

## Intuitionistic Fuzzy Ideals of $M\Gamma$ groups in Near Rings as Maximal Product of Graphs

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**Abstract:** This paper explains about the degree of the vertices in the Maximal product of graphs which represents the IF Ideals of  $M\Gamma$  groups in Near rings. It also describes various theorems about the characteristics of the vertices in calculating the degrees with an example

**Keywords:** IF Ideals of  $M\Gamma$  groups in Near rings, Maximal product of graphs, Degrees of vertices in Maximal product of graphs

### 1. Introduction

In 1983, G. Pilz [11] introduced and explained the concept of Near rings. After the introduction of Fuzzy set by Zadeh L.A[15], many extended the algebraic concept of Near rings to Fuzzy Near rings. In 1996, S. D. KIM & H. S. KIM [4] extended Fuzzy Ideals of Near rings and also explained their various characteristics by theorems. K. T. Atanassov [1], in 1986 extended Fuzzy sets to IF sets by introducing IF sets. Later A. Jianming et al[2] discussed about IF ideals in near rings in 2005, M.G. Karunambigai et al[6] in 2012, explained various properties of IF graphs with its properties. S.K. Mala et al [7] described IF ideals of  $M\Gamma$  groups in Near rings in 2018 and later represented them as graph by S.K. Mala et al[8] by explaining its properties in 2019. In 2019, M. Sitara et al [10] introduced Fuzzy graph structures with applications in detail. The maximal product of graphs of IF Ideals of  $M\Gamma$  groups in Near rings has been discussed by S.K. Mala et al [9] in 2020.

### 2. Preliminaries

#### Definition: 2.1

Near ring is a non-empty set with two binary operation satisfying

- i. Group with respect to first operation
- ii. Semi group with respect to second operation
- iii. Second operation is distributive over the first operation.

#### Definition: 2.2

Fuzzy set is a crisp set with its elements having membership function. If they have non-membership value along with it satisfying the condition that their sum lies between 0 and 1, is called as an Intuitionistic Fuzzy set.

#### Definition: 2.3

A Fuzzy set in a near-ring R is called a fuzzy ideal of R if it satisfies:

- (i)  $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$
- (ii)  $\mu(y + x - y) \geq \mu(x)$
- (iii)  $\mu(xy) \geq \mu(y)$
- iv.  $\mu((x + z) - y - xy) \geq \mu(z)$  for all  $x, y, z \in R$ .

#### Definition: 2.4

An IF set A of a Near ring is said to be Intuitionistic fuzzy ring if it obeys

- (i)  $\mu_A(x - y) > \min\{\mu_A(x), \mu_A(y)\}$
- (ii)  $\mu_A(xy) > \min\{\mu_A(x), \mu_A(y)\}$
- (iii)  $\gamma_A(x - y) < \max\{\gamma_A(x), \gamma_A(y)\}$
- (iv)  $\gamma_A(xy) < \max\{\gamma_A(x), \gamma_A(y)\}$ , for all  $x, y$  in near ring.

#### Definition: 2.5

Let  $GI_1(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$  and  $GI_2(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$  be 2 graphs of intuitionistic fuzzy ideals of  $M\Gamma$  group in near rings (IFIM $\Gamma$ GNR)  $I_1$  and  $I_2$  then  $GI_1 * GI_2 = (V_1, E_1, \mu_1, \gamma_1)$  is called maximal product graph of intuitionistic fuzzy ideal of  $M\Gamma$  group in near rings with structure vertices –

$V_1 = V_{I1} * V_{I2}$  and, edges  $-E_1 = \{(u_1, v_1) (u_2, v_2) / u_1 = u_2 \text{ and } v_1, v_2 \in E_{I2} \text{ (or) } v_1 = v_2 \text{ and } u_1, u_2 \in E_{I1}\}$

Here  $\mu_I(u,v) = \mu_{I1}(u) \vee \mu_{I2}(v)$  for all  $(u,v) \in V_1$   
 and  $\gamma_I(u,v) = \gamma_{I1}(u) \wedge \gamma_{I2}(v)$  for all  $(u,v) \in V_1$ .  
 Also,  $\mu_I((u_1,v_1)(u_2,v_2)) = \{\mu_{I1}(u_1) \vee \mu_{I2}(v_1 v_2) \text{ where } u_1 = u_2 \& v_1 v_2 \in E_{I2}$   
 $\mu_{I2}(v_2) \vee \mu_{I1}(u_1 u_2) \text{ where } v_1=v_2 \& u_1 u_2 \in E_{I1}$   
 and  $\gamma_I((u_1,v_1)(u_2,v_2)) = \{\gamma_{I1}(u_1) \wedge \gamma_{I2}(v_1 v_2) \text{ where } u_1 = u_2 \& v_1 v_2 \in E_{I2}$   
 $\gamma_{I2}(v_2) \wedge \gamma_{I1}(u_1 u_2) \text{ where } v_1=v_2 \& u_1 u_2 \in E_{I1}$ .

