Pre $\tilde{I}$ Generalized Connected Soft Set in a Soft Topological Space with Respect to an Ideal

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Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 20 April 2021

Abstract: Molodstov introduced the concept of soft set as a completely new Mathematical tool with adequate parameterization for dealing with uncertainties. In a soft topological space with the soft ideal $\tilde{I}$ is defined as i) $F_E \in \tilde{I}$ and $G_E \in \tilde{I} \Rightarrow F_E \tilde{\cup} G_E \in \tilde{I}$, ii) $F_E \in \tilde{I}$ and $G_E \tilde{\subseteq} F_E \Rightarrow G_E \in \tilde{I}$ and it is denoted by $(X, \tau, E, \tilde{I})$. We have already defined Pre $\tilde{I}$ generalized closed soft set as a soft set and it satisfies $(P\tilde{I}SCl\ (F_E))\backslash G_E \in \tilde{I}$ whenever $F_E \tilde{\subseteq} G_E$ and G_E is pre $\tilde{I}$ open soft set and also studied its local properties. In this paper, we introduce the concept of pre $\tilde{I}_g$ connected soft set

in the soft topological spaces with respect to an soft ideal. .

2010 Mathematics Subject Classification: Primary 54A40, Secondary 03E72.

Keywords: pre $\tilde{I}_g$ open soft set, pre $\tilde{I}_g$ separated soft set, pre $\tilde{I}_g$ connected soft set

1. Introduction

In topology, connectedness is used to refer to various properties meaning in some sense, "all one piece". When a mathematical object has such a property, we say, it is connected; otherwise it is disconnected. Connectivity occupies very important place in topology.

In 2003 Shi- Zhong Bai [11] introduced P-connectedness. In 2011 R. Santhi and D. Jayanthi [8] introduced semi-pre connectedness in intuitionistic fuzzy topological spaces. In 2012 E.Peyhan, B.Samadi and A.Tayebi [7] introduced soft connectedness in soft topological spaces. In the same year, J.Mahanta and P.K.Das [5] introduced soft semi connectedness in soft topological spaces. In 2013 Deniz Tokat and Ismail Osmanoglu [2] introduced soft connectedness on multi soft topology. In 2014 B. Shanthi Gowri and Gnanambal Illango [10] used connected sets in Medical Image segmentation. In 2020 Benchali, Patil and Dodamani [1] investigated the properties of soft β connected spaces in soft topological spaces.

In this paper, we have introduced the concept of pre $\tilde{I}_g$ separated soft set, pre $\tilde{I}_g$ connected soft set and discussed their properties in the soft topological spaces with respect to an soft ideal.

2. Preliminaries:

In this section, we present the basic definitions and results of soft set theory, soft topological space via soft ideal which will be needed in the sequel.

Definition 2.1 [6] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X, where F is a mapping given by $F: A \to P(X)$. In other words, the soft set over X, is a parameterized family of subsets of the universe X. For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set F_A and if $e \notin A$, then $F(e) = \phi$

i.e. $F_A = \{(e, F(e)): e \in A \subseteq E, F: A \to P(X)\}$.

Definition 2.2 [9] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if i. $\tilde{X}, \tilde{\phi} \in \tau$, ii. the union of any number of soft sets in τ belongs to τ, iii. the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.3 [3] Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with a fixed set of parameters E, then $\tilde{I} \subseteq SS(X)_E$ is called a soft ideal on X with a fixed set E if $F_E \in \tilde{I}$ and $G_E \in \tilde{I} \Rightarrow F_E \tilde{\cup} G_E \in \tilde{I}$,

$F_E \in \tilde{I}$ and $G_E \tilde{\subseteq} F_E \Rightarrow G_E \in \tilde{I}$,

Definition 2.4 [3] Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. Then $(F_E)^*(\tilde{I}, \tau) = \tilde{\cup}\{x_e \in \tilde{X}: O_{x_e} \tilde{\cap} F_E \tilde{\notin} \tilde{I}\ \forall\, O_{x_e} \in \tau\}$ is called the soft local function of F_E with respect to $\tilde{I}$ and τ, where O_{x_e} is a τ-open soft set containing x_e.

