Research Article

# **Review And Analysis Of Optimization Algorithms For Digital Filter Design**

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Abstract: In general Digital Signal Processing (DSP) and more especially filtering is an important and basic requirement for signal systems, computers, and communication networks. The design of Optimal Digital Filters is a intimidating task and it has challenged the scientist, engineers, and researchers for designing the filters with improvised, proficient, and intelligent techniques using the Emerging Evolutionary Computations. Metaheuristics have emerged as the best promising tool and an striking area of research with numerous improvements and advancements in the solution to optimization issues. However, it has not shown clarity to decide the best performing metaheuristic for designing an optimal digital filter. In this paper, a comprehensive review and analysis of various metaheuristics used by researchers for designing an optimal digital filter are carried out. More specifically, "Finite Impulse Response (FIR)" and "Infinite Impulse Response (IIR)" filter design using optimization-based techniques such as nature-inspired "Swarm Intelligence (SI) Swarm Intelligence (SI), Cuckoo Search (CS), Grasshopper Optimization Algorithms,, Particle Swarm Optimization, (PSO), Ant Colony Optimization (ACO Bat Algorithms (BA), Genetic Algorithms (GA), Artificial Bee Colony (ABC), Bacterial foraging optimization (BFO, Biogeography-based optimization (BBO), Harmony search (HS), Krill herd (KH), Social spider optimization (SSO), Symbiotic organisms search (SOS), Firefly algorithm (FA), Gravitational search algorithm (GSA), Grey wolf algorithm (GWO), Teaching-learning-based optimization (TLBO), Whale optimization algorithm (WOA)" are also described. Also, a survey on the origin of twenty-one optimization algorithms is carried out that is being proposed as optimization algorithms in literature for the proposal of digital filters.

**Keywords:**Digital Filter Design, Metaheuristics, Evolutionary Computations, Optimization Algorithms, Nature Inspired Algorithms.

#### 1. Introduction.

Digital Filters and optimal design issues are a very important aspect of research. Unlike analog filters, digital filters are advantageous, proficient, and intelligent for effectively filtering noise from various signals. Mathematical processes are applied to the discrete-time signal to obtain the desired results [1], [2],[3]. The response of FIR filters is mathematically represented as:

$$H(z) = \sum_{n=0}^{N-1} w(n) z^{-1} - - - - (I)$$
  
$$H(z) = w(0) + w(1)z^{-1} + - - - + w(n)z^{-(N-1)} - - - (II)$$

Where N is the filter length of impulse response w(n). Here, the output in time domain y(n) is

u(n) = x(n) \* w(n) - - - - (III)

O(z) output in frequency domain is as below,

$$O(z) = X(z)H(z) - - - (IV)$$

Where the input signals x(n) and X(z) are. Those are referred to as time and frequency domain. Frequency response of 1-D FIR filter;

$$H(qk) = \sum_{n=0}^{N} h(n) e^{-jqk^n} - - - - (V)$$

where  $k = 2\pi k/n$ ;  $H(w_k)$  is Fourier transform. It is known as FIR filter frequency response. The frequency in the  $[0, \pi]$  is saple of N points.

Let us consider the impulse response from this filter is  $h(n_1, n_2)$  where  $0 \le n_1 \le N_1 - 1 \& 0 \le n_2 \le N_2 - 1$ , then 2-D transfer  $H(z_1, z_2)$  function [7],[8] be,

$$H(z_{1}, z_{2}) = \sum_{\nu=0}^{\nu-1} \sum_{\nu=0}^{\nu-1} h(v_{1} - v_{2}) z_{1}^{-\nu_{1}} z_{2}^{-\nu_{2}} - - - - - (VI)$$

The output Y(z1, z2) will be

$$Y(z_1, z_2) = H(z_1, z_2). X(z_1, z_2) - - - - - (VII)$$
  
"Where X(z1, z2) is 2-D inputs."

