VI). Inverse

$[k_1,k_2]^{-1} = [\frac{1}{k_2},\frac{1}{k_1}],\ for 0 \notin [k_1,k_2]$

(VII). $[k_1,k_2]^n = [k_1^{\ n}, k_2^{\ n}], if k_1 \geq 0$

$= [k_2^{\ n}, k_1^{\ n}], if k_2 < 0$

$= [0, \max\{k_1^{\ n}, k_2^{\ n}\}], otherwise$

3. Euler Lagrange's Equations

Euler's equation of motion for a continuous flow along the stream line is essentially a relationship between the rate, friction, and density of a flowing fluid. Euler's motion equation is founded on Newton's second law about motion.

Think a stream movement from location to location, and we've looked at one particular cylindrical segment of the flow.

Let $\left[\underline{y}\ (x), \overline{y}\ (x)\right]$ be the curve joining the interval $\left[\underline{A}, \overline{A}\right], \left[\underline{B}, \overline{B}\right]$ which works the given function I is an extreme $I = \int_{x_1}^{x_2} f\left(\left[\underline{x}, \overline{x}\right], \left[\underline{y}, \overline{y}\right], \left[\underline{y}', \overline{y}'\right]\right)dx$.

Consider a group of curves that are next to each other. $\left[\underline{Y}, \overline{Y}\right] = \left[\underline{y}, \overline{y}\right] + \varepsilon[\underline{\eta(x)}, \overline{\eta(x)}]$ (1)

Where ε is a parameter

$[\underline{\eta(x)}, \overline{\eta(x)}]$ is arbitrary differential function

At the end interval $[\underline{A}, \overline{A}]$ *and* $[\underline{B}, \overline{B}]$,

So that at $[\underline{A}, \overline{A}]$, $[\underline{\eta(x_1)}, \overline{\eta(x_1)}] = [0,0]$ and at $[\underline{B}, \overline{B}]$, $[\underline{\eta(x_2)}, \overline{\eta(x_2)}] = [0,0]$

When $\varepsilon = 0$, Neighboring curve become $[\underline{y}, \overline{y}] = [\underline{y(x)}, \overline{y(x)}]$, this is at the extreme.

The family of comparison feature is a group of neighboring curves. If the interval valued function

$$I = \int_{x_1}^{x_2} f\left(\left[\underline{x}, \overline{x}\right], \left[\underline{y}, \overline{y}\right], \left[\underline{y}', \overline{y}'\right]\right)dx$$

Here replacing $[\underline{y}, \overline{y}] = [\underline{Y}, \overline{Y}]$, So we get

$$\int_{x_1}^{x_2} f\left(\left[\underline{x}, \overline{x}\right], \left[\underline{y}, \overline{y}\right], \left[\underline{y}', \overline{y}'\right]\right)dx = \int_{x_1}^{x_2} f\left(\left[\underline{x}, \overline{x}\right]\left[\underline{y}, \overline{y}\right] + \varepsilon[\underline{\eta(x_2)}, \overline{\eta(x_2)}]\left[\underline{y}', \overline{y}'\right]\right)dx \text{ -----(2)}$$

Which is a function of ε (say $I(\varepsilon)$) is a maximum or minimum for $\varepsilon = 0$

$$I(\varepsilon) = \int_{x_1}^{x_2} f\left(\left[\underline{x}, \overline{x}\right], \left[\underline{y}, \overline{y}\right], \left[\underline{y}', \overline{y}'\right]\right)dx$$

For $\varepsilon = 0$ (The neighboring curves becomes the extremum and extremum for $\eta = 0$)

The necessary Conditions for this $\overline{I}'(\varepsilon) = 0$

Differentiating $\overline{I}$ under the integral sign by Lebinitz rule, we have

$$\tilde{I}'(\varepsilon)=\frac{d\tilde{I}}{d\varepsilon}=\int_{x_1}^{x_2}\left(\frac{\partial f}{\partial[\underline{x},\overline{x}]}\frac{\partial[\underline{x},\overline{x}]}{\partial\varepsilon}+\frac{\partial f}{\partial[\underline{y},\overline{y}]}\frac{\partial[\underline{y},\overline{y}]}{\partial\varepsilon}+\frac{\partial f}{\partial[\underline{y}',\overline{y}']}\frac{\partial[\underline{y}',\overline{y}']}{\partial\varepsilon}\right)dx \text{ --------(3)}$$

But ε being independent of $[\underline{x},\overline{x}]$

(i.e) $\frac{\partial[\underline{x}\,\overline{x}]}{\partial\varepsilon}=0$

$$\tilde{I}'(\varepsilon)=\frac{d\tilde{I}}{d\varepsilon}=\int_{x_1}^{x_2}\left(\frac{\partial f}{\partial[\underline{y},\overline{y}]}\frac{\partial[\underline{y},\overline{y}]}{\partial\varepsilon}+\frac{\partial f}{\partial[\underline{y}',\overline{y}']}\frac{\partial[\underline{y}',\overline{y}']}{\partial\varepsilon}\right)dx$$