Theorem 2.5 [4] Let (X,τ,E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. Then the soft closure operator $cl^*: SS(X)_E \to SS(X)_E$ defined by $cl^*(F_E) = (F_E)\,\tilde{\cup}\,(F_E)^*$ satisfies Kuratowski's axioms.

Definition 2.6 [4] Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and $F_E \tilde{\in} SS(X)_E$. Then F_E is called $\tilde{I}$-open soft if $F_E \tilde{\subseteq} int\,((F_E)^*(\tilde{I},\tau))$.

We denote the set of all $\tilde{I}$-open soft sets by $\tilde{I}OS(X)$.

Definition 2.7 [4] Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and $F_E \tilde{\in} SS(X)_E$. A soft set $F_E$is said to be pre $\tilde{I}$-open soft sets over X if $F_E \tilde{\subseteq} int(cl^*(F_E))$. We denote the set of all pre $\tilde{I}$-open soft sets by $P\tilde{I}OS(X)$. The complement of pre $\tilde{I}$-open soft set is pre $\tilde{I}$ closed soft sets.

Definition 2.8 [4] Let $(X,\tau,E,\tilde{I})$ be a soft topological space over X and $F_E \tilde{\in} SS(X)_E$. Then the Pre $\tilde{I}$ soft closure of F_E denoted by $P\tilde{I}Scl(F_E)$ is defined as the soft intersection of all Pre $\tilde{I}$ closed supersets of soft set F_E. That is $P\tilde{I}Scl(\mathrm{Q_E}) = \tilde{\cap}\{\mathrm{Q_E} : \mathrm{Q_E}$ is Pre $\tilde{I}$ closed soft set and $\mathrm{Q_E} \tilde{\supseteq} \mathrm{F_E}\}$.

3. Pre $\tilde{I}$ Generalized Connected Soft Set in a Soft Topological Space with Respect to an Ideal

In this section, we introduce the concept of pre $\tilde{I}_g$ separated soft set, pre $\tilde{I}_g$ connected soft sets in a soft topological space with respect to an soft ideal. Also, we discuss some of the main results based on the above with illustrations.

Definition 3.1 Let $(X,\tau,E,\tilde{I})$ be a soft ideal topological space over X. A soft set F_E is said to be pre $\tilde{I}$ generalized closed soft set with respect to a soft ideal $\tilde{I}$ in a soft topological space $(X,\tau,E,\tilde{I})$ if $(P\tilde{I}Scl(F_E)) \setminus G_E \tilde{\in} \tilde{I}$ whenever $F_E \tilde{\subseteq} G_E$ and G_E is pre $\tilde{I}$ open soft set. The set of all p*re* $\tilde{I}$ generalized closed soft sets over X is denoted by $P\tilde{I}_gCS(X)$.

Definition 3.2

Let $(X,\tau,E,\tilde{I})$ be a soft topological space with respect to an ideal $\tilde{I}$. Two non empty soft disjoint soft subsets $P_{E_1}$and P_{E_2} of $SS(X)_E$ are called pre $\tilde{I}_g$ separated soft sets over X if $P\tilde{I}_gScl(P_{E_1})\,\tilde{\cap}\,P_{E_2} = P_{E1}\,\tilde{\cap}\,P\tilde{I}_gScl(P_{E_2}) = \emptyset$.

Definition 3.3

A pre $\tilde{I}_g$ soft separation of a soft topological space $(X,\tau,E,\tilde{I})$ with respect to an ideal $\tilde{I}$ is a pair of pre $\tilde{I}_g$ separated soft sets $P_{E_1}$and P_{E_2} whose soft union is $\tilde{X}$.

Example 3.3.1 Let X=$\{x,y\}$, E=$\{e_1,e_2\}$, where $\tilde{X}$ = $\{\,(e_1,\{x,y\})\,,(e_2,\{x,y\})\,\}$.