Substituting the value of  $z_1 = \exp(jw_1)$  and  $z_2 = \exp(jw_2)$  in above equation frequency response  $H(z_1, z_2)$  FIR Filter is

$$H(e^{jw_1}, e^{jw_2}) = \sum_{n=0}^{N-1} \sum_{n=0}^{N-1} h(n_1 - n_2) e^{-jw_1n_1} e^{-jw_2n_2} - - - - - (VIII)$$

The mathematical representation of the one-dimensional IIR Filter [8] is given as

$$y(v) = \sum_{k=0}^{M} p_k x(v-k) - \sum_{k=0}^{V} q_k y(v-k) - - - - (IX)$$

Where co-efficient filters are  $p_k$  and  $q_k$ . x(v) and y(v) are input filters, output in the time domain. M & N are co-efficient number of input and output with  $V \ge M$ . The transfer function H(z) of IIR Filter is

$$H(z) = \frac{\sum_{k=0}^{M} p_k z^{-k}}{1 + \sum_{k=1}^{N} q_k z^{-k}} - - - - - (X)$$

"The mathematical representation of two dimensional IIR Filter [9], [10], when a(0,0)=1, is"

$$H(z_1, z_2) = \frac{\sum_{k_1 k_2 \in R_b} \sum b(k_1 k_2) \frac{1}{z_1^{-k_1} z_2^{-k_2}}}{1 + \sum_{k_1 k_2 \in R_{a(0,0)}} \sum a(k_1 k_2) \frac{1}{z_1^{-k_1} z_2^{-k_2}}} - - - - - (XI)$$

Where, coefficient of the filter are a(k1,k2) and b(k1,k2). "Ra - (0,0) represents the reason of support of a(k1,k2) represents the region of support of b(k1,k2)".

The digital filter design is the determination of the set of filter coefficients for getting a result. These coefficients affect the performance of filters in terms of the "width of pass-band and stop band, attenuation, overall gain, etc. There are several traditional methods such as windowing functions as Butterworth, Chebyshev, Kaiser, etc". "Remez exchange algorithm (REA) of Parks", "McClellan" and "Steepest-descent method" for digital optimization. But all these optimizations are not suitable and fail because of the reasons mentioned in [11].

A brief survey on metaheuristics proposed for Digital Filter Design is described in section 2 related work. Section 3 of this review paper describes optimization algorithms. Section 4 presents a survey on digital filter design. This paper is concluded in Section 5.

### 2. Related Work

A parallel Genetic Algorithm to project a straight system of a finite word length (FWL), is used for "finite impulse response (FIR)" [12]. The two-stage algorithm is cast-off for designing FIR filters that projected sums of signed-powers-of-two (SPT) coefficients. [13].

IIR digital filter design using genetic programming through automatically defined functions is projected. "Digital filter constructions are signified as S-expressions by subroutines, which are adorned directly from the set of transformation equations".[14]. The Ant Colony algorithms are projected for solving optimization problems. Ant System in addition to the Ant Colony System is used for resolving the design matter of IIR filters. [15]. Structured stochastic optimization algorithms are castoff, with particular attention to PSO. The PSO algorithm is applied IIR filter structure[16].

The design is grounded on equiripple resembling polynomial A recursive formula for assessing the impulse retort is presented.[17] The decomposition-based multiobjective evolutionary algorithm is castoff. "A fitness-rate-rank-based multiarmed outlaw is embedded to select the best operative pool by gathering their recently achieved fitness development rates". [18]. Seeker Optimization Algorithm evolutionary technique is used to design digital IIR filter. The enactment is compared with three DE versions, four PSO versions, and GA. Authors titles SOA to be better for IIR filter design.[19]. IIR filter design method with linear phase reply is planned using slight derivative constraints. The optimality of fractional derivative constraints is achieved by the use of the Greedy Based Sorting technique.[20]. FIR filter design issue numerous rules are applied to select indices of possible zero coefficients to be used in 1-norm optimization. [21].The group suspension

unconventionality minimization complicated for IIR filter plan is reframed into an iterative optimization complicated to achieve lower group delay deviation. [22].

For IIR filter with the closely linear-phase response using CSO Algorithm is projected [23]. Optimum values of fractional derivate constraints and orientation points in the passband are gained for "an IIR filter, using different evolutionary methods such as PSO, constraint factor inertia PSO (CFI-PSO), quantum PSO, etc" [24]. A sequential partial optimization technique is obtainable in this paper. The limit result of the successive fractional optimization algorithm has been shown [25]

"Proposed an estimated algorithm that can grip filters with a large number of coefficients using less computational than the exact FDO algorithm and find healthier solutions than existing FDO heuristics.[26]". The interval analysis (IA) to resolve this nonlinear optimization problem is projected. [27]. "A lithe and wild synthesis technique for designing asymmetric interleavers with flat-top, low dispersion, adjustable isolation, and bandwidths at both outputs is proposed."[28].

