Comparison Between Quadratic And Logistic Discrimination Function To Get On The Best Classification With Application

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Abstract: The purpose of this research is to compare between quadratic and logistic discrimination function, also to identify the best method in discrimination which helps us to classify the data correctly. A several stages are starting from stratified sampling; classification errors and error rate if the pre-probability is equal and unequal by containing a few errors. Our choice of the correct path in analyzing the data helped us to identify the reasons that increase the errors in classification. In this study, we used some statistical measurements in terms of, the stratified sample, classification errors and the percentage of errors.
The percentage of errors in the pre-probability is equal and unequal when the accumulation of errors in the square discrimination function was less than the accumulation of errors in the logistic discrimination function when the probability is equal, but when the probability is not equal, the results of the square function were much better than the square discrimination function. However, the accumulation of errors in the quadratic function is less than the accumulation of errors in the logistic function.

Keywords: Quadratic discrimination function, logistic discrimination function, Misclassification, Error rate.

1- Introduction

The discrimination function is one of the methods used to classify data. Therefore, according to studies, societies are made up of several groups. The discrimination function categorizes data into the totals to which it belongs. In this research, the classification process begins with taking data from the community and ends with the classification of data by groups. The classification process takes place through several stages, errors occurs in each stage of classification. The method is chosen when the error rate is few, the classification errors will be less, and the results of classification will be less.

In the previous years, many scientists used discrimination function such as Randles and others (1978), compared the linear discrimination function with quadratic discrimination function. Titterington (1981) discussed the discrimination function in complex data. Schmitz and others (1983) compared the logistic discrimination function with methods within applied data. Bayne and others (1983) studied the estimating parameters for linear Fisher function, logistic function, and quadratic discrimination function. Pohar and others (2004) used regression comparison logistic with linear discrimination function using simulation. Ganeslingam (2006) compared the quadratic discrimination with discrimination function. Al-Rawi (2007) used the canonical discrimination function in distinguishing between digital images. Al-Rawi (2007) used quality control and the discriminative function in applied studies. Rashid and others (2008) studied a student performance in conducting the tasks such as tests, or presentation for duties. Ion (2012) compared between logistic regression function with normal distinction. Tsangaratos and Ilia (2016) also compared logistic regression with Bayesian discrimination.

The research aims to compare the quadratic discrimination function with the logistic discrimination function to reach a better classification for students of the Legal Management Department at the Technical Institute / Nineveh will help us to use methods to communicate with students better. Reducing errors in the distribution of students correctly according to the representative system (Auditory, visual, sensory).

This research was written on several stages it started from the theoretical side in which the methods of calculating the square discrimination function were discussed. Also, the methods of calculating the logistic discrimination function, through which the methods of constructing the quadratic and logistic discrimination function and the identification of error results from the identified functions. As for the practical side, data was collected from students The Legal Department of the Technical Institute of Nineveh to identify the representative systems for students, students were divided into three groups (audio, visual, sensory). The errors resulting from taking the stratified and non-stratified sample were calculated, and in the case of equal pre-probability equal variable. The classification errors were presented for the quadratic and logistic classification functions at the known and unknown pre-probability. Also, the error rate was displayed in the squared and logistic discrimination function if the probability is equal and unequal. It turns out that the quadratic discrimination function is better than the logistic discrimination function and the best is the quadratic discrimination function if the pre-probability is equal.

The article is organized as follows. Multiple quadratic discrimination function is introduced in Section 2. Discrimination using logistic regression is introduced in Section 3. Data Analysis is discussed in Section 4. The results and discussions are given in Section 5. Conclusions and recommendations are discussed in Section 6.