Then the soft subsets over X are $\tilde{X}, \tilde{\emptyset}$, $F_{E_1} = \{(e_1,\{x\})\}$, $F_{E_2} = \{(e_1,\{y\})\}$,
$F_{E_3} = \{(e_1,\{x,y\})\}$, F_{E_4}=$\{(e_2,\{x\})\}$, F_{E_5}=$\{(e_2,\{y\})\}$, F_{E_6}=$\{(e_2,\{x,y\})\}$,
F_{E_7}=$\{(e_1,\{x\}),(e_2,\{x\})\}$, F_{E_8}=$\{(e_1,\{x\}),(e_2,\{y\})\}$, $\mathrm{F_{E_9}}$=$\{(e_1,\{x\}),(e_2,\{x,y\})\}$,
$\mathrm{F_{E_{10}}}$= $\{(e_1,\{y\}),(e_2,\{x\})\}$, $\mathrm{F_{E_{11}}}$=$\{(e_1,\{y\}),(e_2,\{y\})\}$, $F_{E_{12}}$=$\{(e_1,\{y\}),(e_2,\{x,y\})\}$,
$F_{E_{13}}$=$\{(e_1,\{x,y\}),(e_2,\{x\})\}$, $F_{E_{14}}$=$\{(e_1,\{x,y\}),(e_2,\{y\})\}$. So $|SS(X)_E|$ = 2^4 = 16.

Consider the soft topological space $(X,\tau_1,E,\tilde{I})$ with respect to an ideal $\tilde{I}$ where $\tau_1 = \{\tilde{\mathrm{X}},\emptyset,F_{E_7}\}$, $\tau_1{}' = \{\emptyset,\tilde{\mathrm{X}},\mathrm{F_{E_{11}}}\}$, $\tilde{I} = \{\tilde{\phi},F_{E_1},F_{E_2},F_{E_3}\}$, where F_{E_1} ,F_{E_2} and F_{E_3} are soft sets defined by $F_{E_1} = \{(e_1,\{x\})\}$, $F_{E_2} = \{(e_1,\{y\})\}$, $F_{E_3} = \{(e_1,\{x,y\})\}$.

And$P\tilde{I}_gOS(X)$= $\{F_{E_1},F_{E_2},F_{E_3},F_{E_4},F_{E_5},F_{E_6},F_{E_7},F_{E_9},F_{E_{10}},F_{E_{12}},F_{E_{13}},F_{E_{14}},\emptyset,\tilde{X}\}$.

$P\tilde{I}_gCS(X)$= $\{F_{E_1},F_{E_2},F_{E_3},F_{E_4},F_{E_5},F_{E_6},F_{E_8},F_{E_9},F_{E_{11}},F_{E_{12}},\emptyset,\tilde{X}\}$.

Take$\tilde{X} = F_{E_3}\,\tilde{\cup}\,F_{E_6}$,then $P\tilde{I}_gScl(F_{E_3}) = \{(e_2,\{x,y\})\} = F_{E_6}$, $P\tilde{I}_gScl(F_{E_6}) = \{(e_1,\{x,y\})\} = F_{E_3}$. We have $P\tilde{I}_gScl(F_{E_3})\,\tilde{\cap}\,F_{E_6} = F_{E_3}\,\tilde{\cap}\,P\tilde{I}_gScl(F_{E_6}) = \emptyset$. Therefore $F_{E_3}$and F_{E_6} are pre $\tilde{I}_g$ separated soft sets. Hence F_{E_3} and F_{E_6} is a pre $\tilde{I}_g$ soft separation of $\tilde{X}$.

Definition 3.4

Let $(\tilde{X},\tilde{\tau},E)$ be a soft topological space $(X,\tau,E,\tilde{I})$ with respect to an ideal $\tilde{I}$. A pre $\tilde{I}_g$ connected soft set over X is a soft set $F_E \tilde{\in} SS(X)_E$ which does not have a pre $\tilde{I}_g$ soft separation in the soft relative topology induced on the soft subset F_E.

Example 3.4.1

Let X=$\{x,y\}$, E=$\{e_1,e_2\}$, where $\tilde{\mathrm{X}} = \{\,(\mathrm{e}_1,\{x,y\})\,,(\mathrm{e}_2,\{x,y\})\,\}$. Consider the soft

subsets over X that are given in Example 3.3.1. Define $\tau_2 = \{\tilde{\mathrm{X}},\emptyset,F_{E_1},F_{E_3}\}$, $\tau_2{}' = \{\emptyset,\tilde{\mathrm{X}},F_{E_6},F_{E_{12}}\}$, $\tilde{I} = \{\tilde{\phi},F_{E_1},F_{E_2},F_{E_3}\}$, $P\tilde{I}_gOS(X) = \{F_{E_1},F_{E_2},F_{E_3},F_{E_4},F_{E_5},F_{E_{12}},F_{E_{13}},F_{E_{14}},\emptyset,\tilde{X}\}$.

