$(\in\gamma, \in\gamma \vee q\delta)$ - Fuzzy Bi-Ideals of Near-Rings

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Abstract: In this paper, we introduced the concept of $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy bi-ideals and $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ - fuzzy bi-ideals of a near-ring. Some new characterizations are also given. In particular, homomorphic behaviour of $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy bi-ideals are also discussed. New type of fuzzy bi-ideals of near rings is also introduced.

Keywords: Near-ring, $(\in\gamma, \in\gamma \vee q\delta)$ - fuzzy ideal, $(\in\gamma, \in\gamma \vee q\delta)$ - fuzzy bi-ideal, homomorphism, $(\overline{\in}\gamma, \overline{\in}\gamma \vee q\overline{\delta})$ - fuzzy bi-ideal. Subject Classification: 16Y30, 03E72

1. Introduction

A new type of fuzzy subgroup , that is , the $(\in, \in \vee q)$ –fuzzy sub group, was introduced by Bhakat and Das[2] using the combined notions of "belongingness" and "quasicoincidence" of fuzzy points and fuzzy sets. In fact, the $(\in, \in \vee q)$ - fuzzy sub group is an important generalization of Rosenfeld's fuzzy subgroup. It is now natural to investigate similar type of generalizations of the existing fuzzy subsystems with other algebraic structures, see[3, 4, 10-12, 14].In [3], Davvaz introduced the concepts of $(\in, \in \vee q)$ - fuzzy subnear-rings (ideals) of near-rings and investigated some of their related properties. Zhan[11] considered the concept of $(\overline{\in}, \overline{\in} \vee \overline{q})$ - fuzzy subnear-rings (ideals) of near-rings and obtained some of its related properties. Finally, some characterizations of $[\mu]_t$ by means of $(\in, \in \vee q)$ - fuzzy ideals were also given. Zhan and Yin[14] redefined generalized fuzzy subnear-rings (ideals) of near-ring and investigated some of their related properties. Zhan and Yin [15] also introduce $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy subnear-rings (ideals) of a near-rings.

In this paper, the concept of $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy bi-ideals, $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ - fuzzy bi-ideals of a near-rings is given with its equivalent conditions. We give the relationship between $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy bi-ideals and crisp ideals of near-rings. The homomorphism in $(\in_\gamma, \in_\gamma \vee q_\delta)$ - fuzzy bi-ideals of a near-rings is also discussed with its related properties. Also we introduce new type of fuzzy bi-ideals of a near-rings.

2. Preliminaries

Definition 2.1: A non-empty set R with two binary operations "+" and "." is called a left near-ring if it satisfies the following conditions:

1. (R, +) is a group,
2. (R, .) is a semigroup,
3. $x.(y+z) = x.y + x.z$, for all $x, y, z \in R$.

We will use the word "near-ring" to mean "left near-ring" and denote xy instead of x.y.

Definition 2.2: [5] A subgroup B of R is said to be a bi-ideal if $BNB \subseteq B$

Note 2.3: [15] A fuzzy set μ of R of the form

$$\mu(y) = \begin{cases} t(\neq 0) & \text{if } y = x, \\ 0 & \text{if } y \neq x, \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t .A fuzzy point x_t is said to belong to (resp., be quasi-coincident with) a fuzzy set μ, written as $x_t \in \mu$ (resp., $x_t\, q\, \mu$) if $\mu(x) \geq t$ (resp., $\mu(x) + t > 1$). If $x_t \in \mu$ or $x_t\, q\, \mu$, then, we write $x_t \in \vee q\mu$. If $\mu(x) < t$ (resp., $\mu(x) + t \leq 1$) then, we call $x_t \overline{\in} \mu$ (resp., $x_t \overline{q} \mu$). We note that the symbol $\overline{\in \vee q}$ means that $\in \vee q$ does not hold.

Result 2.4: [15] Let $\gamma, \delta \in [0,1]$ be such that $\gamma < \delta$ .For a fuzzy point x_r and a fuzzy set μ of R, we say that

1. $x_r \in_\gamma \mu$ if $\mu(x) \geq r > \gamma$.
2. $x_r\, q_\delta\, \mu$ if $\mu(x) + r > 2\delta$.
3. $x_r \in_\gamma \vee q_\delta \mu$ if $x_r \in_\gamma \mu$ or $x_r\, q_\delta\, \mu$.

