Geometric Mean with Pythagorean Fuzzy Transportation Problem

S.Krishna Prabha[a], S. Sangeetha[b], P. Hema[c] , Muhammed Basheer[d], and G. Veeramala[e]

[a]
Assistant Professor, Department of Mathematics,
PSNA College of Engineering and Technology, Dindigul.
[b]Assistant Professor, Department of Mathematics, Dhanalakshmi Srinivasan College of Arts and Science for women, Autonomous, Perambalur
[c]Assistant Professor, Department of Mathematics, RMK College of Engineering and Technology, Puduvoyal-601206
[d]Assistant Professor, Department of Mathematics, University of Technology,S.of Oman
[e]Assistant Professor,Department of Mathematics, M.Kumarasamy college of Engineering, Karur, Tamilnadu.

Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 16 April 2021

Abstract: Transportation algorithm is one of the powerful configurations to afford the retail to the client in proficient deportment. Recently to deal with ambiguity in optimization problems like transportation problem, Pythagorean fuzzy set (PFS) theory is utilized. Compared with fuzzy sets and intustionistic fuzzy sets, PFS are widely used for modeling haziness and indistinctness. PFSs have meaningful applications in many different fields. Various investigators have introduced different methods like North West corner method, least cost method, Vogel's approximation method, heuristic method etc, to crack the fuzzy transportation problems. In this work geometric mean technique is introduced to resolve a Pythagorean fuzzy transportation problem (PFTP) and an algorithm of the suggested method is presented. A numerical example is illustrated with the new technique and the end result acquired through this method is compared with the extant methods. This suggested technique provides an optimal resolution.

Keywords: Pythagorean Fuzzy Sets, Pythagorean Fuzzy Transportation Problem, Score Function, Geometric Mean, Accuracy Function, Defuzzification.

1. Introduction

Transportation models afford the commodities to the clients in more proficient approaches. Transportation problem promise the proficient advancement and cautious accessibility of raw equipments and finished merchandise. Transportation problems accord with the shipping of a single/multi manufactured article contrived at different sources to number of various destinations. The main goal of transportation problem is to maximize the profit and to minimize the transportation cost. The basic transportation problem was introduced by Hitchcock [5] in 1941. Charnes et al [3] in 1953 suggested the stepping stone method which used the simplex method to solve the transportation problem. Later on the primal simplex transportation scheme was introduced by Dantzig [4] in 1963. In the recent world, all the constraints of the transportation problems may not be known completely due to intractable characteristics. In order to overcome this situation, fuzzy numbers are initiated by Zadeh [16] in 1965 and later developed by Zimmermann [19] in 1978. Yager [14, 15] in 2013, 2014 established an additional category of non-standard fuzzy subset called Pythagorean fuzzy set (PFS), which is a special case used to overcome the situation that if the sum of the member- ship function and non-membership function is greater than one. In PFS the square sum of the membership and the non- membership degrees is equal to or less than one. Pythagorean fuzzy sets (PFS) can be recommend as a better alternative, in some special case when fuzzy sets have some extent of limitations in handling ambiguity and uncertainty. The solution for the transportation problem with fuzzy coefficients which are expressed as fuzzy numbers was proposed by Chanas and Kuchta [1] in 1992. The interval and fuzzy extension of classical transportation problems was explained in detail by Charnas.S, et al, [2] in 1993. A parametric study on transportation problem under fuzzy environment was made by Abbas S.A and Saad O.M.[13] in 2003. A new method for solving transportation problems using trapezoidal fuzzy numbers was suggested by Kadhirvel and Balamurugan [7] in 2012. Narayanamoorthy, et al [11, 12] in 2013 and 2015 have introduced a new procedure for solving fuzzy transportation problems. The extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets was introduced by Zhang et al [18] in 2014. Pythagorean fuzzy multiple criteria group decision making using Similarity measure was presented by Zhang X [17] in 2016. Symmetric Pythagorean fuzzy weighted geo- metric/averaging operators and their application in multi criteria decision-making problems was introduced by Ma Z et al [9] in 2016. A Pythagorean fuzzy analytic hier- archy process to multi-criteria decision making was presented by Mohmed et al [10] in 2017. A Pythagorean fuzzy technique to resolve the transportation problem was introduced by Kumar et al [8] in 2019.Jeyalakshmi et al [6] in 2021, introduced monalisha technique to unravel pythagorean fuzzy transportation problem.