2- Multiple Quadratic Discrimination Function

If we have g groups, and the number of observation n_i of the Group i, and the prior probability Group q_i of the Group i, and that the value of observation x to Discrimination with p dimension of variables, to coincide with the law of discrimination, and that x is bound column that corresponds to a row of original data, and imposing $f_i(x)$ which is a density function of the Group i,Assuming that $P(x/G_i)$ represents the conditional probability to observation x which belong to the group i, and probability prior to the group i which gives x viewing as follows $f(G_i/x)$ according to the following bayes theory:(Remme,1980)

$$f(G_i/x) = \frac{q_i\, f_i(x)}{\sum_{i=1}^{g} q_i\, f_i(x)} \qquad (1)$$

And by replacing $P(x/G_i)$instead of $f_i(x)$, the equation will be as follows:

$$f(G_i/x) = \frac{q_i\, P\left(\frac{x}{G_i}\right)}{\sum_{i=1}^{g} q_i\, P\left(\frac{x}{G_i}\right)} \qquad (2)$$

Quadratic Discriminatory function assume that multivariate groups naturally ,and assuming that S_i represent variance matrix between groups of the group i, and $\overline{x_i}$ represent average of the group i, and square Mahalanobis' distance between observation is:(Wu,1996)

$$D^2 = (x-\overline{x_i})' S_i^{-1}(x-\overline{x_i}) \qquad (3)$$

When replace function 3 in multiple density function normal distribution we will get:

$$f(x/G_i) = (2\pi)^{-p/2}|S_i|^{-1/2}e^{-D_i^2/2} \qquad (4)$$

When replacing the above formula in $f(G_i/x)$ and a simple formula we get:

$$f(G_i/x) = \frac{q_1|S_i|^{-1/2}e^{-D_i^2/2}}{\sum_{j=1}^{g} q_1|S_i|^{-1/2}e^{-D_i^2/2}} \qquad (5)$$

And that the posterior possibility in the analysis Quadratic of discrimination observation x which belong to the group i, and that way calculation square Mahalanobis' distance between the averages of the totals is calculated by the Stata program. The quadratic discriminant function can be derived for square Mahalanobis' distance to impose that $Q_i(X)$is a quadratic discriminant function of group i for observation x.

$$Q_i = -D_i^2/2 - \ln|S_i|/2 + \ln(q_i) \qquad (6)$$

Through the above equation can be seen to the correct classification of the group through posterior possibility or a larger number of observations used to quadratic discriminant function. In the absence of a function is unequal variance, it can be applied to the quadratic discriminant analysis, As in the following equation.(Hamilton,2012)

$$\delta_k(x) = -\frac{1}{2}\log|\sum\nolimits_k| - \frac{1}{2}(x-\mu_k)^T \sum\nolimits_k^{-1}(x-\mu_k) + log\pi_k \qquad (7)$$

3- Discrimination Using Logistic Regression

Assuming that we have a range of observation and that these observation are distributed to the group, if the group $H_1, H_2, \dots H_g$ is a division of aggregates, which is consist of a vector of variables$x^T = (x_0, x_1, \dots, x_g)$, Assuming that the variation $x_0 = 0$ .The logistic function will be written in the form of the discrimination function according to the general formula of the prior probability and as follows

$$pr(H_5/x) = \frac{\exp(z_s)}{\sum_{s=1}^{g}\exp(z_s)} \qquad s = (1,2,\dots,g) \qquad (8)$$

That's where

$$z_s = \alpha_s^T \ .x \ and \ \alpha_s^T = (\alpha_{s0}, \dots, \alpha_{sp}) \ \ (s = 1, \dots, g-1) \qquad (9)$$