Definition 2.5: [15] A fuzzy set μ of R is called an $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy subnear-ring of R if for all $t, r \in (\gamma,1]$ and $x, y, a \in R$

i)a) $x_t \in_\gamma \mu$ and $y_r \in_\gamma \mu$ imply $(x+y)_{t \wedge r} \in_\gamma \vee q_\delta \mu$,

b) $x_t \in_\gamma \mu$ implies $(-x)_t \in_\gamma \vee q_\delta \mu$,

ii) $x_t \in_\gamma \mu$ and $y_r \in_\gamma \mu$ imply $(xy)_{t \wedge r} \in_\gamma \vee q_\delta \mu$,

Moreover, μ is called an $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy ideal of R if μ is $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy subnear-ring of R and

iii) $x_r \in_\gamma \mu$ implies $(y + x - y)_r \in_\gamma \vee q_\delta \mu$,

iv) $y_r \in_\gamma \mu$ and $x \in R$ imply $(xy)_r \in_\gamma \vee q_\delta \mu$,

v) $a_r \in_\gamma \mu$ implies $((x+a)y-xy)_r \in_\gamma \vee q_\delta\, \mu$.

Definition 2.6: [14] Let μ and λ be any two fuzzy sets of R. The product of μ and λ is defined by

$$(\mu \circ \lambda)(x) = \bigvee_{x=ab} (\mu(a) \wedge \lambda(b))$$

and $(\mu \circ \lambda)(x) = 0$ if x cannot be expressed as x = ab

Definition 2.7: [14] Let μ and λ be any two fuzzy sets of R. The sum of μ and λ is defined by

$$(\mu + \lambda)(x) = \bigvee_{x=a+b} (\mu(a) \wedge \lambda(b))$$

and $(\mu + \lambda)(x) = 0$ if x cannot be expressed as x = a + b.

In particular, for any fuzzy set μ of R and any element x of R, the sum of x and μ is given by

$$(x + \mu)(y) = \bigvee_{y=x+a} \mu(a)$$

and $(x + \mu)(y) = 0$ if x cannot be expressed as y = x + a.

. $(\in_\gamma, \in_\gamma \vee q_\delta)$ -Fuzzy bi-ideals of near-rings

Definition 3.1: A fuzzy set μ of R is called an $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy bi-ideal of R if for all $t, r \in (\gamma,1]$ and x, y, z ∈ R

(I1a) $x_t \in_\gamma \mu$ and $y_r \in_\gamma \mu$ imply $(x+y)_{t\wedge r} \in_\gamma \vee q_\delta\, \mu$,

(I1a*) $x_t \in_\gamma \mu$ implies $(-x)_t \in_\gamma \vee q_\delta\, \mu$,

(I1b) $x_t \in_\gamma \mu$ and $y_r \in_\gamma \mu$ imply $(xy)_{t\wedge r} \in_\gamma \vee q_\delta\, \mu$,

(I1c) $x_t \in_\gamma \mu$ implies $(y + x - y)_t \in_\gamma \vee q_\delta\, \mu$,

(I1d) $y_t \in_\gamma \mu$ and $x \in R$ imply $(xy)_t \in_\gamma \vee q_\delta\, \mu$,

(I1e) $z_t \in_\gamma \mu$ implies $((x+z)y-xy)_t \in_\gamma \vee q_\delta\, \mu$,

(I1f) $x_t \in_\gamma \mu$ and $z_r \in_\gamma \mu$ imply $(xyz)_{t\wedge r} \in_\gamma \vee q_\delta\, \mu$,

Example 3.2: If R={0, a, b, c} be klein's four group. Define '+' and '.' in R as follows.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	0	0	0
c	0	a	a	c

Then (R, +, .) is a near ring. Define a fuzzy set μ of R as follows μ(0)=0.7, μ(a)= μ(c)=0.4, μ(b)=0.8. Then μ is a $(\in_{0.1}, \in_{0.1} \vee q_{0.5})$ - fuzzy bi-ideal of R.