A modified algorithm using geometric mean to unravel the pythagorean fuzzy transportation problem is suggested in this work. This paper is structured as bellow; prefaces of the Pythagorean fuzzy sets and geometric mean are provided in section 2. Mathematical model for Pythagorean fuzzy transportation problem is exhibited in section 3. The suggested modified algorithm is deliberated in section 4, a numerical example is illustrated in section 5 and ultimately section 6 concludes the given work.

2. Preliminaries

Definition 2.1 [8]
Let X is a fixed set, a pythagorean fuzzy set is an object having the form $P = \{(x, (\theta_P(x), \delta_P(x))) | x \in X\}$,where the function $\theta_P(x): X \to [0, 1]$ and $\delta_P(x): X \to [0, 1]$ are the degree of membership and non-membership of the element $x \in X$ to P, respectively.
Also for every $x \in X$, it holds that $(\theta_P(x))^2 + (\delta_P(x))^2 \leq 1$.

Definition 2.2 [8]
Let $\breve{a}_1^p = (\theta_i^p, \delta_s^p)$ and $\breve{b}_1^p = (\theta_0^p, \delta_f^p)$ be two Pythagorean Fuzzy Numbers (PFNs). Then the arithmetic operations are as follows:

(i) Additive property: $\breve{a}_1^p \oplus \breve{b}_1^p = \left(\sqrt{(\theta_i^p)^2 + (\theta_0^p)^2 - (\theta_i^p)^2 (\theta_0^p)^2}, \delta_s^p . \delta_f^p\right)$

(ii) Multiplicative property: $\breve{a}_1^p \otimes \breve{b}_1^p = \left(\theta_i^p . \delta_s^p \quad \sqrt{(\delta_s^p)^2 + (\delta_f^p)^2 - (\delta_s^p)^2 (\delta_f^p)^2}\right)$

(iii) Scalar product: $k\breve{a}_1^p = \left(\sqrt{1 - (1 - \theta_i^p)^k}, (\delta_s^p)^k\right)$

where k is nonnegative const..i.e.$k > 0$

Definition 2.3 [8] (*Comparison of two PFNs*) Let $\breve{a}_1^p = (\theta_i^p, \delta_s^p)$ and $\breve{b}_1^p = (\theta_0^p, \delta_f^p)$ be two Pythagorean Fuzzy Numbers such that the score and accuracy function are as follows:

(i) Score function: $S(\breve{a}_1^p) = \frac{1}{2}\left(1 - (\theta_i^p)^2 - (\delta_s^p)^2\right)$ -------------(1)
(ii) Accuracy function: $(\breve{a}_1^p) = (\theta_i^p)^2 + (\delta_s^p)^2$ -----------------(2)

Then the following five cases arise:
Case 1: If $\breve{a}_1^p > \breve{b}_1^p$ iff $S(\breve{a}_1^p) > S(\breve{b}_1^p)$
Case 2: If $\breve{a}_1^p < \breve{b}_1^p$ iff $S(\breve{a}_1^p) < S(\breve{b}_1^p)$
Case 3: If $S(\breve{a}_1^p) = S(\breve{b}_1^p)$ and $H(\breve{a}_1^p) < H(\breve{b}_1^p)$,then $\breve{a}_1^p < \breve{b}_1^p$
Case 4: If $S(\breve{a}_1^p) = S(\breve{b}_1^p)$ and $H(\breve{a}_1^p) > H(\breve{b}_1^p)$,then $\breve{a}_1^p > \breve{b}_1^p$
Case 5: If $S(\breve{a}_1^p) = S(\breve{b}_1^p)$ and $H(\breve{a}_1^p) = H(\breve{b}_1^p)$,then $\breve{a}_1^p = \breve{b}_1^p$

Definition 2.4

The geometric mean is a mean or average, which indicates the central tendency or typical value of a set of numbers by using the product of their values (as opposed to the arithmetic mean which uses their sum).