$P\tilde{I}_gCS(X) = \{F_{E_4},F_{E_5},F_{E_6},F_{E_9},F_{E_{12}},F_{E_{13}},F_{E_{14}},\emptyset,\tilde{X}\}$.

Then $(X,\tau_2,E,\tilde{I})$ is a soft topological space with respect to an ideal $\tilde{I}$. Consider the soft subset F_{E_3}=$\{(e_1,\{x,y\})\}$. Then the soft relative topology induced on the soft set $\mathrm{F_{E_3}}$ is $\tau_2(F_{E_3}) = \{\emptyset,F_{E_1},F_{E_3}\}$. The collection of all pre $\tilde{I}_g$ soft open sets are $P\tilde{I}_gOS(F_{E_3})$=$\{\emptyset,F_{E_1},F_{E_3}\}$. Therefore the soft set F_{E_3} does not have a pre

$\tilde{I}_g$ soft separation in the soft relative topology induced on the soft subset F_{E_3}. Hence F_{E_3} is a pre $\tilde{I}_g$ connected soft subset of a soft topological space $(X, \tau_2, E, \tilde{I})$.

Remark 3.5

In a soft topological space with respect to an ideal $\tilde{I}$ soft empty set is pre $\tilde{I}_g$ connected soft set. There does not exist a pre $\tilde{I}_g$ soft separation in the soft empty set. Hence it is not a pre $\tilde{I}_g$ connected soft set.

(i) In a soft topological space with respect to an ideal $\tilde{I}$ soft singleton set is a pre $\tilde{I}_g$ connected soft set. There does not exist a pre $\tilde{I}_g$ soft separation in the soft singleton. Hence it is not a pre $\tilde{I}_g$ connected soft set.

(ii) In the soft indiscrete topological space with respect to an ideal $\tilde{I}$ all soft subsets are pre $\tilde{I}_g$ connected soft sets.

Proposition 3.6

Every pre $\tilde{I}_g$ connected soft set is a soft connected set.

Proof

Let F_E be a pre $\tilde{I}_g$ connected soft set in the soft topological space $(X, \tau, E, \tilde{I})$ with respect to an ideal $\tilde{I}$. Since F_E is a pre $\tilde{I}_g$ connected soft set, there does not exist a pre $\tilde{I}_g$ soft separation of F_E. Since every open soft set is a pre $\tilde{I}_g$ open soft set, there does not exist a soft separation of F_E. Hence, F_E is a soft connected set in the soft topological space $(X, \tau, E, \tilde{I})$.

Note 3.6.1

A soft connected set need not be a pre $\tilde{I}_g$ connected soft set.

Example 3.6.2

Consider the soft set $\tilde{X}$ and its soft subsets given in Example 3.3.1. Let $(\tilde{X}, \tilde{\tau}_1, E)$ be the soft topological space where $\tau_1 = \{\tilde{X}, \emptyset, F_{E_7}\}$, $\tau_1' = \{\emptyset, \tilde{X}, F_{E_{11}}\}$,

$\tilde{I} = \{\tilde{\phi}, F_{E_1}, F_{E_2}, F_{E_3}\}$, where F_{E_1} , F_{E_2} and F_{E_3} are soft sets defined by $F_{E_1} = \{(e_1, \{x\})\}$, $F_{E_2} = \{(e_1, \{y\})\}$, $F_{E_3} = \{(e_1, \{x, y\})\}$.

$P\tilde{I}_g OS(X)=$

$$\{F_{E_1}, F_{E_2}, F_{E_3}, F_{E_6}, F_{E_7}, F_{E_9}, F_{E_{10}}, F_{E_{12}}, F_{E_{13}}, F_{E_{14}}, \emptyset, \tilde{X}\}.$$

$P\tilde{I}_g CS(X)= \{F_{E_1}, F_{E_2}, F_{E_3}, F_{E_4}, F_{E_5}, F_{E_6}, F_{E_8}, F_{E_9}, F_{E_{11}}, F_{E_{12}}, \emptyset, \tilde{X}\}$.