Theorem 3.3: A fuzzy set μ of R is called an $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy bi-ideal of R if and only if for all $t, r \in (\gamma,1]$ and x, y, z ∈ R.

(I2a) $\mu(x+y) \vee \gamma \geq \mu(x) \wedge \mu(y) \wedge \delta$

(I2a*) $\mu(-x) \vee \gamma \geq \mu(x) \wedge \delta$

(I2b) $\mu(xy) \vee \gamma \geq \mu(x) \wedge \mu(y) \wedge \delta$

(I2c) $\mu(y+x-y) \vee \gamma \geq \mu(x) \wedge \delta$

(I2d) $\mu(xy) \vee \gamma \geq \mu(y) \wedge \delta$

(I2e) $\mu((x+z)y-xy) \vee \gamma \geq \mu(z) \wedge \delta$

(I2f) $\mu(xyz) \vee \gamma \geq \mu(x) \wedge \mu(z) \wedge \delta$

Proof: We only prove (I1f)⇔(I2f). The other proofs are similar.

(I1f) ⇒ (I2f) If there exists x, y, z ∈ R such that $\mu(xyz) \vee \gamma < r = \mu(x) \wedge \mu(z) \wedge \delta$ then $\mu(x) \geq r > \gamma$, $\mu(z) \geq r > \gamma$, $\mu(xyz) < r$ and $\mu(xyz) + r < 2r \leq 2\delta$ that is $x_r \in_\gamma \mu$, $z_r \in_\gamma \mu$ but $(xyz)_r \overline{\in_\gamma \vee q_\delta}\, \mu$, a contradiction. Hence (I2f) holds.

(I2f)⇒ (I1f) If there exists x, y, z ∈ R and $t, r \in (\gamma,1]$ such that $x_t \in_\gamma \mu$, $z_r \in_\gamma \mu$ but $(xyz)_{t\wedge r} \overline{\in_\gamma \vee q_\delta}\, \mu$, then $\mu(x) \geq t$, $\mu(z) \geq r$, $\mu(xyz) < t \wedge r$ and $\mu(xyz) + t \wedge r \leq 2\delta$. It follows that $\mu(xyz) < \delta$ and so $\mu(xyz) \vee \gamma < t \wedge r \wedge \delta \leq \mu(x) \wedge \mu(y) \wedge \delta$, a contradiction. Hence (I1f) holds.

Remark 3.4: For any $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ fuzzy bi-ideal of R, we can conclude that

1. If γ =0 and δ =1, then μ is the fuzzy bi-ideal of R.
2. If γ =0 and δ =0.5, then μ is the $(\in, \in \vee q)$ fuzzy bi-ideal of R.
3. If γ =0.5 and δ =1, then μ is the $(\bar{\in}, \bar{\in} \vee \bar{q})$ fuzzy bi-ideal of R.

Note 3.5: For any fuzzy set μ of R, we define $\mu_r^{\gamma} = \{x \in R / x_r \in_{\gamma} \mu\}$,

$\mu_r^{\delta} = \{x \in R / x_r\, q_{\delta}\, \mu\}$

$[\mu]_r^{\delta} = \{x \in R / x_r \in_{\gamma} \vee q_{\delta}\, \mu\}$ for all r ∈ [0, 1]. It is clear that $[\mu]_r^{\delta} = \mu_r^{\gamma} \cup \mu_r^{\delta}$

The next theorem provides the relationship between $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideals of R and crisp bi-ideals of R.

Theorem 3.6: Let μ be a fuzzy set of R. Then

1) μ is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R if and only if μ_r^{γ} $(\neq \varphi)$ is a bi-ideal of R for all $r \in (\gamma, \delta]$

2) If $2\delta = 1+\gamma$, then μ is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R if and only if μ_r^{δ} $(\neq \varphi)$ is a bi-ideal of R for all $r \in (\delta, 1]$

3) If $2\delta = 1+\gamma$, then μ is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R if and only if $[\mu]_r^{\delta}$ $(\neq \varphi)$ is a bi-ideal of R for all $r \in (\gamma, 1]$

Proof: 1) We only prove last condition. The other proofs are similar.