Here  $E_I$ (edges set) has edges only if either the first coordinates are same, or the second coordinates are same with an edge existing already in  $G_{I1}$  or  $G_{I2}$ .

**3. Intuitionistic Fuzzy Ideals of  $M\Gamma$ groups in Near Rings as Maximal Products of Graph**

Let  $G_{I1}(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$  and  $G_{I2}(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$  be two graphs of IFIM $\Gamma$ GNR  $I_1$  and  $I_2$  in near ring  $N^*$  then,  $G_{I1} * G_{I2} = (V_I, E_I, \mu_I, \gamma_I)$  is called maximal product structure of IFIM $\Gamma$ GNR.

The following theorems explains the degree and total degree of vertices  $V_I$  of  $G_{I1} * G_{I2}$ .

**Theorem: 3.1**

If  $G_{I1}(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$  and  $G_{I2}(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$  are the graphs of IFIM $\Gamma$ GNR such that  $\mu_{I1}(u_i) \leq \mu_{I2}(u_i v_j)$   $\gamma_{I1}(u_i) \geq \gamma_{I2}(v_i v_j)$  therefore, the vertex degree of maximal product

$G_{I1} * G_{I2}(V_I, E_I, \mu_I, \gamma_I)$  is given by  
 $D_{G_{I1} * G_{I2}} \mu_I(u_i, v_j) = D_{G_{I1}} \mu_{I1}(u_i) \mu_{I2}(v_j) + D_{G_{I2}} \mu_{I2}(v_j)$   
 $D_{G_{I1} * G_{I2}} \gamma_I(u_i, v_j) = D_{G_{I1}} \gamma_{I1}(u_i) \gamma_{I2}(v_j) + D_{G_{I2}} \gamma_{I2}(v_j)$

**Proof**

Let  $G_1(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$  and  $G_2(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$  are the graphs of IFIM $\Gamma$ GNR such that  $\mu_{I1}(u_i) \leq \mu_{I2}(v_i v_j)$  then  $\mu_{I1}(u_i v_j) \leq \mu_{I2}(v_j)$  and  $\gamma_{I1}(u_i) \geq \gamma_{I2}(v_i v_j)$  then  $\gamma_{I1}(u_i v_j) \geq \gamma_{I2}(v_j)$  for  $u_i \in V_{I1}, u_i v_j \in E_{I1}, v_i \in V_{I2}, v_i v_j \in E_{I2}$ .

Therefore, the vertex degree of  $G_{I1} * G_{I2}$  maximal product are:  
 $D_{G_{I1} * G_{I2}} \mu_I(u_i, v_j) = \sum \mu_{I1}(u_i v_j) \vee \mu_{I2}(v_j) + \sum \mu_{I2}(v_i v_j) \vee \mu_{I1}(u_i)$  and  
 $D_{G_{I1} * G_{I2}} \gamma_I(u_i, v_j) = \sum \gamma_{I1}(u_i v_j) \wedge \gamma_{I2}(v_j) + \sum \gamma_{I2}(v_i v_j) \wedge \gamma_{I1}(u_i)$   
 $\Rightarrow D_{G_{I1} * G_{I2}} \mu_I(u_i, v_j) = \sum \mu_{I2}(v_j) + \sum \mu_{I2}(v_i v_j)$  and  
 $D_{G_{I1} * G_{I2}} \gamma_I(u_i, v_j) = \sum \gamma_{I2}(v_j) + \sum \gamma_{I2}(v_i v_j)$   
 $\Rightarrow D_{G_{I1} * G_{I2}} \mu_I(u_i, v_j) = D_{G_{I1}} \mu_{I1}(u_i) \mu_{I2}(v_j) + D_{G_{I2}} \mu_{I2}(v_j)$  and  
 $D_{G_{I1} * G_{I2}} \gamma_I(u_i, v_j) = D_{G_{I1}} \gamma_{I1}(u_i) \gamma_{I2}(v_j) + D_{G_{I2}} \gamma_{I2}(v_j)$