It is clear that the soft set $\tilde{X}$ is soft connected, Now we show that it is not pre $\tilde{I}_g$ connected soft.

Here $\tilde{X} = F_{E_3} \tilde{\cup} F_{E_6}$,then $P\tilde{I}_g Scl(F_{E_3}) = \{(e_2, \{x, y\})\} = F_{E_6}$,$P\tilde{I}_g Scl(F_{E_6}) = \{(e_1, \{x, y\})\}$ $= F_{E_3}$.We have

$P\tilde{I}_g Scl(F_{E_3}) \tilde{\cap} F_{E_6} = F_{E_3} \tilde{\cap} P\tilde{I}_g Scl(F_{E_6}) = \emptyset$. Therefore $F_{E_3}$and F_{E_6} are pre $\tilde{I}_g$ separated soft sets. Hence F_{E_3} and F_{E_6} is a pre $\tilde{I}_g$ soft separation of $\tilde{X}$.

Hence $\tilde{X}$ can be expressed as a soft union of two pre $\tilde{I}_g$ separated soft sets F_{E_3} and F_{E_6}. Hence $\tilde{X}$ is not pre $\tilde{I}_g$ connected soft set.

Theorem 3.7

Let $(X, \tau, E, \tilde{I})$ be a soft topological space with respect to an ideal $\tilde{I}$ and F_E be a pre $\tilde{I}_g$ connected soft set. Let P_{E_1} and $P_{E_2}$are pre $\tilde{I}_g$ soft separated sets. If $F_E \tilde{\subseteq} P_{E_1} \tilde{\cup} P_{E_2}$ .Then either $F_E \tilde{\subseteq} P_{E_1}$ or $F_E \tilde{\subseteq} P_{E_2}$.

Proof

Let $(X, \tau, E, \tilde{I})$ be a soft topological space with respect to an ideal $\tilde{I}$ and $F_E \tilde{\in} SS(X)_E$ be a pre $\tilde{I}_g$ connected soft set. Let P_{E_1} and P_{E_2} are pre $\tilde{I}_g$ soft separated sets such that $F_E \tilde{\subseteq} P_{E_1} \tilde{\cup} P_{E_2}$.

We have to prove either $F_E \tilde{\subseteq} P_{E_1}$ or $F_E \tilde{\subseteq} P_{E_2}$.

Suppose not. Then $F_E \tilde{\nsubseteq} P_{E_1}$ and $F_E \tilde{\nsubseteq} P_{E_2}$.

Then, $G_E = P_{E_1} \tilde{\cap} F_E \neq \emptyset$ and $H_E = P_{E_2} \tilde{\cap} F_E \neq \emptyset$ and $F_E = G_E \tilde{\cup} H_E$.

Since $G_E \tilde{\subseteq} P_{E_1}$ implies that $P\tilde{I}_g Scl(G_E) \tilde{\subseteq} P\tilde{I}_g Scl(P_{E_1})$.

Since P_{E_1}, P_{E_2} are pre $\tilde{I}_g$ soft separation sets, we have $P\tilde{I}_g Scl(P_{E_1}) \tilde{\cap} P_{E_2} = \emptyset$.

Therefore, $P\tilde{I}_g Scl(P_{E_1}) \tilde{\cap} P_{E_2} = P\tilde{I}_g Scl(G_E) \tilde{\cap} P_{E_2} = P\tilde{I}_g Scl(G_E) \tilde{\cap} H_E = \emptyset$.

Again $H_E \tilde{\subseteq} P_{E_2}$, implies $P\tilde{I}_g Scl(H_E) \tilde{\subseteq} P\tilde{I}_g Scl(P_{E_2})$.

Since P_{E_1}, P_{E_2} are pre $\tilde{I}_g$ soft separation sets, we have $P_{E_1} \tilde{\cap} P\tilde{I}_g Scl(P_{E_2}) = \emptyset$.

Therefore, $P_{E_1} \tilde{\cap} P\tilde{I}_g Scl(P_{E_2}) = P_{E_1} \tilde{\cap} P\tilde{I}_g Scl(H_E) = G_E \tilde{\cap} P\tilde{I}_g Scl(H_E) = \emptyset$.