(I2f) Let μ be an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R and x, y, z ∈ μ_r^{γ} for all $r \in (\gamma, \delta]$, then $\mu(x) \geq r > \gamma$, $\mu(y) \geq r > \gamma$, $\mu(z) \geq r > \gamma$. It follows that $\mu(xyz) \vee \gamma \geq \mu(x) \wedge \mu(z) \wedge \delta \geq r \wedge \delta = r$. (i.e) xyz $\in \mu_r^{\gamma}$. Similarly, we can prove the other conditions of bi-ideals hold. Hence μ_r^{γ} is an bi-ideal of R for all $r \in (\gamma, \delta]$. Conversely, assume that μ_r^{γ} is a bi-ideal of R for all $r \in (\gamma, \delta]$. Let x, y ∈ R. If $\mu(xyz) \vee \gamma < r = \mu(x) \wedge \mu(z) \wedge \delta$ then $x_r \in_{\gamma} \mu$, $z_r \in_{\gamma} \mu$ but $(xyz)_r\, \overline{\in_{\gamma} \vee q_{\delta}}\, \mu$ that is x, y, z ∈ μ_r^{γ}, since μ_r^{γ} is an bi-ideal, we have xyz ∈ μ_r^{γ}, a contradiction. Hence $\mu(xyz) \vee \gamma \geq \mu(x) \wedge \mu(z) \wedge \delta$. Thus μ be an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R.

The proof of (2) is similar to the proof of (1)

3) We only prove last condition. The other proofs are similar.

(I2f) Let μ be an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R and $r \in (\gamma, 1]$. Then for all x, y, z $\in [\mu]_r^{\delta}$, we have $x_r \in_{\gamma} \vee q_{\delta}\, \mu$, $y_r \in_{\gamma} \vee q_{\delta}\, \mu$ and $z_r \in_{\gamma} \vee q_{\delta}\, \mu$, that is $\mu(x) \geq r > \gamma$ (or) $\mu(x) > 2\delta\text{-}r > 2\delta\text{-}1 = \gamma$, $\mu(y) \geq r > \gamma$ (or) $\mu(y) > 2\delta\text{-}r > 2\delta\text{-}1 = \gamma$ and $\mu(z) \geq r > \gamma$ (or) $\mu(z) > 2\delta\text{-}r > 2\delta\text{-}1 = \gamma$. Since μ is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R, then $\mu(xyz) \vee \gamma \geq \mu(x) \wedge \mu(z) \wedge \delta$ and so $\mu(xyz) \geq \mu(x) \wedge \mu(z) \wedge \delta$.

Case(i): $r \in (\gamma, \delta]$ then $2\delta\text{-}r \geq \delta \geq \gamma$ and so $\mu(xyz) \geq r \wedge r \wedge \delta = r$ (or) $\mu(xyz) \geq r \wedge (2\delta\text{-}r) \wedge \delta = r$ (or) $\mu(xyz) \geq (2\delta\text{-}r) \wedge (2\delta\text{-}r) \wedge \delta = \delta \geq r$. Hence $(xyz)_r \in_{\gamma} \mu$.