IIR filter design to attenuate or abolish the undesired measurement noise is projected [29]. In IIR filter design optimization of fitness, the purpose is proposed [30]. Evolutionary optimization algorithms applying bandwidth adaptive harmony search (BAHS) algorithm for 1-dimensional IIR filters are proposed [31]. Fractional Order Digital Differentiator in rapports of IIR filter based on a metaheuristic optimization method [32]. ABC is used for the design of IIR) filter founded on nonlinear minimization of mean square error. [33]. The ABC algorithm is used for the min-max design of linear phase FIR full-band digital differentiators. [34]. Rational approximations in terms of infinite impulse reply for the full band digital differentiator founded [35]. A better version of PSO named Restart PSO for a project of linear stage low pass FIR filter is proposed [36]. Fractional Order Integrators (FOIs) based on nature enthused metaheuristic optimization method motivated named Crow Search Algorithm [37].

"Particle Swarm Optimization (PSO) and the Bat Algorithm (BA)" are cast-off for FIR design. Results show that the "proposed filter design approach using the BA algorithm outstrips Parks and McClellan filter" [38]. FIR filters design using memetic algorithm along with a weighted fitness function is projected [39]. The finite bit length design technique of variable digital filters by variable multiple elements of stopband is projected [40]. A constrained Genetic Algorithm centered approach is planned [41].

Fractional order filters are intended using SI based evolutionary optimization algorithm. [42]. FIR filter is designed using a multiobjective ABC algorithm. [43]. An innovative population-based metaheuristic optimization algorithm Adolescent Identity Search Algorithm (AISA), is demonstrated mathematically to resolve optimization problems.[44]

In paper [45], the optimal design of a digital fractional-order Butterworth filter using a single-step, discretization is presented. In paper [46], authors present a survey and define that to overwhelmed the sub-optimality. In the paper [47], a multipurpose digital linear phase double band filter is planned.

The paper [48] presents the efficacy of employing the "swarm intelligence (SI) based and population-based evolutionary computing".

The paper [49], deals by the problem of digital IIR filter. Two original changes are proposed to "Particle Swarm Optimization and authenticated through novel submission for the project of IIR filter". and new research opportunities are investigated [50].

The "Grasshopper Optimization Algorithm (GOA)" to project "a linear phase finite impulse response (FIR) low pass, high pass, bandpass, and bandstop filters are used" [51]. In paper [52] "nine nature-inspired optimization algorithms, five broadminded alternatives of Differential Evolution (DE), three advanced variants of PSO, and an efficient evolutionary strategy method (CMA-ES-RIS) are working to strategy the fractional-step low pass Butterworth filter (FLBF)".

## 3. Optimization Algorithms:

It is a process executed iteratively to obtain a reasonably optimal solution. Out of two optimization methods i.e. deterministic and stochastic, the deterministic approach may fail with an increase in problem size and computation. Therefore, stochastic approaches are preferred because of metaheuristics and independently work on for giving an optimal solution to the problem. These metaheuristics are useful by researchers, in conclusion, the optimal solution to almost all the engineering and applied science glitches. They are also stated as Evolutionary Algorithms or nature-inspired algorithms:

3.1 "Artificial Bee Colony (ABC)": It is a SI centered meta-heuristic algorithm. It is industrialized by Karaboga D in 2005 [53]. An algorithm is built on the foraging behavior of honey bees. The first two mechanisms, employed and unemployed foraging It is proposed by researchers for a project of Fractional Order Butterworth Filter in [33,34,42,43].

3.2 "Ant Colony Optimization (ACO)": It is industrialized by Marco Dorigo in 1992 [56]. It is the first Swarm Intelligence founded Algorithms. The algorithm is based on the hunting behavior of social ants. Pheromone is

dumped by apiece ant and it evaporates gradually with time. It is applied for IIR filter design as described in the paper [15].

3.3 "Bacteria Foraging Optimization (BFO)": It is an evolutionary computation technique resulting from the social foraging performance of Escherichia coli bacteria. Kevin M. Passino was invented in the year 2000 [54]. It is a globally acknowledged algorithm and used for the optimization of composite engineering problems.