But $F_E = G_E \tilde{\cup} H_E$. Therefore, there exists a pre $\tilde{I}_g$ soft separation of F_E. Hence, F_E is not a pre $\tilde{I}_g$ connected soft set. This is a contradiction. Therefore, either $F_E \tilde{\subseteq} P_{E_1}$ or $F_E \tilde{\subseteq} P_{E_2}$.

Theorem 3.8

If F_E is pre $\tilde{I}_g$ connected soft set, then $P\tilde{I}_gScl(F_E)$ is pre $\tilde{I}_g$ connected soft set.

Proof

Let F_E be a pre $\tilde{I}_g$ connected soft set in a soft topological space $(X,\tau,E,\tilde{I})$.

We have to prove $P\tilde{I}_gScl(F_E)$ is a pre $\tilde{I}_g$ connected soft set.

Suppose not. Then there exist a pre $\tilde{I}_g$ soft separation of $P\tilde{I}_gScl(F_E)$.

Therefore there exist a pair of pre $\tilde{I}_g$ soft separated sets $P_{E_1}$and P_{E_2} such that $P\tilde{I}_gScl(F_E) = P_{E_1} \tilde{\cup} P_{E_2}$. But $F_E \tilde{\subseteq} P\tilde{I}_gScl(F_E) = P_{E_1} \tilde{\cup} P_{E2}$.

Since F_E is pre $\tilde{I}_g$ connected soft set, then by Theorem 3.7 either $F_E \tilde{\subseteq} P_{E_1}$ or $F_E \tilde{\subseteq} P_{E_2}$.

If $F_E \tilde{\subseteq} P_{E_1}$ then $P\tilde{I}_gScl(F_E) \tilde{\subseteq} P\tilde{I}_gScl(P_{E_1})$.

Since P_{E_1} and P_{E_2} are pre $\tilde{I}_g$ soft separated sets , $P\tilde{I}_gScl(P_{E_1}) \tilde{\cap} P_{E_2} = \emptyset$.

Hence, $P\tilde{I}_gScl(F_E) \tilde{\cap} P_{E_2} = \emptyset$. Since $P_{E_2} \tilde{\subseteq} P\tilde{I}_gScl(F_E)$,then $P_{E_2} = \emptyset$. This is a contradiction.

Similarly if $F_E \tilde{\subseteq} P_{E_2}$, we can prove $P_{E_1} = \emptyset$ which is a contradiction. Therefore, there does not exist a pre $\tilde{I}_g$ soft separation of $P\tilde{I}_gScl(F_E)$. Hence $P\tilde{I}_gScl(F_E)$ is a pre $\tilde{I}_g$ connected soft set.

Theorem 3.9

If F_E is pre $\tilde{I}_g$ connected soft set and $F_E \tilde{\subseteq} G_E \tilde{\subseteq} P\tilde{I}_gScl(F_E)$, then G_E is pre $\tilde{I}_g$ connected soft set.

Proof

Let $F_E \tilde{\in} SS(X)_E$ be a pre $\tilde{I}_g$ connected soft set such that $F_E \tilde{\subseteq} G_E \tilde{\subseteq} p(\overline{F_E})$.

We have to prove G_E is pre $\tilde{I}_g$ connected soft set.

Suppose G_E is not a pre $\tilde{I}_g$ connected soft set. Then there exists a pair of pre $\tilde{I}_g$ soft separated sets $P_{E_1}$and P_{E_2} such that $G_E = P_{E_1} \tilde{\cup} P_{E_2}$.

Since $F_\mathrm{E} \tilde{\subseteq} G_\mathrm{E}$, $F_\mathrm{E} \tilde{\subseteq} P_{E_1} \tilde{\cup} P_{E_2}$.

We claim that either $F_E \tilde{\subseteq} P_{E_1}$ or $F_E \tilde{\subseteq} P_{E_2}$.

For, $F_E \tilde{\cap} P_{E_1} \neq \emptyset$ and $F_E \tilde{\cap} P_{E_2} \neq \emptyset$.

Then $F_E = \left(F_E \tilde{\cap} P_{E_1}\right) \tilde{\cup} \left(F_E \tilde{\cap} P_{E_2}\right)$.

But $F_E \tilde{\cap} P_{E_1}$ and $F_E \tilde{\cap} P_{E_2}$ are pre $\tilde{I}_g$ soft separated sets.