Case(ii): $r \in (\delta, 1]$ then $2\delta\text{-}r < \delta < \gamma$ and so $\mu(xyz) \geq r \wedge r \wedge \delta = \delta > 2\delta\text{-}r$ (or) $\mu(xyz) > r \wedge (2\delta\text{-}r) \wedge \delta = 2\delta\text{-}r$ (or) $\mu(xyz) > (2\delta\text{-}r) \wedge (2\delta\text{-}r) \wedge \delta = 2\delta\text{-}r$. Hence $(xyz)_r\, q_{\delta}\, \mu$.Thus in any case $(xyz)_r \in_{\gamma} \vee q_{\delta}\, \mu$ that is xyz $\in [\mu]_r^{\delta}$. Hence $[\mu]_r^{\delta}$ is a bi-ideal of R. Conversely, assume that $[\mu]_r^{\delta}$ is a bi-ideal of R for all $r \in (, \delta]$. Let x, y ∈ R. If $\mu(x+y) \vee \gamma < r = \mu(x) \wedge \mu(y) \wedge \delta$ then $x_r \in_{\gamma} \mu$, $y_r \in_{\gamma} \mu$ but $(x+y)_r\, \overline{\in_{\gamma} \vee q_{\delta}}\, \mu$ that is x, y $\in [\mu]_r^{\delta}$, since $[\mu]_r^{\delta}$ is a bi-ideal, we have x+y $\in [\mu]_r^{\delta}$, a contradiction. Hence $\mu(x+y) \vee \gamma \geq \mu(x) \wedge \mu(y) \wedge \delta$.Similarly, Let x, y, z ∈ R. If $\mu(xyz) \vee \gamma < r = \mu(x) \wedge \mu(z) \wedge \delta$ then $x_r \in_{\gamma} \mu$, $z_r \in_{\gamma} \mu$ but $(xyz)_r\, \overline{\in_{\gamma} \vee q_{\delta}}\, \mu$ that is x, z $\in [\mu]_r^{\delta}$, since$[\mu]_r^{\delta}$ is a bi-ideal, we have xyz $\in [\mu]_r^{\delta}$, a contradiction. Hence $\mu(xyz) \vee \gamma \geq \mu(x) \wedge \mu(z) \wedge \delta$. Similarly, we can prove other results. Thus μ is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R.

Theorem 3.7: The intersection of any family of $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R.

Proof: Let $\{\mu_i\}_{i \in I}$ be a family of $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R and x, y, z ∈ R. Then $(\wedge_{i \in I} \mu_i)(xyz) \vee \gamma = \wedge_{i \in I} (\mu_i(xyz)) \vee \gamma$. Since each μ_i is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R, so $\mu(xyz) \vee \gamma \geq \mu(x) \wedge \mu(z) \wedge \delta$.

Now, $(\wedge_{i \in I} \mu_i)(xyz) \vee \gamma = \wedge_{i \in I} (\mu_i(xyz)) \vee \gamma \geq \wedge_{i \in I} (\mu_i(x) \wedge \mu_i(z) \wedge \delta) = (\wedge_{i \in I} \mu_i)(x) \wedge (\wedge_{i \in I} \mu_i)(z) \wedge \delta$. Hence, $(\wedge_{i \in I} \mu_i)$ is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R.

Theorem 3.8: The union of any family of $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R.

Proof: It can be easily verified.

Theorem 3.9: Let f : R→S be a onto homomorphism of near rings if B be a $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of S, then the pre image of $f^{-1}(\mathbf{B})$ of B under f in R is also an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R.

Proof: For all x, y ∈ R. Then $f^{-1}(\mu)(x+y) \vee \gamma = \mu(f(x+y)) \vee \gamma = \mu(f(x)+f(y)) \vee \gamma \geq \mu(f(x)) \wedge \mu(f(y)) \wedge \delta = f^{-1}(\mu)(x) \wedge f^{-1}(\mu)(y) \wedge \delta$

Also, for all x, y, z ∈ R. Then $f^{-1}(\mu)(xyz) \vee \gamma = \mu(f(xyz)) \vee \gamma = \mu(f(x)\, f(y)\, f(z)) \vee \gamma \geq \mu(f(x)) \wedge \mu(f(z)) \wedge \delta = f^{-1}(\mu)(x) \wedge f^{-1}(\mu)(z) \wedge \delta$. Hence, $f^{-1}(\mathbf{B})$ is an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R.

Theorem 3.10: Let f: R→S be an onto homomorphism of near rings then B be a $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of S, if $f^{-1}(\mathbf{B})$ of B under f in R is also an $(\in_{\gamma}, \in_{\gamma} \vee q_{\delta})$ -fuzzy bi-ideal of R.

Proof: It can be easily verified.