**Example: 3.2**

Consider  $G_{I1}(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$  for  $I_1 = \{0\}$  of  $Z_3$  and  $G_{I2}(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$  for  $I_2 = \{0\}$  of  $Z_4$  therefore,  $G_I = G_{I1} * G_{I2}$  is a maximal product of  $G_{I1}$  and  $G_{I2}$ . This is explained in the following example.

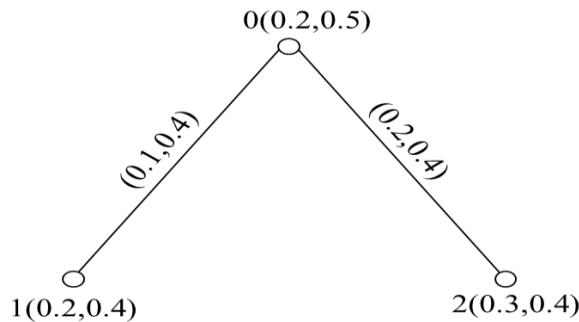


Figure 1: Graph  $G_{I1}$

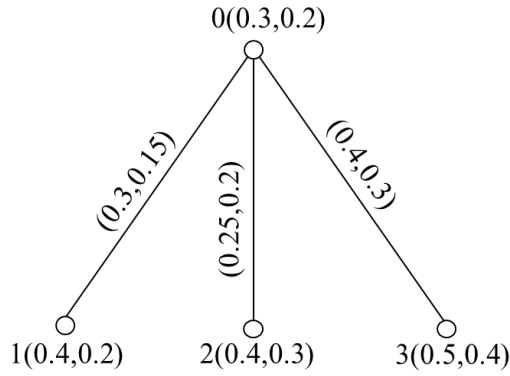


Figure 2: Graph GI2

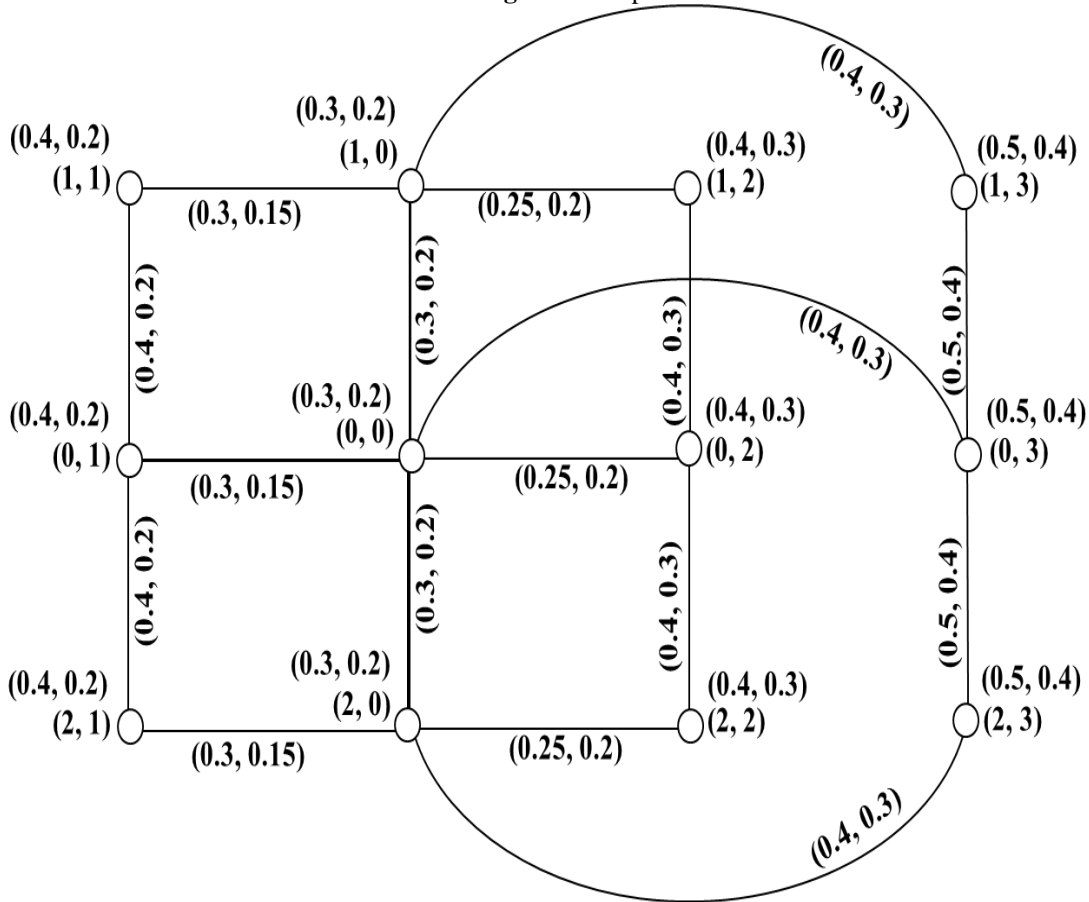


Figure 3: Graph GI1 \* GI2

The  $\mu_l$  value for  $GI_1 * GI_2$  both by theorem and direct calculation are obtained as follows.

By theorem,

$$D_{GI_1 * GI_2} \mu_l(0, 0) = D_{GI_1}^* (0) \mu_{l_2} (0) + D_{GI_2} \mu_{l_2} (0) \\ = 2(0.3) + (0.3 + 0.25 + 0.4) = 1.55$$

$$D_{GI_1 * GI_2} \mu_l(0, 1) = D_{GI_1}^* (0) \mu_{l_2} (1) + D_{GI_2} \mu_{l_2} (1) \\ = 2(0.4) + (0.3) = 1.1$$

$$D_{GI_1 * GI_2} \mu_l(0, 2) = D_{GI_1}^* (0) \mu_{l_2} (2) + D_{GI_2} \mu_{l_2} (2) \\ = 2(0.4) + (0.25) = 1.05$$

$$D_{GI_1 * GI_2} \mu_l(0, 3) = D_{GI_1}^* (0) \mu_{l_2} (3) + D_{GI_2} \mu_{l_2} (3) \\ = 2(0.5) + (0.4) = 1.4$$