This is a contradiction to the pre $\tilde{I}_g$ soft connectivity of F_E.

Hence our claim.

Suppose $F_\mathrm{E} \tilde{\subseteq} P_{E_1}$, then $P\tilde{I}_gScl(F_E) \tilde{\subseteq} P\tilde{I}_gScl(P_{E_1})$

Since P_{E_1} andP_{E_2} are pre $\tilde{I}_g$ soft separated sets,$P\tilde{I}_gScl(P_{E_1}) \tilde{\cap} P_{E_2} = \emptyset$.

Therefore , $P\tilde{I}_gScl(F_E) \tilde{\cap} P_{E_2} = \emptyset$

But $P_{E_2} \tilde{\subseteq} G_\mathrm{E}$. Then by hypothesis $P_{E_2} \tilde{\subseteq} G_\mathrm{E} \tilde{\subseteq} P\tilde{I}_gScl(F_E)$.

Therefore, $P\tilde{I}_gScl(F_E) \tilde{\cap} P_{E_2} = P_{E_2}$.

Thus, we have $P\tilde{I}_gScl(F_E) \tilde{\cap} P_{E_2} = \emptyset$ and $P\tilde{I}_gScl(F_E) \tilde{\cap} P_{E_2} = P_{E_2}$.

Hence $P_{E_2} = \emptyset$, which is a contradiction.

Similarly if $F_E \tilde{\subseteq} P_{E_2}$, then we can prove $P_{E_1} = \emptyset$. This is a contradiction.

Therefore, there does not exist a pre $\tilde{I}_g$ soft separation of G_E.

Hence, G_E is a pre $\tilde{I}_g$ connected soft set.

Theorem 3.10

The soft union F_E of any family $\{F_{Ei}: i \in I\}$ of pre $\tilde{I}_g$ connected soft sets having a non –empty soft intersection is pre $\tilde{I}_g$ connected soft set.

Proof

Let F_E be a soft union of any family of pre $\tilde{I}_g$ connected soft sets having a non-empty soft intersection. Suppose that $F_E = P_{E_1} \tilde{\cup} P_{E_2}$, where P_{E_1}andP_{E_2} form a pre $\tilde{I}_g$ soft separation of F_E. By hypothesis, we may choose a soft point $x_e \tilde{\in} \tilde{\cap}_{i\in I} F_{E_i}$. Then $x_e \tilde{\in} F_{E_i}$ for all $i \in I$. If $x_e \tilde{\in} F_E$, then either $x_e \tilde{\in} P_{E_1}$ or $x_e \tilde{\in} P_{E_2}$ but not both. Since, $P_{E_1}$and P_{E_2} are soft disjoint, we must have $F_{E_i} \tilde{\subseteq} P_{E_1}$, since F_{E_i} is pre $\tilde{I}_g$ connected soft and it is true for all $i \in I$, and so $F_E \tilde{\subseteq} P_{E_1}$. From this we obtain that $P_{E_2} = \emptyset$, which is a contradiction. Thus, there does not exist a pre $\tilde{I}_g$ soft separation of F_E. Therefore, F_E is pre $\tilde{I}_g$ connected soft set.

4. Conclusion:

In this paper, we introduced pre $\tilde{I}_g$ soft separation and pre $\tilde{I}_g$ connected soft set . We compared with connected soft set in soft topological space with respect to an soft ideal. Also we derived "Let $(X,\tau,E,\tilde{I})$ be a soft topological space with respect to an ideal $\tilde{I}$ and F_E be a pre $\tilde{I}_g$ connected soft set. Let P_{E_1} and $P_{E_2}$are pre $\tilde{I}_g$ soft separated sets. If $F_E \tilde{\subseteq} P_{E_1} \tilde{\cup} P_{E_2}$ .Then either $F_E \tilde{\subseteq} P_{E_1}$ or $F_E \tilde{\subseteq} P_{E_2}$", "If F_E is pre $\tilde{I}_g$ connected soft set, then $P\tilde{I}_g Scl(F_E)$ is pre $\tilde{I}_g$ connected soft set" and "If F_E is pre $\tilde{I}_g$ connected soft set and $F_E \cong G_E \cong P\tilde{I}_g Scl(F_E)$, then G_E is pre $\tilde{I}_g$ connected soft set."

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