4. $(\bar{\in}_{\gamma}, \bar{\in}_{\gamma} \vee \bar{q}_{\delta})$ –Fuzzy bi-ideals of near-rings

Definition 4.1: A fuzzy set μ of R is called an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ -fuzzy bi-ideal of R if for all t, r ∈ $(\gamma,1]$ and x, y, z ∈ R

(I3a) $(x+y)_{t\wedge r} \overline{\in}_\gamma$ μ implies $x_t \overline{\in}_\gamma \vee \overline{q}_\delta$ μ or $y_r \overline{\in}_\gamma \vee \overline{q}_\delta$ μ,

(I3a*) $(-x)_t \overline{\in}_\gamma$ μ implies $x_t \overline{\in}_\gamma \vee \overline{q}_\delta$ μ,

(I3b) $(xy)_{t\wedge r} \overline{\in}_\gamma$ μ implies $x_t \overline{\in}_\gamma \vee \overline{q}_\delta$ μ or $y_r \overline{\in}_\gamma \vee \overline{q}_\delta$ μ,

(I3c) $(y + x - y)_t \overline{\in}_\gamma$ μ implies $x_t \overline{\in}_\gamma \vee \overline{q}_\delta$ μ,

(I3d) $(xy)_t \overline{\in}_\gamma$ μ and x ∈ R implies $y_t \overline{\in}_\gamma \vee \overline{q}_\delta$ μ,

(I3e) $((x+z)y-xy)_t \overline{\in}_\gamma$ μ implies $z_t \overline{\in}_\gamma \vee \overline{q}_\delta$ μ,

(I3f) $(xyz)_{t\wedge r} \overline{\in}_\gamma$ μ implies $x_t \overline{\in}_\gamma \vee \overline{q}_\delta$ μ or $z_r \overline{\in}_\gamma \vee \overline{q}_\delta$ μ

Theorem 4.2: A fuzzy set μ of R is called an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ -fuzzy bi-ideal of R if and only if for all t, r ∈ $(\gamma,1]$ and x, y, z ∈ R.

(I4a) $\mu(x+y) \vee \delta \geq \mu(x) \wedge \mu(y)$

(I4a*) $\mu(-x) \vee \delta \geq \mu(x)$

(I4b) $\mu(xy) \vee \delta \geq \mu(x) \wedge \mu(y)$

(I4c) $\mu(y+x-y) \vee \delta \geq \mu(x)$

(I4d) $\mu(xy) \vee \delta \geq \mu(y)$

(I4e) $\mu((x+z)y-xy) \vee \delta \geq \mu(z)$

(I4f) $\mu(xyz) \vee \delta \geq \mu(x) \wedge \mu(z)$

Proof: The proof is similar to the proof of theorem 3.3

Theorem 4.3: Let μ be a fuzzy set of R. Then

1) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ -fuzzy bi-ideal of R if and only if μ_r^γ (≠ φ) is a bi-ideal of R for all r ∈ $(\delta, 1]$

2) μ is an $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ -fuzzy bi-ideal of R if and only if μ_r^δ (≠ φ) is a bi-ideal of R for all r ∈ $(\gamma, \delta]$

Proof: The proof is similar to the proof of theorem 3.6

5. New type fuzzy bi-ideal of near-rings

Let μ and λ be any two fuzzy sets of R. If $x_t \in_\gamma$ μ implies $x_t \in_\gamma \vee q_\delta$ λ for all x ∈ R and t ∈ (0, 1], then we write $\mu \subseteq \vee q_\delta \lambda$. If $x_t \overline{\in}_\gamma$ μ implies $x_t \overline{\in}_\gamma \vee \overline{q}_\delta$ λ for all x ∈ R and t ∈ (0, 1], then we write $\mu \supseteq \vee \overline{q}_\delta \lambda$.

Definition 5.1: A fuzzy set μ of R is called a new $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy bi-ideal of R if it satisfies:

(I5a) $(\mu + \mu) \subseteq \vee q_\delta \mu$

(I5a*) $\mu^{-1} \subseteq \vee q_\delta \mu$

(I5b) $(\mu \circ \mu) \subseteq \vee q_\delta \mu$

(I5c) $(y + \mu - y) \subseteq \vee q_\delta \mu$

(I5d) $(\chi_R \circ \mu) \subseteq \vee q_\delta \mu$

(I5e) $((x + \mu) \circ y - xy) \subseteq \vee q_\delta \mu$

(I5f) $(\mu \circ \Upsilon_R \circ \mu) \subseteq \vee q_\delta \mu$

Theorem 5.2: A fuzzy set μ of R is a new $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy bi-ideal of R if and only if it satisfies (I2a), (I2a*), (I2b), (I2c), (I2d), (I2e), (I2f)

Proof: Let μ be a new $(\in_\gamma, \in_\gamma \vee q_\delta)$ -fuzzy bi-ideal of R. We only prove (I5f). The others are similar.