$$\begin{aligned} D_{G_1 * G_2} \mu_l(1, 0) &= D_{G_1} * (1) \mu_{l_2}(0) + D_{G_2} \mu_{l_2}(0) \\ &= 1(0.3) + (0.3 + 0.25 + 0.4) = 1.25 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \mu_l(1, 1) &= D_{G_1} * (1) \mu_{l_2}(1) + D_{G_2} \mu_{l_2}(1) \\ &= 1(0.4) + (0.3) = 0.7 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \mu_l(1, 2) &= D_{G_1} * (1) \mu_{l_2}(2) + D_{G_2} \mu_{l_2}(2) \\ &= 1(0.4) + (0.25) = 1.65 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \mu_l(1, 3) &= D_{G_1} * (1) \mu_{l_2}(3) + D_{G_2} \mu_{l_2}(3) \\ &= 1(0.5) + (0.4) = 0.9 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \mu_l(2, 0) &= D_{G_1} * (2) \mu_{l_2}(0) + D_{G_2} \mu_{l_2}(0) \\ &= 1(0.3) + (0.3 + 0.25 + 0.4) = 1.25 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \mu_l(2, 1) &= D_{G_1} * (2) \mu_{l_2}(1) + D_{G_2} \mu_{l_2}(1) \\ &= 1(0.4) + (0.3) = 0.7 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \mu_l(2, 2) &= D_{G_1} * (2) \mu_{l_2}(2) + D_{G_2} \mu_{l_2}(2) \\ &= 1(0.4) + (0.25) = 1.65 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \mu_l(2, 3) &= D_{G_1} * (2) \mu_{l_2}(3) + D_{G_2} \mu_{l_2}(3) \\ &= 1(0.5) + (0.4) = 0.9 \end{aligned}$$