If $\alpha_g = 0$ that $z_g = 0$.
Easy to identify the p-value which represents the prior probability for groups H_s where (s=1,2,...,g).(Efron,1975)

$$\alpha_{s0} = \beta_{s0} + \log(P_s/P_g) \qquad (10)$$

$$\alpha_{j0} = \beta_{sj} (j = 1,2, \dots, p) \qquad (11)$$

The parameter vector $\beta_S^T = (\beta_{s0}, \beta_{s1}, \dots, \beta_{sp})$can be calculator retail according to the following formula:

$$log \frac{pr(x/H_s)}{pr(x/H_p)} = \beta_S^T . x. \qquad (12)$$

The logistic function of the ratio in equation 12 is necessary and important to apply in equation 8. It can also be said that the ratio in the equation 12 must be calculated for the difference between any two groups s ≠ t ≠ g

$$log \frac{pr(x/H_s)}{pr(x/H_g)} = \ log \frac{pr(x/H_s)}{pr(x/H_g)} - log \frac{pr(x/H_t)}{pr(x/H_g)}$$
$$= (\beta_s - \beta_t)^T . x. \qquad (13)$$

The first equation imply that H_s which depend on the group basis, but the results of the previous equation is used, And the selection of the second group requires us to calculate parameters so as not to affect the prior probability, z_s equation 9 is the Linear combinations between the variables and called (score),or discriminate function. Which represent the values$z^T = (z_1, \beta_2, \dots, \beta_{g-1})$ comprises scalars values or function of discrimination, and to discriminate between the g of the g-1 should be group of values. Using the theoretical ideal (Rao) and that the law division is ideal for observation x classification to H_s group If the following condition is met:

$$pr(H_s|x) \geq \ pr(H_t|x) \quad (t = 1,2, \dots, g) \quad (14)$$

Equation 8 can be replaced by equation 14.

$$z_t \geq z_t \qquad (15)$$

This equation is called the law of the optimal division which divides the sample space R^{g-1} with the knowledge that the sample space is R^p(Albert, 1986).

4- Data Analysis

The data was collected from students of the legal administration department of the first stage for the academic year 2019-2020 through a questionnaire adopted on 10 questions, through this community classify study questions to three groups depending on the answer issued diplomas that neither allowed students who first answer the four questions or more than they are classified as visual representation system, who were their answer The second answer four questions and more so they are classified within the audiovisual representation system, students who were their answer third answer four questions and more understanding are classified as kinesthetic representation system, as for the students whose answers were the third answer to four or more questions, they are classified within the system of kinesthetic representation, (1 represents the visual system, 2 represents the Audiovisual, 3 represents the kinesthetic system).

5- Results and Discussion

Table 1 represents the function used to distinguish the first, second, and third group and a function commonly used in discrimination and in both cases quadratic and logistic, which have been relied upon in the classification of data to (audio, visual, kinesthetic).

Table (1) represents a function of discrimination

Coef. of Variation	Estimation Sample discrm. QDA				Estimation Sample discrm. Logistic			
	1	2	3	Total	1	2	3	Total
Var1	0.491	0.453	0.500	0.540	2.875	0.991	1.000	1.391
Var2	0.431	0.415	0.338	0.402	0.807	0.908	0.571	0.741
Var3	0.266	0.178	0.286	0.263	0.517	0.242	0.498	0.495
Var4	0.266	0.167	0.195	0.220	0.517	0.333	0.371	0.428
Var5	0.531	0.436	0.465	0.485	1.283	0.895	0.899	1.029
Var6	0.517	0.471	0.379	0.460	1.123	0.943	0.655	0.888
Var7	0.366	0.520	0.481	0.557	3.479	1.138	0.917	1.434

Var8	0.550	0.312	0.458	0.470	1.651	0.595	0.949	1.063
Var9	0.480	0.419	0.464	0.483	1.501	1.083	0.913	1.165
Var10	0.327	0.255	0.141	0.277	0.564	0.498	0.216	0.463

Table (2) represents the ratio of incorrect equal prior probability

Error Rate	Quadratic function				Logistic function			
	1	2	3	Total	1	2	3	Total
Stratified	.0975	.0354	.0450	0.0593	0.1174	0.1948	0.1309	0.14772
Unstratified	-0.1002	0.2948	-0.0184	0.0587	-0.0647	0.3745	0.1084	0.1394
Priors	0.33	0.33	0.33		0.33	0.33	0.33	