On differentiating (1) with respect to $[\underline{x},\overline{x}]$

$$\frac{d[\underline{y},\overline{y}]}{d[\underline{x},\overline{x}]}=\left[\underline{y'(x)},\overline{y'(x)}\right]+\varepsilon[\underline{\eta'(x)},\overline{\eta'(x)}]$$

Again differentiating with respect to ε

$$\frac{\partial[\underline{y}',\overline{y}']}{\partial\varepsilon}=[\underline{\eta'(x)},\overline{\eta'(x)}]$$

Differentiating (1) with respect to ε

$$\frac{\partial[\underline{y},\overline{y}]}{\partial\varepsilon}=[\underline{\eta(x)},\overline{\eta(x)}]$$

Now (3) becomes

$$\tilde{I}'(\varepsilon)=\frac{d\tilde{I}}{d\varepsilon}=\int_{x_1}^{x_2}\left(\frac{\partial f}{\partial[\underline{y},\overline{y}]}[\underline{\eta(x)},\overline{\eta(x)}]+\frac{\partial f}{\partial[\underline{y}',\overline{y}']}[\underline{\eta'(x)},\overline{\eta'(x)}]\right)dx \text{ -------(4)}$$

Integrating the second term on the right by parts , we get

$$\tilde{I}'(\varepsilon)=\int_{x_1}^{x_2}\frac{\partial f}{\partial[\underline{y},\overline{y}]}[\underline{\eta(x)},\overline{\eta(x)}]dx+\int_{x_1}^{x_2}\frac{\partial f}{\partial[\underline{y}',\overline{y}']}[\underline{\eta'(x)},\overline{\eta'(x)}]dx$$

$$\tilde{I}'(\varepsilon)=\tilde{I}_1(\varepsilon)+\tilde{I}_2(\varepsilon) \text{ -------(5)}$$

Where $\tilde{I}_1(\varepsilon)=\int_{x_1}^{x_2}\frac{\partial f}{\partial[\underline{y},\overline{y}]}[\underline{\eta(x)},\overline{\eta(x)}]dx$

$$\tilde{I}_2(\varepsilon)=\int_{x_1}^{x_2}\frac{\partial f}{\partial[\underline{y}',\overline{y}']}[\underline{\eta'(x)},\overline{\eta'(x)}]dx$$

Now, we take $\tilde{I}_2$

$$\tilde{I}_2(\varepsilon)=\int_{x_1}^{x_2}\frac{\partial f}{\partial[\underline{y}',\overline{y}']}[\underline{\eta'(x)},\overline{\eta'(x)}]dx$$

$$\tilde{I}_2(\varepsilon)=\left[\frac{\partial f}{\partial[\underline{y}',\overline{y}']}[\underline{\eta(x)},\overline{\eta(x)}]\right]_{x_1}^{x_2}-\int_{x_1}^{x_2}\frac{d}{d[\underline{x},\overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}',\overline{y}']}[\underline{\eta(x)},\overline{\eta(x)}]\right]dx$$

$$\tilde{I}_2(\varepsilon)=\left[\left(\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}[\underline{\eta(x_2)},\overline{\eta(x_2)}]\right)-\left(\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}[\underline{\eta(x_1)},\overline{\eta(x_1)}]\right)\right]-\int_{x_1}^{x_2}\frac{d}{d[\underline{x},\overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}[\underline{\eta(x)},\overline{\eta(x)}]\right]dx$$

$$[\underline{\eta(x_2)},\overline{\eta(x_2)}]=[\underline{\eta(x_1)},\overline{\eta(x_1)}]=[0,\ 0]$$

$$\tilde{I}_2(\varepsilon)=-\int_{x_1}^{x_2}\frac{d}{d[\underline{x},\overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}[\underline{\eta(x)},\overline{\eta(x)}]\right]dx$$

Using in (5)

$$\tilde{I}'(\varepsilon)=\tilde{I}_1(\varepsilon)-\int_{x_1}^{x_2}\frac{d}{d[\underline{x},\overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}[\underline{\eta(x)},\overline{\eta(x)}]\right]dx$$

$$\tilde{I}'(\varepsilon)=\int_{x_1}^{x_2}\frac{\partial f}{\partial[\underline{y},\overline{y}]}[\underline{\eta(x)},\overline{\eta(x)}]dx-\int_{x_1}^{x_2}\frac{d}{d[\underline{x},\overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}[\underline{\eta(x)},\overline{\eta(x)}]\right]dx$$

$$\tilde{I}'(\varepsilon)=\int_{x_1}^{x_2}\left(\frac{\partial f}{\partial[\underline{y},\overline{y}]}-\frac{d}{d[\underline{x},\overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}\right]\right)[\underline{\eta(x)},\overline{\eta(x)}]\ dx$$