By direct calculation,

$$D_{G_1 * G_2} \mu_l(0, 0) = 0.25 + 0.3 + 0.4 + 0.3 + 0.3 = 1.55$$

$$D_{G_1 * G_2} \mu_l(0, 1) = 0.3 + 0.4 + 0.4 = 0.1$$

$$D_{G_1 * G_2} \mu_l(0, 2) = 0.4 + 0.25 + 0.4 = 1.05$$

$$D_{G_1 * G_2} \mu_l(0, 3) = 0.5 + 0.5 + 0.4 = 1.4$$

$$D_{G_1 * G_2} \mu_l(1, 0) = 0.25 + 0.4 + 0.3 + 0.3 = 1.25$$

$$D_{G_1 * G_2} \mu_l(1, 1) = 0.3 + 0.4 = 0.7$$

$$D_{G_1 * G_2} \mu_l(1, 2) = 0.25 + 0.4 = 0.65$$

$$D_{G_1 * G_2} \mu_l(1, 3) = 0.4 + 0.5 = 0.9$$

$$D_{G_1 * G_2} \mu_l(2, 0) = 0.4 + 0.25 + 0.3 + 0.3 = 1.25$$

$$D_{G_1 * G_2} \mu_l(2, 1) = 0.4 + 0.3 = 0.7$$

$$D_{G_1 * G_2} \mu_l(2, 2) = 0.4 + 0.25 = 0.65$$

$$D_{G_1 * G_2} \mu_l(2, 3) = 0.5 + 0.4 = 0.9$$

Similarly, the  $\gamma_l$  value for  $G_1 * G_2$  both by theorem and direct calculation are obtained as follows:

By theorem,

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(0, 0) &= D_{G_1} * (0) \gamma_{l_2}(0) + D_{G_2} \gamma_{l_2}(0) \\ &= 2(0.2) + (0.15 + 0.2 + 0.3) = 1.05 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(0, 1) &= D_{G_1} * (0) \gamma_{l_2}(1) + D_{G_2} \gamma_{l_2}(1) \\ &= 2(0.2) + (0.15) = 0.55 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(0, 2) &= D_{G_1} * (0) \gamma_{l_2}(2) + D_{G_2} \gamma_{l_2}(2) \\ &= 2(0.3) + (0.2) \\ &= 0.8 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(0, 3) &= D_{G_1} * (0) \gamma_{l_2}(3) + D_{G_2} \gamma_{l_2}(3) \\ &= 2(0.4) + (0.3) = 1.1 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(1, 0) &= D_{G_1} * (1) \gamma_{l_2}(0) + D_{G_2} \gamma_{l_2}(0) \\ &= 1(0.2) + (0.15 + 0.2 + 0.3) = 0.85 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(1, 1) &= D_{G_1} * (1) \gamma_{l_2}(1) + D_{G_2} \gamma_{l_2}(1) \\ &= 1(0.2) + (0.15) = 0.35 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(1, 2) &= D_{G_1} * (1) \gamma_{l_2}(2) + D_{G_2} \gamma_{l_2}(2) \\ &= 1(0.3) + (0.2) = 1.5 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(1, 3) &= D_{G_1} * (1) \gamma_{l_2}(3) + D_{G_2} \gamma_{l_2}(3) \\ &= 1(0.4) + (0.3) = 0.7 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(2, 0) &= D_{G_1} * (2) \gamma_{l_2}(0) + D_{G_2} \gamma_{l_2}(0) \\ &= 1(0.2) + (0.15 + 0.2 + 0.3) = 0.85 \end{aligned}$$

$$\begin{aligned} D_{G_1 * G_2} \gamma_l(2, 1) &= d_{G_1} * (2) \gamma_{l_2}(1) + d_{G_2} \gamma_{l_2}(1) \\ &= 1(0.2) + (0.15) = 0.35 \end{aligned}$$

$$D_{G_1 * G_2} \gamma_l(2, 2) = d_{G_1} * (2) \gamma_{l_2}(2) + d_{G_2} \gamma_{l_2}(2)$$