In general the geometric mean is defined as the *n*th root of the product of *n* numbers, i.e., for a set of numbers $x_1, x_2, ..., x_n$, the geometric mean is defined as

$$\left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}} = \sqrt[n]{x_1 x_2 x_3 \dots \dots x_n} \quad \text{------------(3)}$$

3. Model of Pythagorean fuzzy transportation problem[6]

The balanced pythagorean fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:
c_{ij}^p =The pythagorean fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination.
a_{ij}^p= the Pythagorean fuzzy availability of the product at i^{th} source
b_{ij}^p= the Pythagorean fuzzy demand of the product at j^{th} destination
x_{ij} = the fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost.
Pythagorean fuzzy transportation problem is given by

$$minimize \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^p * x_{ij} \text{------------------------- (4)}$$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i^p$, i = 1,2,3,………,m
$\sum_{i=1}^{m} x_{ij} = b_j^p$,j = 1,2,3,………,n
$\sum_{i=1}^{m} a_i^p = \sum_{j=1}^{n} b_j^p$

4. Algorithm using GM to solve PFTP

Step-1: Make sure whether the TP is balanced or not, if not, make it balanced.
Step-2: Acquire the GM using (3) for every row and column.
Step-3: Choose the utmost geometric mean value from step 2, and assign the min (supply or demand) at the place of lowest value of consequent row or column.
Step-4: Repeat step2 and step3 till the demand and supply are fatigued.
Step-5: Compute the total transportation cost of the PFTP.

5. Numerical Example:

The input data for Pythagorean fuzzy transportation problem is given bellow. The optimal aim of the process is to minimize the transportation cost and maximize the profit. The same problem used in [6] is taken for verification.

Table.1. Pythagorean fuzzy transportation problem

	D_1	**D_2**	**D_3**	**D_4**	**Supply**
O_1	(0.3, 0.6)	(0.4, 0.6)	(0.8, 0.4)	(0.6, 0.4)	25
O_2	(0.4, 0.3)	(0.7, 0.4)	(0.5, 0.7)	(0.7, 0.4)	26
O_3	(0.6, 0.2)	(0.8, 0.2)	(0.7, 0.3)	(0.9, 0.1)	29
Demand	18	22	27	13	

In the given table the total supply is equal to the total demand which is equal to 80.Hence the transportation problem is balanced transportation problem.

Step-1: Determine the cost table from the given problem. Here total supply equals total demand, hence we can proceed to step 2. By the definition 2.2 the score function is given by, Score function, $S = \frac{1}{2}\left(1 - \left(\theta_i^\rho\right)^2 - \left(\delta_i^\rho\right)^2\right)$ by (1)

Here we use the score function for converting the Pythagorean fuzzy numbers into crisp numbers.

$S\,(C_{11}) = \frac{1}{2}\left(1 - \left(\theta_i^\rho\right)^2 - \left(\delta_i^\rho\right)^2\right)$

$= \frac{1}{2}(1 - (0.3)^2 - (0.6)^2) = \frac{1}{2}(1 - 0.09 - 0.36) = \frac{0.55}{2} = 0.275$

Applying the score function to all the values, we convert all the Pythagorean fuzzy numbers into crisp numbers. The defuzzified Pythagorean fuzzy transportation problem is given bellow.

Table.2. Defuzzified Pythagorean fuzzy transportation problem

	D_1	**D_2**	**D_3**	**D_4**	**Supply**
O_1	0.275	0.24	0.1	0.24	25
O_2	0.375	0.175	0.13	0.175	26
O_3	0.3	0.16	0.21	0.09	29
Demand	18	22	27	13	

Step-2: Find the GM using (3) for every row and column and write it below the corresponding rows and columns.