The ratio of the error in the sample selection of class equal prior probability and unequal:

From Table 2, which shows the ratio of the error in the event of taking the sample stratified ratio of pre-equal, and we will talk about each group separately, as follows:

1. The ratio of error in the stratified sampling of the first group with the equal prior probability of a quadratic discrimination function is 0.0975 which is less than the ratio of the classify of the first group for the logistic discrimination function of about 0.1174. The ratio of the error in the non-stratified sample of the first group of prior probability equal quadratic discrimination function is -0.1002 which is less than the ratio of logistic discrimination function about -0.0647 the first group.
2. The ratio of error in the stratified sampling of the second group with the equal prior probability of a quadratic discrimination function is 0.0354 which is less than the ratio of the classify of the second group for the logistic discrimination function is about 0.1948. The ratio of the error in the non-stratified sample of the second group of prior probability equal quadratic discrimination function is 0.2948 which is less than the ratio of logistic discrimination function is about 0.3745.
3. The ratio of error in the stratified sampling of the third group with the equal prior probability of a quadratic discrimination function is 0.0450 which is less than the ratio of the classify of the third group for the logistic discrimination function about 0.1309 The ratio of the error in the non-stratified sample of the third group of prior probability equal quadratic discrimination function is -0.0647 which is less than the ratio of logistic discrimination function is about 0.1084.

Table (3) represents the ratio of incorrect unequal prior probability

Error Rate	Quadratic function				Logistic function			
	1	2	3	Total	1	2	3	Total
Stratified	0.1040	0.0523	0.0199	0.0623	0.0876	0.3010	0.0776	0.1335
Un-stratified	0.1040	0.0523	0.0199	0.0623	0.0876	0.3010	0.0776	0.1335
Priors	0.4146	0.2317	0.3536		0.4146	0.2317	0.3536	

From Table No. (3) which shows the percentage of incorrect if stratified sampling prior probability not equal, we will talk about each group separately as follows:

1. The ratio of error in the stratified sampling of the first group with the unequal prior probability of a quadratic discrimination function is 0.1040 which is bigger than the ratio of the classify of the first group for the logistic discrimination function about 0.0876 It is the only case where discrimination logistic function less than the value of a discriminant quadratic function. The non-stratified sample It is similar to the stratified sample.

2. The ratio of error in the stratified sampling of the second group with the unequal prior probability of a quadratic discrimination function is 0.0523 which is less than the ratio of the classification of the second group for the logistic discrimination function about 0.3010. The non-stratified sample is similar to the stratified sample.
3. The ratio of error in the stratified sampling of the second group with the unequal prior probability of a quadratic discrimination function 0.0199 which is less than the ratio of the classify of the second group for the logistic discrimination function of about 0.0776. The non-stratified sample is similar to the stratified sample.

The Ratio of Errors

From Table 4, which represents the ratio of errors in the functions of discrimination, as it follows: In the first function, the ratio of the error in quadratic discrimination function is (0.0882), which is less than the ratio of the error in logistic discrimination function, which about 0.1176,the ratio of the error in quadratic discrimination function is(0.0526), which is less than the second logistic discrimination function, about to (0.2105) and a significant difference between the two ratios are clearly, In the third function the ratio of the error in quadratic discrimination function is 0.0345,which is less than the third logistic discrimination function about to 0. 2105, If we compare the ratio of error in total of quadratic discrimination function for group which about to 0.05845 with the ratio of logistic discrimination function for group about 0.1554 which is equivalent to almost three times the ratio of the error in discrimination quadratic function, that all of the above, depending on the prior probability for three groups was 0.33.