For Extremum value $\tilde{I}'(\varepsilon)=0$

$$0=\int_{x_1}^{x_2}\left(\frac{\partial f}{\partial[\underline{y},\overline{y}]}-\frac{d}{d[\underline{x},\overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}\right]\right)[\underline{\eta(x)},\overline{\eta(x)}]\ dx$$

$[\underline{\eta(x)},\overline{\eta(x)}]$ is an arbitrary function

$$\frac{\partial f}{\partial[\underline{y},\overline{y}]}-\frac{d}{d[\underline{x},\overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}\right]=0$$

Which is an interval valued Euler's Equation of Motion

4. Numerical Examples

Prove that the shortest distance between two points in a plane is a straight line.

Proof:

The shortest path between two points isn't necessarily the straight line. That smallest distance between two clusters is determined by the object's or surface's geometry. A line is the shortest distance for flat surfaces, but great-circle distances reflect the real shortest distance for circular surfaces, such as Earth.

Let $[\underline{A},\overline{A}]$ and $[\underline{B},\overline{B}]$ be the given points and $\tilde{s}$ is the interval arc length of a curve connected them.

Then $\tilde{s}=\int_{x_1}^{x_2}d\,\tilde{s}=\int_{x_1}^{x_2}\sqrt{1+[\underline{y'}\,\overline{y'}]^2}\ d\,x$

Now S will be minimum if it satisfies interval valued Euler's Equation

$$\frac{\partial f}{\partial[\underline{y},\overline{y}]}-\frac{d}{d[\underline{x},\overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}',\overline{y'}]}\right]=0\text{-----}(1)$$

Here $\tilde{f} = \sqrt{1+[\underline{y}' \overline{y}']^2}$, this is independent of $[\underline{y}, \overline{y}]$

$(i.e) \qquad \dfrac{\partial f}{\partial[\underline{y}, \overline{y}]} = 0$

Using in (1)

$$\frac{d}{d[\underline{x},\ \overline{x}]}\left[\frac{\partial f}{\partial[\underline{y}', \overline{y}']}\right] = 0$$

Differentiating $\tilde{f} = \sqrt{1+[\underline{y}' \overline{y}']^2}$ with respect to $[\underline{y}, \overline{y}]$

$$\frac{\partial f}{\partial[\underline{y}', \overline{y}']} = \frac{[\underline{y}', \overline{y}']}{\sqrt{1+[\underline{y}' \overline{y}']^2}}$$

$$\frac{d}{dx}\left(\frac{\partial f}{\partial[\underline{y}', \overline{y}']}\right) = \frac{d}{dx}\left(\frac{[\underline{y}', \overline{y}']}{\sqrt{1+[\underline{y}' \overline{y}']^2}}\right)$$

We take integration $\displaystyle\int \frac{d}{dx}\left(\frac{[\underline{y}', \overline{y}']}{\sqrt{1+[\underline{y}' \overline{y}']^2}}\right) d\,x$

$$\frac{[\underline{y}', \overline{y}']}{\sqrt{1+[\underline{y}' \overline{y}']^2}} = cons\tan t$$

$$[\underline{y}', \overline{y}'] = C\sqrt{1+[\underline{y}' \overline{y}']^2}$$

$$[\underline{y}', \overline{y}'] = Cons\tan t \quad (Say\ \ m)$$

Again Integration

$$\int [\underline{y}', \overline{y}']\, d\tilde{x} = \int m\, d\,\tilde{x}$$

$$[\underline{y}, \overline{y}] = m\ [\underline{x},\ \ \overline{x}] + C$$

$\underline{y} = m\ \ \underline{x} + C$ and $\overline{y} = m\ \ \overline{x} + C$ are the straight line

The constant m and C are determinant from the fact that the straight line passing through $[\underline{A}, \overline{A}]$ to $[\underline{B}, \overline{B}]$.

Conclusion:

The suggested interval valued Euler's Equations problems produce neat and precise interval system graphs. The Euler equations of motion, and also interval valued Euler equations of integrals that depend upon functions for just a single variable, are determined, and the shortest path among two points in a plane is shown to be a straight line. This method is easy to study and comprehend, and it is effective in solving real-world problems. Euler's Equations can be written with interval values have high accuracy at a low computing expense. When the outcomes are compared to all those obtained using equivalent or analogous methods, we see that the interval results achieved have a strong degree of compatibility.

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