3.4 "Bat Algorithms (BA)": The Bat Algorithm was established by Xin-She Yang in 2010 [57]. The main physiognomies in the BA are based on the echolocation performance of micro-bats. it is found that the design of the FIR filter is used in the research paper [38].

3.5 "Biogeography Based Optimization (BBO)": It is the training of the geographical distribution of biological organisms. "BBO has structures in common with other biology-founded optimization methods, such as GAs and PSO". [58].

*3.6* "Crow Search Algorithm (CSA)": It is also an optimization algorithm heuristic and metaheuristic in nature. It is one of the metaheuristic optimization algorithm presented by Askarzadeh in the year 2016 [55]

3.7 "Cuckoo Search (CS)": It is optimization algorithm industrialized by "Xin She Yang and Suash Deb in 2009" [59]. It is enthused by cuckoo species that lay their eggs in the nests of other species. [23,48,51].

3.8 "Firefly algorithm (FA)": It is developed by Yang in 2007 [60]. It is based on the broken patterns and conduct of fireflies. [47].

3.9 "Genetic Algorithms (GA)": It is advanced by John Holland and his traitors in the 1960s and 1970s [4,5]. GA is a concept of biological fruition based on Charles Darwin's model of natural selection [8,28,40,41]. There are many variants of GA industrialized and applied for a large number of optimization matters.

3.10 "Grasshopper Optimization Algorithms (GOA)": It is developed by Shahrzad Saremi in 2017 [72] and practical to challenging optimization problems. The algorithm's mathematical models are based on the behavior of grasshopper swarms in nature. "The GOA algorithm is first benchmarked on a set of test problems including CEC2005 to test and verify its recital qualitatively and quantitatively" [51].

3.11 "Gravitational Search Algorithm (GSA)": It is established by Rashedi in the year 2009 [64]. GSA is a new optimization algorithm centered on the law of gravity and mass interactions. [35].

3.12 "Grey Wolf Algorithm (GWA)": It is a recent metaheuristic planned by Mirjalili in 2014 [62]. GWO is inspired by the grey wolf's social ladder and hunting mechanism.

3.13 "Harmony Search (HS)": It is a singularity mimicking metaheuristic presented in 2001 by Zong Woo Geem, Joong Hoon Kim, and G. V. Loganathan.[65]. It is inspired by the makeshift process of jazz musicians

3.14 "Krill Herd (KH)": It is developed by Gandomi and Alavi in the year 2012 It is a biologically enthused algorithm and applied for solving optimization problems. It is based on the herding behavior of krill individuals. [67].

3.15 "Memetic Algorithm (MA)": It is developed by Pablo Moscato in the year 1989 [66]. The pseudo-code for MA is Procedure Memetic Algorithm, Set, Generate an initial population; while Stopping state of affairs are not satisfied [39].

3.16 "Particle Swarm Optimization (PSO)": It is originally credited to Kennedy, Eberhart, and Shi in the year 1995 [4],[5]. It is envisioned for simulating the social behavior, and movement of entities in a bird flock or fish school. [16,19,24,29,30,36,38,48,52].

3.17 "Social Spider Optimization (SSO)": It is proposed by Erik in the year 2014 [71]. An algorithm is inspired by the cooperative behavior of social animals.

3.18 "Swarm Intelligence (SI)": It is the collective behavior of distributed, self-organized, natural, or artificial systems based on nature stirred and used on artificial optimization issues. [70]. There is no centralized control structure to concern commands to agents. Agents follow meek rules in local, and to certain degree random, connections between them arise into the optimal global behavior, unknown to the separate agents. [24,30,36,38,48,49]

3.19 "Symbiotic Organisms Search (SOS)": It is one of the very auspicious recent development in the field of metaheuristic algorithms projected by Min-Yuan Cheng in the year 2014. It is also nature stirred metaheuristic and is analogous to the collaborating behavior among organisms in nature.

3.20 "Teaching Learning Based Optimization (TLBO)": It is proposed by Rao in the year 2011. It is a wellorganized optimization method. Like other nature enthused algorithms, this method is inspired by the effect of a teacher on learners. It has two chunks: Teacher Phase' and Learner Phase.

3.21 "Whale Optimization Algorithm (WOA)": This is a nature-inspired metaheuristic optimization algorithm, called Whale Optimization Algorithm (WOA), which impersonators the social conduct of humpback whales. [61].