$$\begin{aligned}
 &= 1(0.3) + (0.2) = 0.5 \\
 D_{GI_1 * GI_2} \gamma_l(2, 3) &= d_{GI_1}^* (2) \gamma_l(3) + d_{GI_2} \gamma_l(3) \\
 &= 1(0.4) + (0.3) = 0.7 \\
 \text{By direct calculation,} \\
 D_{GI_1 * GI_2} \gamma_l(0, 0) &= 0.2 + 0.3 + 0.2 + 0.2 + 0.15 = 1.05 \\
 D_{GI_1 * GI_2} \gamma_l(0, 1) &= 0.2 + 0.2 + 0.15 = 0.55 \\
 D_{GI_1 * GI_2} \gamma_l(0, 2) &= 0.3 + 0.3 + 0.2 = 0.8 \\
 D_{GI_1 * GI_2} \gamma_l(0, 3) &= 0.3 + 0.4 + 0.4 = 1.1 \\
 D_{GI_1 * GI_2} \gamma_l(1, 0) &= 0.2 + 0.2 + 0.3 + 0.15 = 1.85 \\
 D_{GI_1 * GI_2} \gamma_l(1, 1) &= 0.15 + 0.2 = 0.35 \\
 D_{GI_1 * GI_2} \gamma_l(1, 2) &= 0.2 + 0.3 = 0.5 \\
 D_{GI_1 * GI_2} \gamma_l(1, 3) &= 0.4 + 0.3 = 0.7 \\
 D_{GI_1 * GI_2} \gamma_l(2, 0) &= 0.15 + 0.2 + 0.2 + 0.3 = 0.85 \\
 D_{GI_1 * GI_2} \gamma_l(2, 1) &= 0.2 + 0.15 = 0.35 \\
 D_{GI_1 * GI_2} \gamma_l(2, 2) &= 0.3 + 0.2 = 0.5 \\
 D_{GI_1 * GI_2} \gamma_l(2, 3) &= 0.3 + 0.4 = 0.7
 \end{aligned}$$

**Theorem:3.3**

If  $GI_1 (V_{I_1}, E_{I_1}, \mu_{I_1}, \gamma_{I_1})$  and  $GI_2 (V_{I_2}, E_{I_2}, \mu_{I_2}, \gamma_{I_2})$  are the graphs of IFIM $\Gamma$ GNR such that  $\mu_{I_1}(u_i) \leq \mu_{I_2}(v_i v_j)$   $\gamma_{I_1}(u_i) \geq \gamma_{I_2}(v_i v_j)$  and function  $\mu_{I_2}(v_j)$  is a constant “C<sub>1</sub>” and function  $\gamma_{I_2}$  is a constant “C<sub>2</sub>”. Therefore, the vertex degree in  $GI_1 * GI_2$  maximal product graph of IFIM $\Gamma$ GNR is given by  $D_{GI_1 * GI_2} \mu_l(u_i, v_j) = D_{GI_1}^* \mu_{I_1}(u_i) \cdot C_1 + D_{GI_2} \mu_{I_2}(v_j)$  and  $D_{GI_1 * GI_2} \gamma_l(u_i, v_j) = D_{GI_1}^* \gamma_{I_1}(u_i) \cdot C_2 + D_{GI_2} \gamma_{I_2}(v_j)$

**Proof**

If  $GI_1 (V_{I_1}, E_{I_1}, \mu_{I_1}, \gamma_{I_1})$  and  $GI_2 (V_{I_2}, E_{I_2}, \mu_{I_2}, \gamma_{I_2})$  are the graphs of IFIM $\Gamma$ GNR in such a way that  $\mu_{I_1}(u_i) \leq \mu_{I_2}(v_i v_j)$  and  $\gamma_{I_1}(u_i) \geq \gamma_{I_2}(v_i v_j)$  for  $i, j = 1$  to  $n$ . Here  $\mu_{I_2}(v_j) = C_1$  and  $\gamma_{I_2}(v_j) = C_2$ . Also,  $\mu_{I_1}(u_i) \leq \mu_{I_2}(v_i v_j) \implies \mu_{I_1}(u_i u_j) \leq \mu_{I_2}(v_j)$  and  $\gamma_{I_1}(u_i) \geq \gamma_{I_2}(v_i v_j) \implies \gamma_{I_1}(u_i u_j) \geq \gamma_{I_2}(v_j)$  for  $i, j = 1$  to  $n$ .