Table.3. Geometric mean for the cost values

	D_1	**D_2**	**D_3**	**D_4**	**Supply**	**GM**
O_1	0.275	0.24	0.1	0.24	25	0.1995
O_2	0.375	0.175	0.13	0.175	26	0.19657
O_3	0.3	0.16	0.21	0.09	29	0.17355
Demand	18	22	27	13		
GM	0.31393	0.18871	0.13976	0.15577		

Step-3: Choose the maximum geometric mean value from table 3, and assign the min (supply or demand) at the place of lowest value of consequent row or column. The maximum geometric mean value is at the first column and the minimum allocation is at the cell (1, 1)

Table.4. First allocation by GM

	D_1	**D_2**	**D_3**	**D_4**	**Supply**	
O_1	0.275 **18**	0.24	0.1	0.24	25 **7**	0.1995
O_2	0.375	0.175	0.13	0.175	26	0.19657
O_3	0.3	0.16	0.21	0.09	29	0.17355
Demand	18 **0**	22	27	13		
	0.31393	0.18871	0.13976	0.15577		

Step-4: Repeating the procedure until all the rim requirements is satisfied.

Table.5. Optimal solution of Pythagorean fuzzy transportation problem

	D_1	**D_2**	**D_3**	**D_4**	**Supply**
O_1	0.275 **(18)**	0.24	0.1 **(7)**	0.24	25
O_2	0.375	0.175	0.13 **(20)**	0.175 **(6)**	26
O_3	0.3	0.16 **(22)**	0.21	0.09 **(7)**	29
Demand	18	22	27	13	

The above table satisfies the rim conditions with (m + n-1) non negative allocations at independent positions.
Thus the optimal allocation is: X_{11}=18, x_{13}=7, x_{23}=20, x_{24}=6, x_{32}=22, x_{34}=7.
The transportation cost according to the MAM's method is:
Total Cost

=(0.275*18)+(0.1*7)+(0.13*20)+(0.175*6)+(0.16*22)+(0.09*7)

= 4.95+0.7+2.6+1.05+3.52+0.63=13.45

Total minimum cost will be Rs.13.45

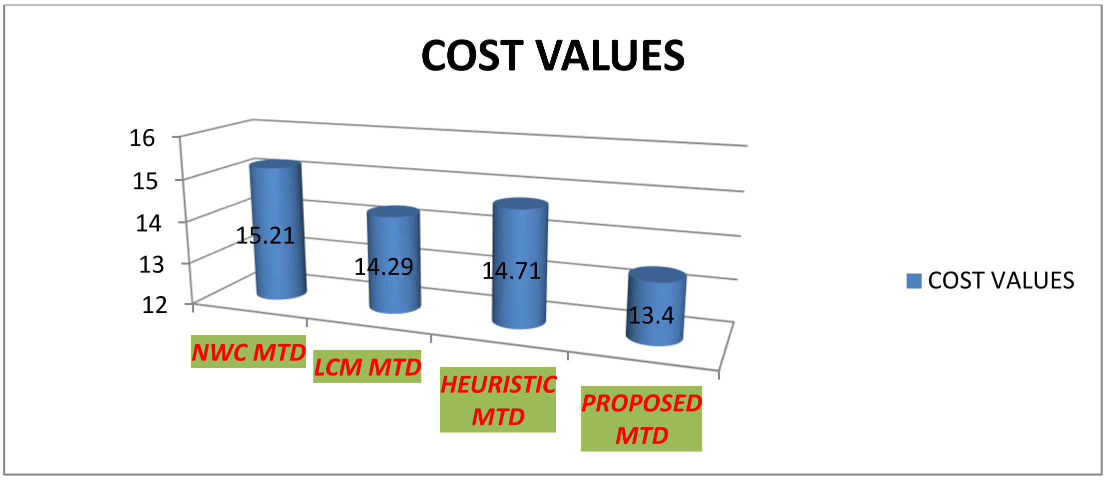
In order to show the efficiency of the proposed method, the same problem is solved with various methods like North West corner (NWC), Least Cost (LCM) and Heuristics method (HM). We get the following results after solving the problem.

Table 6: Comparison Table

NWC METHOD	**LCM METHOD**	**HEURISTIC METHOD**	**PROPOSED METHOD**
15.21	14.29	14.71	13.4

By comparing the proposed method with the other methods, the proposed method gives a better optimum solution. This method is more efficient to reduce the transportation cost than the other existing methods.