#### 4. Survey on Optimization Algorithms

A comprehensive survey is carried out on literature available on the proposed optimization algorithm for the proposal of the digital filter. Standard abbreviations, original authors, and year of origin of optimization algorithms and their application of FIR, IIR filter is presented in table.1.

Sl.	Name of Optimization Algorithm and	Standard	Original Author	Year	Filter
No.	applied in research papers for digital filter	Abbreviations	_	of	Туре
	design			origin	
1	Artificial Bee Colony [33,34,42,43]	ABC	Karaboga D [53]	2005	IIR
					FIR
2	Ant Colony Optimization [15]	ACO	Marco Dorigo [56]	1992	IIR
3	Bacterial Foraging Optimization [54]	BFO	Kevin M Passiono	2000	IIR
			[54]		FIR
4	Bat Algorithms ([38]	BA	Xin-She Yang [57]	2010	FIR
5	Biogeography Based Optimization [58]	BBO	Dan Simon [58]	2008	IIR
6	Crow Search Algorithm [37]	CSA	Askarzadeh, A[55]	2016	FIR
7	Cuckoo Search [23,48,51]	CS	Xin-She Yang,S	2009	IIR
			Deb[59]		FIR
8	Firefly Algorithm [47]	FA	Xin-She Yang [60]	2007	FIR
9	Genetic Algorithms [8,28,40,41]	GA	John Holland[4],[5]	1960	FIR
					IIR
10	Grasshooper Optimization Algorithms [51]	GOA	Shahrzad Saremi [72]	2017	FIR
11	Gravitational Search Algorithm [35]	GSA	Rashedi [64]	2009	IIR
12	Grey Wolf Algorithm [62]	GWA	Mirjalili [62]	2014	FIR
13	Harmony Search [31]	HS	Geem, Kim [65]	2001	IIR
i14	Krill Herd [67]	KH	Gandomi [67]	2012	IIR
15	Memetic Algorithm [39]	MA	Pablo Moscato [66]	1989	FIR
16	Particle Swarm Optimization	PSO	Kennedy j [63]	1995	IIR
	[16,19,24,29,30,36,38,48,52]				FIR
17	Social Spider Optimization [71]	SSO	.Erik [71]	2014	IIR
18	Swarm Intelligence [24,30,36,38,48,49]	SI	Beni and J Wang [70]	1989	IIR
					FIR
19	Symbiotic Organisms Search [69]	SOS	Min-Yuan Cheng [69]	2014	FIR
20	Teaching Learning Based Optimization [68]	TLBO	Rao, [68]	2011	FIR
21	Whale Optimization Algorithm [61]	WOA	S. Mirjalili [61]	2016	IIR

## Table.1: Survey on Optimization Algorithms.

On analyzing the data presented in the table.1, it is found that a large number of a research paper is based on nature-inspired optimization algorithms.

## 5. Conclusions

This paper is presented with a review and analysis of numerous optimization algorithms projected by researchers for the optimal design of digital filters. More precisely, Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filter design using optimization-based techniques such as nature-inspired "Ant Colony Optimization (ACO), Swarm Intelligence (SI) Swarm Intelligence (SI), Genetic Algorithms (GA), Particle Swarm Optimization, (PSO), Bat Algorithms (BA), Grasshopper Optimization Algorithms, Cuckoo Search (CS), Artificial Bee Colony (ABC), Bacterial foraging optimization (BFO, Biogeography-based optimization (BBO), Firefly algorithm (FA), Gravitational search algorithm (GSA), Grey wolf algorithm (GWO), Harmony search (HS), Krill herd (KH), Social spider optimization (SSO), Teaching-learning-based optimization (TLBO), Symbiotic organisms search (SOS), Whale optimization algorithm (WOA)" are described with the nature in which they improve the solutions. Also, a survey on the origin of twenty-one optimization algorithms is conceded out that are being applied by researchers as optimization algorithms in literature for the policy of digital filters. It is found that most nature-inspired optimization algorithms perform sound in finding optimal solutions. However, in literature, it has also found that individually PSO, BAT, GA, and GSA are found to provide the improved solution, but by uniting the two of the algorithms i.e. hybridizing the algorithms elasticity

the best solution. The use of two optimization algorithms synergizes the resolutions and leads to enhance optimization; foremost to be more correct. Therefore, hybrid algorithms may be favored in digital filter design.

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