Table 4, which represents the ratio of errors in functions								
Quadratic function					Logistic function			
	1	2	3	Total	1	2	3	Total
Error Rate	0.0882	0.0526	0.0345	.05845	0. 1176	0. 2105	0. 1379	0.1554
Priors	0.33	0.33	0.33		0.33	0.33	0.33	

Misclassification in Case of Equal Prior Probability

From Table 5, which represents the classification table data and misclassification can be described by the table as follows:

The first group data are 34 observation and using the quadratic discrimination function, the 31 observation has been properly classified to the first group by 91.18%, 2 observation classified to the second group of errors 5.88%, one observation was classified to the third set of errors 2.94%, Using the logistic discrimination function of (34) observation has been classified into as follows: 30 observation have been classified into the first group properly by 88.24, and 3 observations were classified incorrectly to the second group which belongs to the first group with error 8.82%, one observation has been classified into the third group which belongs to the first group with error 2.94%.

The second group is the 19 observation using a quadratic discrimination function, 18 observations were classified to the second group properly by 94.74%, 1 observation classified to the first group that belongs to the second group with error 5.26%, there is no observation has been classified into the third group incorrectly that the percentage of the error equal to zero, by using a function of logistic discriminate 15 observations were classified correctly to the second group properly by 78.95%,2 observations were classified to the first group that belongs to the second group with error 10.53%, two observations were classified to the third group that belongs to the second group with error 10.53%.

The third group is the 29 observations using a quadratic discrimination function, 28 observations were classified to the third group that belongs to the third group properly by 89.46%, and no observation has been classified incorrectly to the first group, 1 observation has been classified into the second group which belongs to the third group with error 3.45%, using discrimination logistic function the 25 observations have been properly classified to the third group by 86.21%, there are no observations have been classified into the first group incorrectly by error 0%,4 observations were classified to the second group that belongs to the third group with error 13.79%.

All of the above prior probability of 0.33 for each group, which means with equal prior probability.

Table (5) represents the correct classification and misclassification

Quadratic discriminate function					Logistic discriminate function				
Ture var.	1	2	3	Total	Ture var.	1	2	3	Total

1	31	2	1	34	1	30	3	1	34
	91.18	5.88	2.94	100		88.24	8.82	2.94	100
2	1	18	0	19	2	2	15	2	19
	5.26	94.74	0	100		10.53	78.95	10.53	100
3	0	1	28	29	3	0	4	25	29
	0	3.45	96.55	100		0	13.79	86.21	100
Total	32	21	29	82	Total	32	22	28	82
	39.02	25.61	35.37	100		39.02	26.83	34.15	100
Priors	0.333	0.333	0.333		Priors	0.3333	0.3333	0.3333	

Misclassification in Case of Unequal Prior Probability

In the preceding paragraph was discussed misclassification of quadratic discrimination function with logistic discrimination function in case of equal probability prior and in this paragraph will be discussed not equal prior probability, As can be noted that their ratios have changed clearly, As it will be mentioned differences, which rose in Table 6 for Table 5 only.

1. In the second group of quadratic discrimination function, 17 observations have been classified into the second group which belongs to the second property by 89.47%, 1 observation has been classified into the first group which belongs to the second group with error 5.26%, 1 observation has been classified into the third group which belongs to the second group with error 5.26%.
2. In the second group of logistic discrimination function, 12 observations have been classified into the second group which belongs to the second property by 63.161%, 4 observation have been classified into the first group which belong to the second group with error 21.051%, 3 observation have been classified into the third group which belongs to the second group with error 15.79%.
3. Logistic discriminate function in the third group, 27 observations have been classified into the third group property by logistic discriminate function 93.10%, it does not have to observation the first group is classified incorrectly, 2 observations have been classified into the second group which belongs to the third group with error 6.90%.

The above is based on unequal prior probability, that is, according to the proportion of data in the community.