Therefore, the vertex degree in  $GI_1 * GI_2$  maximal product is

$$\begin{aligned}
 D_{GI_1 * GI_2} \mu_l(u_i, v_j) &= \sum \mu_{I_1}(u_i u_j) \vee \mu_{I_2}(v_j) + \sum \mu_{I_2}(v_i, v_j) \vee \mu_{I_1}(u_i) \\
 &= \sum \mu_{I_2}(v_j) + \sum \mu_{I_2}(v_i v_j) \\
 &= D_{GI_1}^* \mu_{I_1}(u_i) \cdot C_1 + D_{GI_2} \mu_{I_2}(v_j)
 \end{aligned}$$

$$\begin{aligned}
 \text{Also, } D_{GI_1 * GI_2} \gamma_l(u_i, v_j) &= \sum \gamma_{I_1}(u_i u_j) \wedge \gamma_{I_2}(v_j) + \sum \gamma_{I_2}(v_i, v_j) \wedge \gamma_{I_1}(u_i) \\
 &= \sum \gamma_{I_2}(v_j) + \sum \gamma_{I_2}(v_i v_j) = D_{GI_1}^* \gamma_{I_1}(u_i) \cdot C_2 + D_{GI_2} \gamma_{I_2}(v_j)
 \end{aligned}$$

**Theorem :3.4**

If  $GI_1 (V_{I_1}, E_{I_1}, \mu_{I_1}, \gamma_{I_1})$  and  $GI_2 (V_{I_2}, E_{I_2}, \mu_{I_2}, \gamma_{I_2})$  are the graphs of IFIM $\Gamma$ GNR,  $I_1$  and  $I_2$  such that  $\mu_{I_2}(v_j) \leq \mu_{I_1}(u_i v_j)$  and  $\gamma_{I_2}(v_j) \geq \gamma_{I_1}(v_i v_j)$  Therefore, the vertex degree in  $GI_1 * GI_2$  maximal product graph is given by

$$D_{GI_1 * GI_2} \mu_l(u_i, v_j) = D_{GI_2}^* \mu_{I_2}(v_j) \mu_{I_1}(u_i) + D_{GI_1} \mu_{I_1}(u_i)$$

$$D_{GI_1 * GI_2} \gamma_l(u_i, v_j) = D_{GI_2}^* \gamma_{I_2}(v_j) \gamma_{I_1}(u_i) + D_{GI_1} \gamma_{I_1}(u_i)$$

**Proof**

If  $GI_1 (V_{I_1}, E_{I_1}, \mu_{I_1}, \gamma_{I_1})$  and  $GI_2 (V_{I_2}, E_{I_2}, \mu_{I_2}, \gamma_{I_2})$  are the graphs of IFIM $\Gamma$ GNR,  $I_1$  and  $I_2$  so that  $\mu_{I_2}(u_j) \leq \mu_{I_1}(u_i u_k) \implies \mu_{I_2}(v_i v_j) \leq \mu_{I_1}(u_i)$  for  $i, j = 1$  to  $n$  and

$$\gamma_{I_2}(u_j) \geq \gamma_{I_1}(u_i u_k) \implies \gamma_{I_2}(v_i v_j) \geq \gamma_{I_1}(u_i).$$

Therefore, the vertex degree of maximal product  $GI_1 * GI_2$  is

$$\begin{aligned}
 D_{GI_1 * GI_2} \mu_l(u_i, v_j) &= \sum \mu_{I_1}(u_i u_k) \vee \mu_{I_2}(v_j) + \sum \mu_{I_2}(v_i v_j) \vee \mu_{I_1}(u_i) \\
 \implies D_{GI_1 * GI_2} \mu_l(u_i, v_j) &= \sum \mu_{I_1}(u_i u_k) + \sum \mu_{I_1}(u_i) \\
 &= D_{GI_1} \mu_{I_1}(u_i) + D_{GI_2}^* \mu_{I_2}(v_j) \mu_{I_1}(u_i)
 \end{aligned}$$

$$\text{Similarly, } D_{GI_1 * GI_2} \gamma_l(u_i, v_j) = \sum \gamma_{I_1}(u_i u_k) \wedge \gamma_{I_2}(v_j) + \sum \gamma_{I_2}(v_i v_j) \wedge \gamma_{I_1}(u_i)$$

$$\begin{aligned}
 \implies D_{GI_1 * GI_2} \gamma_l(u_i, v_j) &= \sum \gamma_{I_1}(u_i u_k) + \sum \gamma_{I_1}(u_i) \\
 &= D_{GI_1} \gamma_{I_1}(u_i) + D_{GI_2}^* \gamma_{I_2}(v_j) \gamma_{I_1}(u_i)
 \end{aligned}$$

**4. Conclusion**

The degree of maximal product of two graphs of IF Ideals of M $\Gamma$ groups in Near rings are explained by theorems and verified through an example. This can be further extended in Intuitionistic fuzzy graph which has wider applications in the modern scientific world.

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