Fig.1: Comparison Chart



6. Conclusion

In this work instead of normal methods which are prevailing already, geometric mean method is applied to solve a pythagorean fuzzy transportation problem. By defuzzifying with score function the crisp transportation problem is framed, for which the suggested algorithm is applied. When compared with other existing methods the proposed algorithm gives a best optimal solution. In future, the proposed method can be applied to various applications of pythagorean fuzzy problems which deal with real world problems.

Reference:

1. Chanas.S , Kuchta.D ,“ A concept of the optimal solution of the transportation problem with fuzzy cost coefficients”, *Fuzzy Sets and Systems* 82, pp 299-305,1992.
2. Charnas S.Delgado.M., Verdegy J.L., Vila M.A., “Interval and fuzzy extension of classical transportation problems”, *Transporting planning technol*.17 , PP.203-218,1993.
3. Charnes. A, Cooper W. W.and Henderson. A, “An introduction to Linear Programming”, *Wiley*,New Work, 1953.
4. Dantzig G.B, “Linear programming and extensions”, *Princeton University Press*, NJ, 1963.
5. Hitchcock. F.L, “The distribution of a product from several sources to numerous localities”,*Journal of mathematical physics* ,pp 224-230,1941.
6. Jeyalakshmi.K, Chitra.L, Veeramalai.G Krishna Prabha S. and Sangeetha.S, “Pythagorean Fuzzy Transportation Problem Via Monalisha Technique”, *Annals Of R.S.C.B*, Vol. 25, Issue 3, pp 2078– 2086,2021.
7. Kadhirvel. K, Balamurugan. K, “Method for solving transportation problems using trapezoidal fuzzy numbers”, *International Journal of Engineering Research and Applications* (IJERA) ISSN: 2248-9622, Vol. 2, Issue 5, pp.2154-2158, 2012.
8. Kumar R,Edalatpanah.S.A,Jha.S,Singh.R, “A Pythagorean fuzzy approach to the transportation problem”,*Complex and Intelleigent System*. 5, 255–263, https://doi.org/10.1007/s40747-019-0108-1,2019.
9. Ma Z, Xu Z, “Symmetric Pythagorean fuzzy weighted geo- metric/averaging operators and their application in multicriteria decision-making problems”,. *Int J Intell Syst 31:1198–1219,2016.*
10. Mohd WRW, Lazim A , “Pythagorean fuzzy analytic hier- archy process to multi-criteria decision making”. *AIP Conf Proc* 1905:040020, 2017.
11. Narayanamoorthy.S and Kalyani.S, “Finding the initial basic feasible solution of a fuzzy transportation problem by a new method”, *International Journal of Pure and Applied Mathematics*, Volume 101 No. 5 , 687-692, 2015.
12. Narayanamoorthy.S,.Saranya.S &.Maheswari.S., “ A Method for Solving Fuzzy Transportation Problem (FTP) using Fuzzy Russell’s Method”, I.J. *Intelligent Systems and Applications*, 02, 71-75, 2013.
13. Saad O.M. and AbbasS.A., “A parametric study on transportation problem under fuzzy environment” ,*The Journal of Fuzzy Mathematics* ,pp 115-124, 2003.
14. Yager RR., “Pythagorean fuzzy subsets. In: 2013 joint IFSA world congress and NAFIPS

annual meeting (IFSA/NAFIPS) ", pp 57–61, 2013.

15. Yager RR ., "Pythagorean membership grades in multicriteria decision making", *IEEE Trans Fuzzy Syst 22:958–965, 2014.*
16. Zadeh, L. A ,"Fuzzy sets", *Information and control* , vol 8 ,pp 338-353,1965.
17. Zhang X , "A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making". *Int J Intell* Syst 31:593–611, 2016.
18. Zhang X, Xu Z , "Extension of TOPSIS to multiple crite- ria decision making with Pythagorean fuzzy sets". *Int J Intell Syst* 29:1061–1078, 2014.
19. Zimmermann H.J., "Fuzzy programming and linear programming with several objective functions", *Fuzzy Sets and Systems*, pp 45-55, 1978.