Table (6) represents the correct classification and misclassification

Quadratic discriminate function					Logistic discriminate function				
Ture var.	1	2	3	Total	Ture var.	1	2	3	Total
1	31	2	1	34	1	30	3	1	34
	91.18	5.88	2.94	100		88.24	8.82	2.94	100
2	1	17	1	19	2	4	12	3	19
	5.26	89.47	5.26	100		21.05	63.16	15.79	100
3	0	1	28	29	3	0	2	27	29
	0.0	3.45	96.55	100		0	6.90	93.10	100
Total	32	20	30		Total	34	17	31	82
	39.02	24.39	36.59	100		0.4146	20.73	37.80	100
Priors	0.4146	0.2317	0.3537		Priors	0.4146	0.2317	0.3537	

It is through the Figure 1 which represents the data distribution as a quadratic discrimination function and logistic discrimination function, which shows clearly that the errors of a function of discrimination quadratic less compared with the errors is a function of discrimination logistic **.**

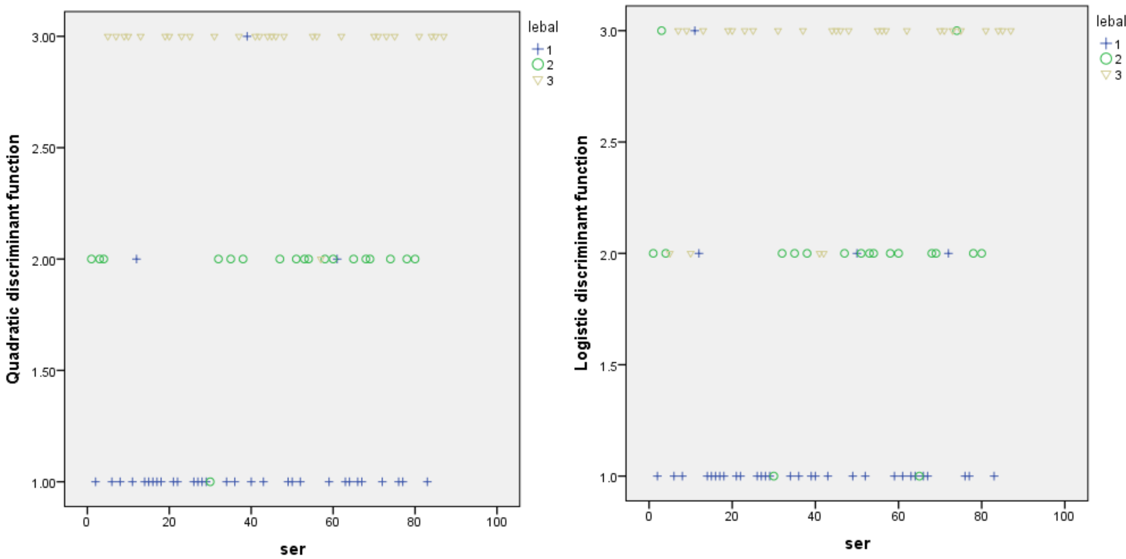

Figure 1 show classification for quadratic and logistic discrimination

6- Conclusions and Recommendations

The above-mentioned cases are a function of discrimination quadratic function and discrimination logistic in the case of being equal and unequal prior probability the results were as follows:

1. For stratified sampling it was applied in two cases, first in case of an equality of the stratified sampling method, and the second in the case of inequality in stratified sampling for Model quadrature and logistic discrimination state show that in the case of equal the prior probability it was clear the percentage of the error less than the case of non-equal probability, The rate of error in quadratic discrimination is lower than the rate of error in logistic discrimination based on the equal and unequal class sample.
2. The ratio of classification errors in the quadratic discrimination function is lower than the ratio of classification errors in the logistic discrimination function.
3. It was also observed that the misclassification in the quadratic discrimination function less than misclassification of a function of discrimination logistic clearly and in three aggregates and uneven, which can be observed clearly in the second group.
4. The misclassification in the quadratic discrimination function in the case of equal prior possibility was better than the results of the quadratic discrimination function in the case of unequal prior possibility.
5. Misclassification in logistic discrimination in the case of equal prior probability was better than the consequences of logistic discrimination in the case of unequal prior probability.

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