If there exists x, y, z ∈ R such that $\mu(xyz) \vee \gamma < t = \mu(x) \wedge \mu(z) \wedge \delta$ then $t < \delta$, $x_t \in_\gamma \mu$, $y_t \in_\gamma \mu$, $z_t \in_\gamma \mu$ but $(xyz)_t \overline{\in_\gamma \vee q_\delta}\, \mu$.

Since,

$$(\mu \circ \Upsilon_R \circ \mu)\,(xyz) = \bigvee_{(xyz)=(abc)} (\mu(a) \wedge \mu(c))$$

$\geq \mu(x) \wedge \mu(z) \geq t,$

we have $(xyz)_t \in_\gamma (\mu \circ \Upsilon_R \circ \mu)$. Thus $(xyz)_t \in_\gamma \vee q_\delta \mu$, a contradiction. This proves (I2f) holds.

Conversely, assume that the conditions hold. We only prove (I5f) holds. The others are similar. Let x, y, z ∈ R and t ∈ (0, 1] be such that $x_t \in_\gamma (\mu \circ \Upsilon_R \circ \mu)$ but $x_t \overline{\in_\gamma \vee q_\delta}$ μ. Then $\mu(x) < t$ and $\mu(x) < \delta$. By definition

$$(\mu \circ \Upsilon_R \circ \mu)(x) = \bigvee_{x=(abc)} \big(\mu(a) \wedge \mu(c)\big)$$

Since $\delta > \mu(x) = \mu(abc) = \mu(abc) \vee \gamma \geq \mu(a) \wedge \mu(c) \wedge \delta$ and so $\mu(x) \geq \mu(a) \wedge \mu(c)$. Thus

$$t \leq (\mu \circ \Upsilon_R \circ \mu)(x) \leq \bigvee_{x=(abc)} \mu(x) = \mu(x)$$

that is $\mu(x) \geq t$, a contradiction. This proves (I5f) holds.

Definition 5.3: A fuzzy set μ of R is called an new $(\overline{\in}_\gamma, \overline{\in}_\gamma \vee \overline{q}_\delta)$ -fuzzy bi-ideal of R if it satisfies:

(I6a) $\mu \supseteq \vee \overline{q}_\delta (\mu + \mu)$

(I6a*) $\mu \supseteq \vee \overline{q}_\delta \mu^{-1}$

(I6b) $\mu \supseteq \vee \overline{q}_\delta (\mu \circ \mu)$

(I6c) $\mu \supseteq \vee \bar{q}_{\delta}$ (y + μ -y)
(I6d) $\mu \supseteq \vee \bar{q}_{\delta}$ (χ_R o μ)
(I6e) $\mu \supseteq \vee \bar{q}_{\delta}$ ((x + μ) o y-xy)
(I6f) $\mu \supseteq \vee \bar{q}_{\delta}$ (μ o Υ_R o μ)

Theorem 5.4: A fuzzy set μ of R is a new ($\bar{\in}_{\gamma}$, $\bar{\in}_{\gamma} \vee \bar{q}_{\delta}$) -fuzzy bi-ideal of R if and only if it satisfies (I4a), (I4a*), (I4b), (I4c), (I4d), (I4e), (I4f)

Proof: The proof is similar to the proof of Theorem 5.2

6. Conclusion:

In this paper, we discussed the concept of ($\in_{\gamma}$, $\in_{\gamma} \vee q_{\delta}$) - fuzzy bi-ideals of a near-rings and gave several characterizations. Also we gave the homomorphism condition in ($\in_{\gamma}$, $\in_{\gamma} \vee q_{\delta}$) -fuzzy bi-ideals of near-rings. Our definitions probably can be applied in other kinds of near-rings.

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